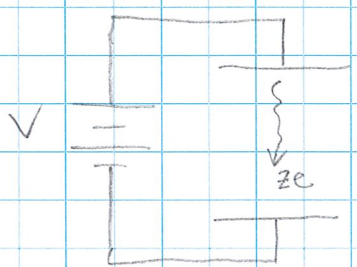
Particle Accelerators

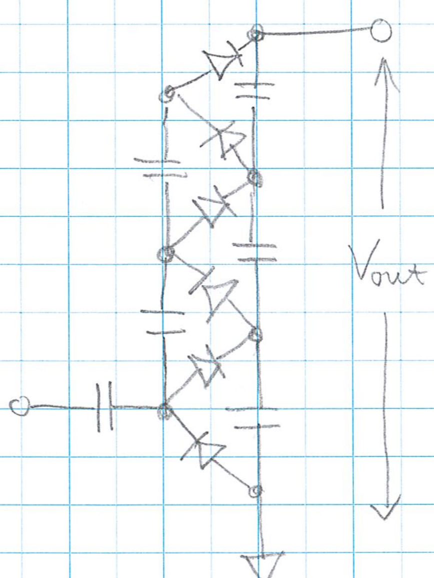
Consider a charged particle accelerated by an electric field:



It gains kinetic energy $V \cdot ze$

How to get large potential differences?

- Lots of batteries
- Van de Graaf generators
- Cockroft - Walton



1932 protons accelerated to 600 keV

$p + Li \rightarrow He + He$

$m(p) = 938.272 \text{ MeV}/c^2$

$m({}^7Li) = (7.016004)(931.494 \text{ MeV}/c^2) = 6535.366 \text{ MeV}/c^2$

$m(He) = (4.0026002)(931.494 \text{ MeV}/c^2) = 3728.398 \text{ MeV}/c^2$

$m(p) + m({}^7Li) = 7473.638 \text{ MeV}/c^2$

$2\,m(He) = 7456.796 \text{ MeV}/c^2$

Work needed to move a positive charge to the surface of a charged sphere:

$$W = -\int_{\infty}^{R} \vec{F} \cdot d\vec{r} = -Ze^2 \int_{\infty}^{R} \frac{dr}{r^2}$$

$$= \frac{Ze^2}{R}$$

For Lithium, $R \sim r_0 A^{1/3}$, $r_0 = 1.25\ fm$

$$R \sim (1.25 \times 10^{-15}\ m)(7)^{1/3} = 2.4 \times 10^{-15}\ m$$

$$e^2 = \frac{\hbar c}{137}$$

$$W = \left(\frac{3 \cdot \hbar c}{137}\right)\left(\frac{1}{2.4\ fm}\right) = \frac{3 \cdot 197.327\ MeV \cdot fm}{137 \cdot 2.4\ fm}$$

$$= 1.800\ MeV$$

(Energy scale needed to overcome the potential barrier)

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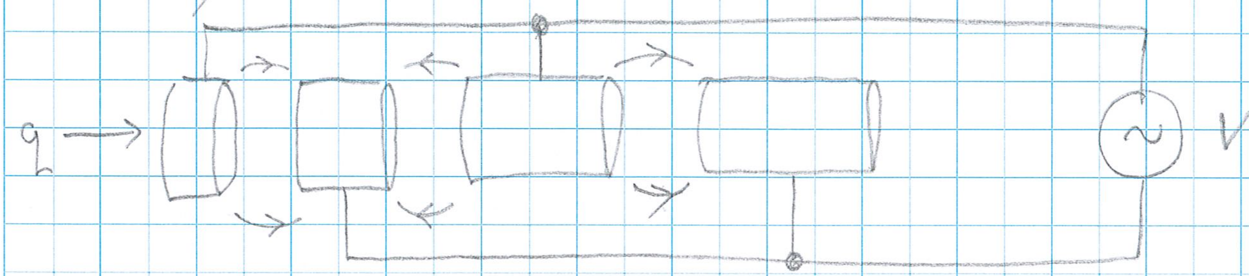
Linear Accelerators

No electric field inside a conducting cavity



Non-relativistic

$$\beta = \sqrt{\frac{2T}{mc^2}}$$

RF frequency f, wavelength $\lambda = c/f$.

Distance traveled in one period:

$$\Delta z = \beta c t = \beta c/f = \beta c \cdot \frac{\lambda}{c} = \beta \lambda$$

Particle gains energy eV across each gap.

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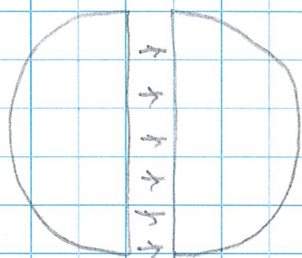
Cyclotron



Energy increases by V each time it crosses the gap.

$$F_c = \frac{mv^2}{r} = qvB$$

$$v = \frac{qBr}{m}$$

period of orbit

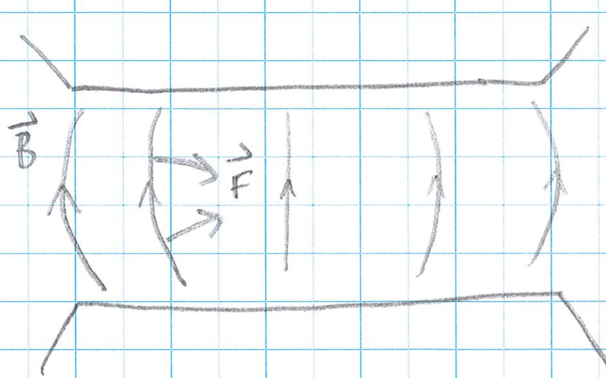
$$T = 2\pi r/v = \frac{2\pi m}{qB}$$

(Constant)

$$E = \frac{q^2B^2r^2}{2m}$$

Energies up to several hundred MeV.

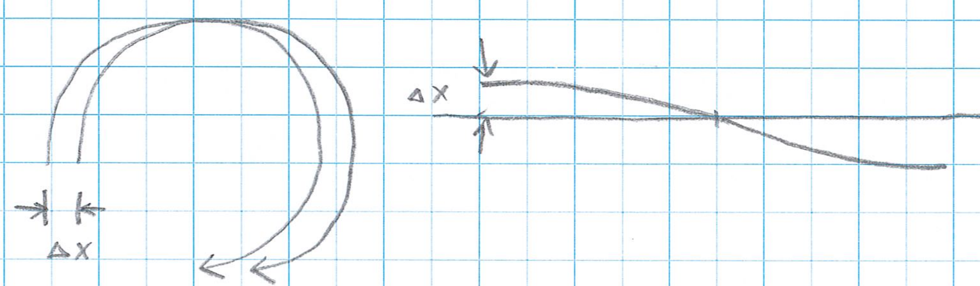
Stability considerations: If the magnetic field is uniform, then the paths of the particles will be helices. They will eventually hit the top or bottom.



$$\frac{dB}{dz} \neq 0$$

Weak focusing

A uniform magnetic field does have a focusing effect on a displaced beam



The size of the required aperture scales linearly with the beam energy.

Examples: Brookhaven Cosmotron 3 GeV
6" × 26" beam pipe.

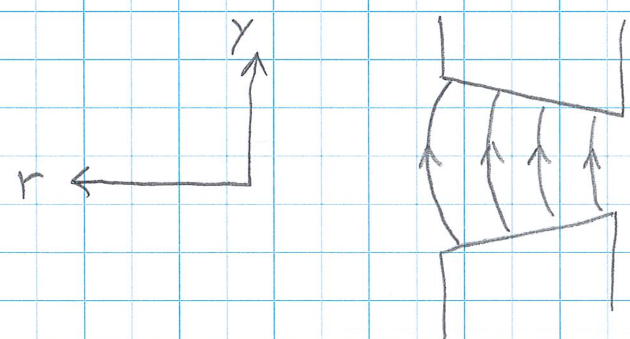
Berkeley Bevatron 6 GeV
12" × 48" beam pipe.

This is not cost effective for higher energies.

Betatron Oscillations

To first order, magnets are constructed so that $B(r) = B_y = \text{const}$.

It is better to have tapered pole pieces



Then $B_y = B_0 \left(\frac{r}{R}\right)^{-n}$

Introduce a change of variables,

$$\zeta = \frac{y}{R} \qquad \rho = \frac{r-R}{R}$$

Then $B_y = B_0 (1+\rho)^{-n} \approx B_0 (1 - n\rho)$

Maxwell's equations in free space for static fields

$$\nabla \times \vec{B} = 0$$

$$(\nabla \times \vec{B})_z = \frac{\partial B_r}{\partial y} - \frac{\partial B_y}{\partial r} = 0$$

$$\left.\frac{\partial B_y}{\partial r}\right|_R = -\frac{B_0 n}{R} = \frac{\partial B_r}{\partial y}$$

So $B_r = -\frac{B_0 n}{R} \cdot y$

Lorentz force, $\vec{F} = q\,\vec{v}_z \times \vec{B}$

$$F_r = q\,v_z\,B_r$$

$$F_y = -q\,v_z \frac{B_0\, n}{R} y = m\ddot{y}$$

This describes simple harmonic motion in the vertical direction.

Most accelerators now use strong focusing:

Use two sets of magnets with opposite tapers.

(a) with n large and > 0
(b) with n large and < 0.

(a) focuses in y and defocuses in r
(b) focuses in r and defocuses in y.

The combination focuses in both r and y.

Example: Brookhaven AGS ~ 33 GeV
CERN PS ~ 28 GeV

These had beam aperatures that were 3″ × 7″ and 7 cm × 15 cm, respectively. Much smaller than what would be needed for weak focusing.