Neutral Meson Mixing

There are four possible neutral mesons (besides the π^0)

$$K^0 = d\bar{s} \qquad \bar{K}^0 = s\bar{d}$$

$$D^0 = c\bar{u} \qquad \bar{D}^0 = u\bar{c}$$

$$B^0 = d\bar{b} \qquad \bar{B}^0 = b\bar{d}$$

$$B_s^0 = s\bar{b} \qquad \bar{B}_s^0 = b\bar{s}$$

These decay weakly. Recall that there are no right handed neutrinos nor left-handed antineutrinos. Apparently, parity is not conserved by weak interactions but CP appears to be.

The K^0 and $\bar{K}^0$ are strong eigenstates

$$C|K^0\rangle = C|d\bar{s}\rangle = |\bar{d}s\rangle$$

$$CP|K^0\rangle = CP|d\bar{s}\rangle = C|\bar{s}d\rangle = |s\bar{d}\rangle = |\bar{K}^0\rangle$$

Likewise

$$CP|\bar{K}^0\rangle = |K^0\rangle$$

We can form linear combinations with distinct CP quantum numbers as follows

$$|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\bar{K}^0\rangle\right)$$

$$|K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\bar{K}^0\rangle\right)$$

$$CP|K^0_1\rangle = +|K^0_1\rangle$$

$$CP|K^0_2\rangle = -|K^0_2\rangle$$

The allowed final states are different:

$K^0_1 \to 2\pi$ $\qquad$ $K^0_1 \not\to 3\pi$ $\quad (CP = -1)$

$K^0_2 \to 3\pi$ $\qquad$ $K^0_2 \not\to 2\pi$ $\quad (CP = +1)$

There is much less energy released in the decay of $K^0_2 \to 3\pi$ so there is less phase space and consequently the decay rate is lower. Lifetime is longer.

$$K^0_S \approx K^0_1$$

$$K^0_L \approx K^0_2 .$$

Consider a K^0 produced in a strong interaction at time $t=0$. How does this state evolve with time?

Physical masses m_S, m_L and decay rates Γ_S, Γ_L.

$$|K^0_S(t)\rangle = e^{im_S t}|K^0_S\rangle$$

$$|K^0_L(t)\rangle = e^{im_L t}|K^0_L\rangle$$

$$\frac{dN_{K^0_S}}{dt} = -\Gamma_S N_{K_S}(t) \qquad N_{K_S}(t) = N_{K_S}(t=0)\,e^{-\Gamma_S t}$$

$$\frac{dN_{K^0_L}}{dt} = -\Gamma_L N_{K^0_L}(t) \qquad N_{K_L}(t) = N_{K_L}(t=0)\,e^{-\Gamma_L t}$$

So, if $\psi_0(t=0) = K^0 = \frac{1}{\sqrt{2}}\left(K^0_S + K^0_L\right)$

then

$$\psi(t) = \frac{1}{2}\Big((K^0 + \bar{K}^0)\, e^{-im_S t - \Gamma_S t/2}$$
$$+ (K^0 - \bar{K}^0)\, e^{-im_L t - \Gamma_L t/2} \Big)$$

What is the probability of finding a K^0 at time t?

$$P(K^0|K^0) = |\langle K^0|\psi(t)\rangle|^2$$

$$= \frac{1}{4}\left| e^{-im_S t} e^{-\Gamma_S t/2} + e^{-im_L t} e^{-\Gamma_L t/2} \right|^2$$

$$= \frac{1}{4}\Big(e^{-\Gamma_S t} + e^{-\Gamma_L t/2}$$
$$+ 2\,\mathrm{Re}\left(e^{-im_S t - \Gamma_S t/2}\, e^{im_L t - \Gamma_L t/2} \right) \Big)$$

$$= \frac{1}{4}\Big(e^{-\Gamma_S t} + e^{-\Gamma_L t}$$
$$+ 2 e^{-\left(\frac{\Gamma_S + \Gamma_L}{2}\right)t} \cos\left((m_L - m_S)\, t \right) \Big)$$

$$= \frac{1}{4}\Big(e^{-\Gamma_S t} + e^{-\Gamma_L t}$$
$$+ 2 e^{-\bar{\Gamma} t} \cos(\Delta m t) \Big)$$

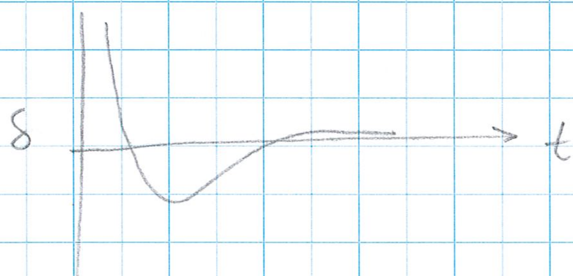
Likewise,

$$P(\bar{K}^0|K^0) = \frac{1}{4}\Big(e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2 e^{-\bar{\Gamma} t} \cos(\Delta m t) \Big)$$

Semileptonic decays are sensitive to the strangeness at the time of decay

$$P(\ell^+\pi^-) - P(\ell^-\pi^+) = e^{-\bar{\Gamma}t}\cos(\Delta m t) \approx e^{-\Gamma_s t/2}\cos(\Delta m t)$$

This is just a damped oscillation



Regeneration

K^0 and $\bar{K}^0$ have different interaction cross sections with matter:

$$\bar{K}^0 + p \to \Lambda + \pi^+ \quad ; \quad \Sigma^0 + \pi^+$$
$$\bar{K}^0 + n \to \Lambda + \pi^0 \quad ; \quad \Sigma^0 + \pi^0$$

but K^0 can't interact. Thus matter will absorb the $\bar{K}^0$ component and transmit the K^0 component.

This is like polarized light...

CP violation.

It was observed that K^0_L do have a small branching fraction to $\pi^+\pi^-$:

$$Br(K^0_L \to \pi^+\pi^-) = 2 \times 10^{-3}.$$

$$|K^0_S\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K^0_1\rangle + \varepsilon|K^0_2\rangle\right)$$

$$|K^0_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\varepsilon|K^0_1\rangle + |K^0_2\rangle\right)$$