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Differential cross section for scattering electrons off spin $\frac{1}{2}$ particles at rest:

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha^2}{4E^2 \sin^4 \theta/2}\right)\frac{E'}{E}\left(\cos^2 \theta/2 - \frac{q^2}{2M^2}\sin^2 \theta/2\right)$$

θ is the electron scattering angle.
E is the initial electron energy
E' is the energy of the scattered electron
$\frac{-q^2}{2M} = E - E' \equiv \nu$ is the energy transferred to the particle of mass M.

If the target was spin 0 the differential cross section would be

$$\frac{d\sigma}{d\Omega} = \left(\frac{\alpha^2}{4E^2 \sin^4 \theta/2}\right)\frac{E'}{E}\cos^2 \theta/2$$

The angular distribution of the scattered electron is sensitive to the spin of the target particle.

At high energies, the electron scatters of charged quarks / antiquarks (partons) inside the proton.

We can tell that they are spin $\frac{1}{2}$.

Detailed analysis.

Quarks are quite light so we can neglect their masses in the c.m. frame

Recall $e^-\mu^- \to e^-\mu^-$ had

$$|\overline{\mathcal{M}}|^2 = 2e^4\left(\frac{s^2+u^2}{t^2}\right)$$

$$d\sigma = \frac{|\overline{\mathcal{M}}|^2\, dQ}{F}$$

In the cm frame, $F = 4|\vec{p}_i|\sqrt{s} = 2s$

$$dQ = \frac{1}{(4\pi)^2}\frac{|\vec{p}_f|}{\sqrt{s}}\, d\Omega$$

$$= \frac{1}{2(4\pi)^2}\, d\Omega$$

So
$$\frac{d\sigma}{d\Omega} = \frac{e^4}{(4\pi)^2}\cdot\frac{1}{2s}\left(\frac{s^2+u^2}{t^2}\right)$$

$$= \frac{\alpha^2}{2s}\left(\frac{s^2+u^2}{t^2}\right)$$

$$= \frac{\alpha^2 s}{2t^2}\left(1 + u^2/s^2\right)$$

But experiments are done in the lab so we want to transform this expression from the c.m. frame to the lab frame.

What variables can we use?

E = initial electron energy

E' = final electron energy

θ = electron scattering angle in the lab.

$$t = (p_e - p_e')^2 = q^2 = -2p_e \cdot p_e' = -2EE'(1-\cos\theta)$$

In the c.m. frame the electron 4-vectors are

$$p_e^* = (E^*, 0, 0, E^*)$$

$$p_e'^* = (E^*, 0, E^*\sin\theta^*, E^*\cos\theta^*)$$

$$p_\mu'^* = (E^*, 0, -E^*\sin\theta^*, -E^*\cos\theta^*)$$

$$\frac{d\sigma}{d\Omega^*} = \frac{\alpha^2 s}{2q^4}\left(1 + u^2/s^2\right)$$

$$u = (p_e^* - p_\mu'^*)^2 = -2p_e^* \cdot p_\mu'^* = -2E^{*2}(1+\cos\theta^*)$$

$$= -4E^{*2}\cos^2\theta/2$$

$$s = (2E^*)^2$$

So $u^2/s^2 = \cos^4\theta^*/2$

$$\frac{d\sigma}{d\Omega^*} = \frac{\alpha^2 s}{2q^4}\left(1 + \cos^4\theta^*/2\right)$$

If γ is the relativistic factor that transforms the lab frame into the c.m. frame, ie, the cm frame moves with velocity β in the lab where

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\beta \approx 1$$

then $E_e = \gamma E^* + \gamma\beta E^* \approx 2\gamma E^*$

$$E'_\mu = \gamma E^* - \gamma\beta E^* \cos\theta^* = \gamma E^*(1 - \cos\theta^*)$$

$$\text{So } \frac{E'_\mu}{E_e} = \frac{E'_e - E_e}{E_e} = \frac{1}{2}(1 - \cos\theta^*) \equiv y$$

$$= \sin^2\theta^*/2$$

y is the fractional electron energy loss.

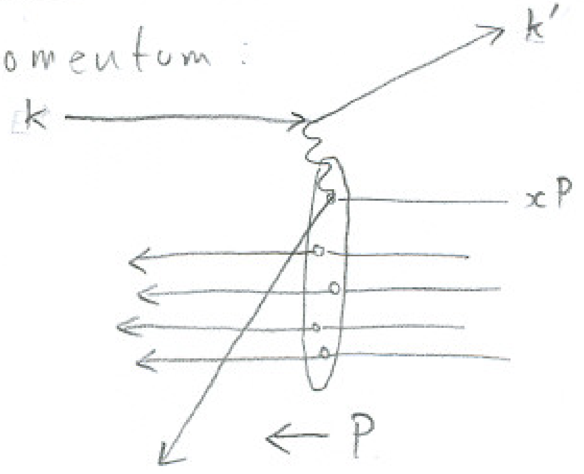
Also, $1 - y = 1 - \sin^2\theta^*/2 = \cos^2\theta^*/2$

$$d\Omega^* = 2\pi\, d(\cos\theta^*) = 4\pi\, dy$$

So $$\frac{d\sigma}{dy} = \frac{2\pi\alpha^2 s}{q^4}\left(1 + (1-y)^2\right)$$

Now all the variables can be measured in the lab.

The proton can be viewed in its relativistic limit as a "pancake" of static constituents, each of which carry a fraction of its momentum:



The constituents were originally called "partons" because it was not obvious that they were actually quarks.

If the proton has momentum $p^\mu = (P, 0, 0, -P)$ then the struck parton has momentum

$$x p^\mu = (xP, 0, 0, -xP).$$

If $s = (k+P)^2 \approx 2k\cdot P$ ep cm energy squared

$\hat{s} = (k + xP)^2 \approx 2x\, k\cdot P = xs$ e"parton" cm energy squared.

The probability of observing a parton with momentum fraction between x and $x+dx$ is denoted $f(x)\,dx$.

The cross section in the lab is then

$$\frac{d\sigma}{dy\,dx} = \frac{2\pi\alpha^2}{q^4}\, s x f(x) \left(1 + (1-y)^2 \right)$$

But this assumed that all partons had charge (-1) like the "muon".

If we assume that the proton contains up and down quarks then we can write

$$\begin{aligned} f(x) &= (Q_u)^2 u(x) + (Q_d)^2 d(x) \\ &= \left(\tfrac{2}{3}\right)^2 u(x) + \left(\tfrac{1}{3}\right)^2 d(x). \end{aligned}$$

But maybe there are also antiquarks and maybe strange quarks in the proton ...

$$\begin{aligned} f(x) &= \left(\tfrac{2}{3}\right)^2 \left(u(x) + \bar{u}(x) \right) + \left(\tfrac{1}{3}\right)^2 \left(d(x) + \bar{d}(x) + s(x) + \bar{s}(x) \right) \\ &= \sum_q Q_q^2\, q(x) \end{aligned}$$

What can we learn from this?

It might be reasonable to assume that

$$s(x) = \bar{s}(x)$$

since virtual $s\bar{s}$ quarks are produced in pairs.

Since the virtual $u\bar{u}$ and $d\bar{d}$ quarks are also produced in pairs we might assume that

$$\bar{u}(x) = \bar{d}(x).$$

Since u and d quarks are both very light.

We can't assume that $u(x) = d(x)$ since the proton has 2 "valence" up quarks and 1 "valence" down quark.

But we can write

$$u_p(x) = u_p^v(x) + u_p^s(x)$$
$$d_p(x) = d_p^v(x) + d_p^s(x).$$

Now we can assume that

$$\bar{u}_p^s(x) = u_p^s(x).$$
$$\bar{d}_p^s(x) = d_p^s(x).$$

$$f_p(x) = \left(\frac{2}{3}\right)^2 u_p^v(x) + \left(\frac{1}{3}\right)^2 d_p^v(x) + 2\left(\frac{2}{3}\right)^2 u_p^s(x) + 2\left(\frac{1}{3}\right)^2 d_p^s(x) + 2\left(\frac{1}{3}\right) s_p^s(x)$$

For the neutron

$$f_n(x) = \left(\frac{2}{3}\right)^2 u_n^v(x) + \left(\frac{1}{3}\right)^2 d_n^v(x) + (\text{same stuff})$$

We might also assume that

$$u_p^v(x) = d_n^v(x)$$
$$d_p^v(x) = u_n^v(x)$$

If we measure $\frac{d\sigma_{ep}}{dx\,dy}$ and $\frac{d\sigma_{en}}{dx\,dy}$

we can extract $f_p(x)$ and $f_n(x)$.

Actually we might measure $\frac{d\sigma_{ed}}{dx\,dy} = \frac{d\sigma_{ep}}{dx\,dy} + \frac{d\sigma_{en}}{dx\,dy}$.

We expect that
$$\int_0^1 u_p^v(x)\,dx = 2$$
$$\int d_p^v(x)\,dx = 1$$

So $\int_0^1 (f_p(x) - f_n(x))\,dx$ should subtract off most of the contributions from non-valence quarks.

$$\cancel{\int_0^1 \left(f_p(x) - f_n(x)\right) = \int \frac{4}{9}\left(u_p(x) + \bar{u}_p(x)\right) + \frac{1}{9}\left(d(x) + \bar{d}(x)\right) - \frac{4}{9}\left(d(x) + d\right.}$$

$$\cancel{\int}\ f_p(x) = \frac{4}{9}\left(u_p(x) + \bar{u}_p(x)\right) + \frac{1}{9}\left(d_p(x) + \bar{d}_p(x)\right) + \frac{1}{9}\left(s_p(x) + \bar{s}_p(x)\right)$$

$$f_n(x) = \frac{4}{9}\left(u_n(x) + \bar{u}_n(x)\right) + \frac{1}{9}\left(d_n(x) + \bar{d}_n(x)\right) + \frac{1}{9}\left(s_n(x) + \bar{s}_n(x)\right)$$

Isospin symmetry:

$$u_p(x) = d_n(x)$$
$$\bar{u}_p(x) = \bar{d}_n(x)$$

$$d_p(x) = u_n(x)$$
$$\bar{d}_p(x) = \bar{u}_n(x)$$

$$s_p(x) = s_n(x)$$
$$\bar{s}_p(x) = \bar{s}_n(x)$$

Sum rules:

$$\int_0^1 \left(f_p(x) - f_n(x)\right) dx = \int_0^1 \left[\frac{4}{9}\left(u_p(x) + \bar{u}_p(x)\right) + \frac{1}{9}\left(d_p(x) + \bar{d}_p(x)\right) - \frac{4}{9}\left(u_n(x) + \bar{u}_n(x)\right) - \frac{1}{9}\left(d_n(x) + \bar{d}_n(x)\right)\right] dx$$

$$= \int_0^1 \left[\frac{4}{9}\left(u_p(x) + \bar{u}_p(x)\right) - \frac{1}{9}\left(u_p(x) + \bar{u}_p(x)\right) + \frac{1}{9}\left(d_p(x) + \bar{d}_p(x)\right) - \frac{4}{9}\left(d_p(x) + \bar{d}_p(x)\right)\right] dx$$

$$= \int_0^1 \left[\frac{1}{3}\left(u_p(x) + \bar{u}_p(x)\right) - \frac{1}{3}\left(d_p(x) + \bar{d}_p(x)\right)\right] dx$$

Up/down quarks can be "valence" quarks or "sea" quarks: $q\bar{q}$ pairs popped from the vacuum.

$$u_p(x) = u_p^v(x) + u_p^s(x)$$
$$d_p(x) = d_p^v(x) + d_p^s(x)$$

Expect that $u_p^s(x) = \bar{u}_p^s(x) = \bar{u}_p(x)$

$$d_p^s(x) = \bar{d}_p^s(x) = \bar{d}_p(x)$$

So $$\int_0^1 \left(f_p(x) - f_n(x)\right) dx = \int_0^1 \left[\frac{1}{3}\left(u_p^v(x) - d_p^v(x)\right) + \frac{2}{3}\left(\bar{u}_p(x) - \bar{d}_p(x)\right) \right] dx$$

There are 2 valence u-quarks and 1 valence d-quarks.

$$\Rightarrow \int_0^1 u_p^v(x)\, dx = 2 \qquad \int_0^1 d_p^v(x)\, dx = 1$$

$$\int_0^1 \left(f_p(x) - f_n(x)\right) dx = \frac{1}{3} + \frac{2}{3}\int_0^1 \left(\bar{u}_p(x) - \bar{d}_p(x)\right) dx$$

What do we expect this to be? Do we expect that the sea quarks are flavor symmetric?

Measured value: 0.24 ± 0.03.

$$\Rightarrow \int_0^1 \left(\bar{u}_p(x) - \bar{d}_p(x)\right) dx \sim -0.14$$

Slightly more $d\bar{d}$ pairs in the proton than $u\bar{u}$ pairs?