27 Oct 2005

Electron - muon scattering:

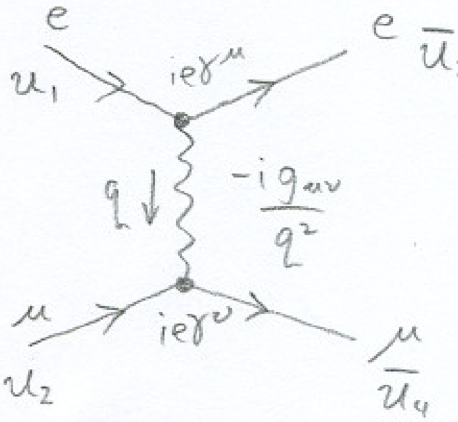
$e^- \mu^- \to e^- \mu^-$

why on earth do we want to do this calculation?

Because it is the same as

$e^- q \to e^- q$

which is used to study the structure of the proton.



$$-i\mathcal{M} = (ie\bar{u}_3\gamma^\mu u_1)\left(\frac{-ig_{\mu\nu}}{q^2}\right)(ie\bar{u}_4\gamma^\nu u_2)$$

$$= \frac{ie^2}{q^2}(\bar{u}_3\gamma^\mu u_1)(\bar{u}_4\gamma_\mu u_2)$$

$$|\overline{\mathcal{M}}|^2 = \frac{e^4}{4q^4}\sum_{spins}\left[(\bar{u}_3\gamma^\mu u_1)(\bar{u}_3\gamma^\nu u_1)^\dagger(\bar{u}_4\gamma_\mu u_2)(\bar{u}_4\gamma_\nu u_2)^\dagger\right.$$

$$= \frac{e^4}{4q^4} L_e^{\mu\nu} L^{muon}_{\mu\nu}$$

where

$$L_e^{\mu\nu} = \frac{1}{2}\sum_{\text{spins}} (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_3 \gamma^\nu u_1)^\dagger$$

$$= \frac{1}{2}\sum_{\text{spins}} (\bar{u}_3 \gamma^\mu u_1)(\bar{u}_1 \gamma^\nu u_3)$$

$$= \frac{1}{2}\sum_{\text{spins}} \mathrm{Tr}(u_3 \bar{u}_3 \gamma^\mu u_1 \bar{u}_1 \gamma^\nu)$$

$$= \frac{1}{2}\mathrm{Tr}((\not{p}_3 + m)\gamma^\mu(\not{p}_1 + m)\gamma^\nu)$$

We will ignore the electron mass but not the muon mass:

$$L_e^{\mu\nu} \approx \frac{1}{2}\mathrm{Tr}(\not{p}_3 \gamma^\mu \not{p}_1 \gamma^\nu)$$

$$= 2(p_3^\mu p_1^\nu + p_1^\mu p_3^\nu - p_1 \cdot p_3\, g^{\mu\nu})$$

For the muon,

$$L^{\text{muon}}_{\mu\nu} = \frac{1}{2}\sum_{\text{spins}} \mathrm{Tr}(\bar{u}_4 \gamma_\mu u_2 \bar{u}_2 \gamma_\nu u_4)$$

$$= \frac{1}{2}\sum_{\text{spins}} \mathrm{Tr}(u_4 \bar{u}_4 \gamma_\mu u_2 \bar{u}_2 \gamma_\nu)$$

$$= \frac{1}{2}\mathrm{Tr}((\not{p}_4 + M)\gamma_\mu(\not{p}_2 + M)\gamma_\nu)$$

$$= \frac{1}{2}\mathrm{Tr}(\not{p}_4 \gamma_\mu \not{p}_2 \gamma_\nu) + \frac{M^2}{2}\mathrm{Tr}(\gamma_\mu \gamma_\nu)$$

$$= 2(p_{4\mu} p_{2\nu} + p_{2\mu} p_{4\nu} - p_2 \cdot p_4\, g_{\mu\nu}) + 2M^2 g_{\mu\nu}$$

So $$|\overline{\mathcal{M}}|^2 = \frac{4e^4}{q^4}\left(p_3^\mu p_1^\nu + p_1^\mu p_3^\nu - p_1 \cdot p_3\, g^{\mu\nu}\right) \times \left(p_{4\mu} p_{2\nu} + p_{2\mu} p_{4\nu} - g_{\mu\nu}(p_2 \cdot p_4 - M^2)\right)$$

$$= \frac{4e^4}{q^4}\Big(2(p_1\cdot p_2)(p_3\cdot p_4) + 2(p_1\cdot p_4)(p_2\cdot p_3) - 2(p_1\cdot p_3)(p_2\cdot p_4) - 2(p_1\cdot p_3)(p_2\cdot p_4) + 4(p_1\cdot p_3)(p_2\cdot p_4) + 2(p_1\cdot p_3)M^2 - 4(p_1\cdot p_3)M^2 \Big)$$

$$= \frac{8e^4}{q^4}\left((p_1\cdot p_2)(p_3\cdot p_4) + (p_1\cdot p_4)(p_2\cdot p_3) - M^2\, p_1\cdot p_3\right)$$

In the c.m. frame,

$p_1 = (E, \vec{p})$ where $|\vec{p}| = E$

$p_2 = (M, 0)$

$p_3 = (E', \vec{p}')$ where $|\vec{p}'| = E'$

$p_4 = p_2 + q$

$q = p_1 - p_3$

So $p_1 \cdot p_2 = EM$

$p_3 \cdot p_4 = p_3 \cdot (p_2 + p_1 - p_3) = E'M + p_1\cdot p_3 = E'(E+M)$

$p_1 \cdot p_4 = p_1 \cdot (p_2 + p_1 - p_3) = EM - p_1\cdot p_3$

$p_2 \cdot p_3 = E'M$

$-\frac{1}{2}q^2 = p_1 \cdot p_3 = EE' - EE'\cos\theta = EE'(1-\cos\theta) = 2EE'\sin^2\frac{\theta}{2}$

So $|\bar{\mathcal{M}}|^2 = \frac{4e^4}{q^4}\left(p_3^\mu p_1^\nu + p_1^\mu p_3^\nu - p_1 \cdot p_3\, g^{\mu\nu}\right) \times$

$$\left(p_{4\mu} p_{2\nu} + p_{2\mu} p_{4\nu} - g_{\mu\nu}(p_2 \cdot p_4 - M^2)\right)$$

$$= \frac{4e^4}{q^4}\Big(2(p_1 \cdot p_2)(p_3 \cdot p_4) + 2(p_1 \cdot p_4)(p_2 \cdot p_3)$$
$$- 2(p_1 \cdot p_3)(p_2 \cdot p_4) - 2(p_1 \cdot p_3)(p_2 \cdot p_4)$$
$$+ 4(p_1 \cdot p_3)(p_2 \cdot p_4) + 2(p_1 \cdot p_3) M^2$$
$$- 4(p_1 \cdot p_3) M^2 \Big)$$

$$= \frac{8e^4}{q^4}\left((p_1 \cdot p_2)(p_3 \cdot p_4) + (p_1 \cdot p_4)(p_2 \cdot p_3) - M^2\, p_1 \cdot p_3 \right)$$

In the c.m. frame,

$p_1 = (E, \vec{p})$ where $|\vec{p}| = E$

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$p_4 = p_2 + q$

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$p_2 \cdot p_3 = E'M$

$-\frac{1}{2} q^2 = p_1 \cdot p_3 = EE' - EE' \cos\theta = EE'(1 - \cos\theta) = 2EE' \sin^2\frac{\theta}{2}$

$$|\bar{\mathcal{M}}|^2 = \frac{8e^4}{q^4}\left(EM(E'M + p_1\cdot p_3) + (EM - p_1\cdot p_3)E'M - M^2 p_1\cdot p_3 \right)$$

$$= \frac{8e^4}{q^4}\left(M(p_1\cdot p_3)(E - E') + 2M^2 EE' - M^2 p_1\cdot p_3 \right)$$

But $p_1\cdot p_3 = -\frac{1}{2}q^2$

$$|\bar{\mathcal{M}}|^2 = \frac{8e^4}{q^4}\left(-\frac{1}{2}Mq^2(E-E') + 2M^2EE' + \frac{1}{2}M^2q^2 \right)$$

$$= \frac{8e^4}{q^4}\cdot 2M^2EE'\left(1 + \frac{q^2}{4EE'} - \frac{q^2}{2M^2}\frac{M(E-E')}{2EE'} \right)$$

But $q^2 = -2p_1\cdot p_3 = -2EE'(1+\cos\theta)$

$$= -4EE'\sin^2\theta/2$$

We can also write

$$p_4 = p_2 + q$$

So $m^2 = M^2 + 2p_2\cdot q + q^2$

$$q^2 = -2p_2\cdot q = -2M(E - E')$$

$$\Rightarrow \quad M(E-E') = -\frac{1}{2}q^2 = +2EE'\sin^2\theta/2$$

So

$$|\bar{\mathcal{M}}|^2 = \frac{8e^4}{q^4}2M^2EE'\left(\cos^2\theta/2 - \frac{q^2}{2M^2}\sin^2\theta/2 \right)$$

Including flux factor and phase space leads to

$$d\sigma = \frac{|\bar{\mathcal{M}}|^2}{F} dQ$$

$$= \frac{\alpha^2}{(4E^2 \sin^4 \theta/2)} \frac{E'}{E} \left(\cos^2 \frac{\theta}{2} - \frac{q^2}{2M^2} \sin^2 \frac{\theta}{2} \right)$$

Experimental setup: