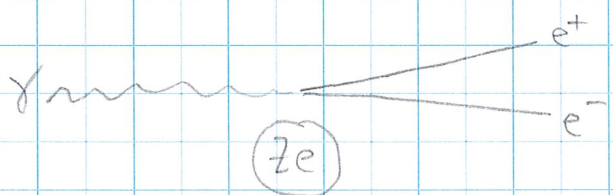
Lecture # 5

Compton scattering

$$\frac{1}{E'} - \frac{1}{E} = \frac{1-\cos\theta}{m_e}$$

Pair production : A photon with sufficient energy can turn into an electron - positron pair in the electric Field of a nucleus.

This can't happen in isolation, as this would violate momentum conservation.



When $x = E_{e^-}/E_\gamma$ the differential cross section is approximately

$$\frac{d\sigma}{dx} = \frac{A}{X_0 N_A}\left(1 - \frac{4}{3}x(1-x)\right)$$

where A is the atomic mass (g/mol) and X_0 is the radiation length of the material.

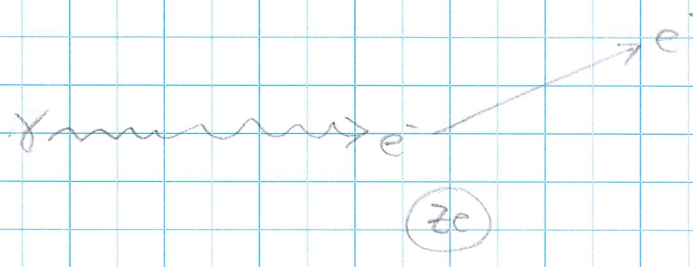
The total cross section approaches a constant at high energies,

$$\sigma_{e^+e^-} = \frac{7}{9}\frac{A}{X_0 N_A}$$

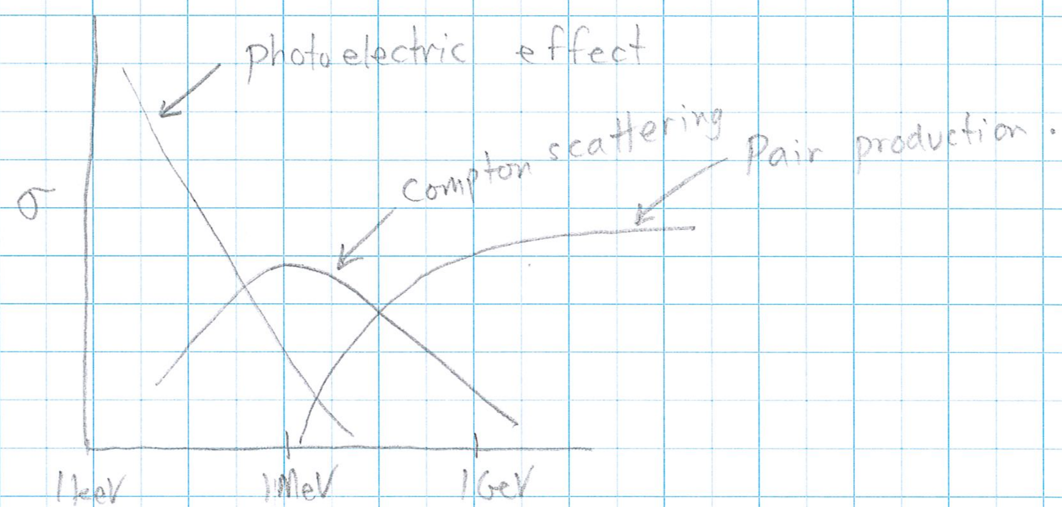
valid for $E_\gamma > 1$ GeV, especially for high-Z materials.

Ionization:

At lower energies, photons can ionize matter via the photoelectric effect.



These are the three dominant mechanisms by which photons lose energy as they interact with matter:



Interaction of charged particles with matter.

All charged particles interact with the electrons in material.

The mean rate of energy loss is given by the Bethe equation:

$$\left\langle -\frac{dE}{dx} \right\rangle = K z^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\frac{1}{2} \log\left(\frac{2 m_e c^2 \beta^2 \gamma^2 W_{max}}{I^2} \right) - \beta^2 - \frac{\delta(\beta\gamma)}{2} \right]$$

$K = 4\pi N_A r_e^2 m_e c^2 = 0.307075 \text{ cm}^2/\text{mol}$.
A = atomic mass (g/mol)

$$W_{max} = \frac{2 m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma m_e/M + (m_e/M)^2}$$

where the incident particle has mass M. This is the maximum energy that can be transferred in a single collision.

I is the "mean excitation energy" in electron-volts.

$$\frac{I}{Z} \sim 9 \text{ to } 11 \text{ eV} \quad \text{for } Z > 20.$$

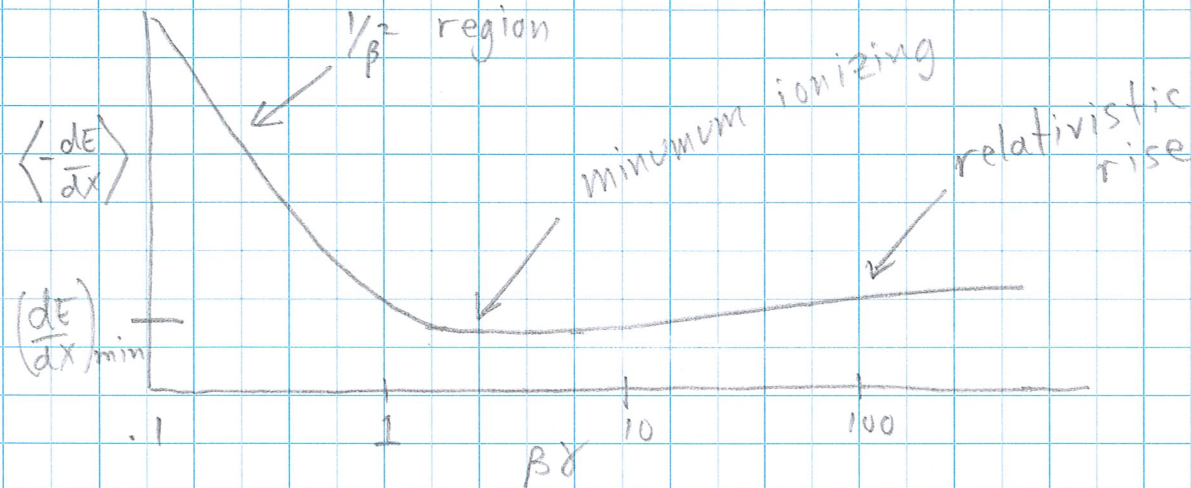
Tables of I exist for various materials.

$\delta(\beta\gamma)$ is the "density effect" correction. The medium becomes polarized and screens the distance over which it can interact with the incident particle.

$$\frac{\delta}{2} \to \log\left(\frac{\hbar\omega_p}{I}\right) + \log\beta\gamma - \frac{1}{2}$$

$\hbar\omega_p = (28.816 \text{ eV})\sqrt{\rho \langle Z/A \rangle}$ where ρ is in g/cm^3.

The rate of energy loss depends almost exclusively on $\beta\gamma$ and is often called "the universal energy loss curve".



The minimum is almost exactly at $\beta\gamma = 3$ for all types of particles.

Depth-dose curves.

As the particle slows down, $\left\langle -\frac{dE}{dx} \right\rangle$ increases. This causes it to slow down even faster.

$\left(\frac{dE}{dx}\right)_{min} = 1.5 - 2 \ MeV \cdot cm^2/g$ (scaled by the density)

Highest rate of energy loss is in the last little bit of path length.

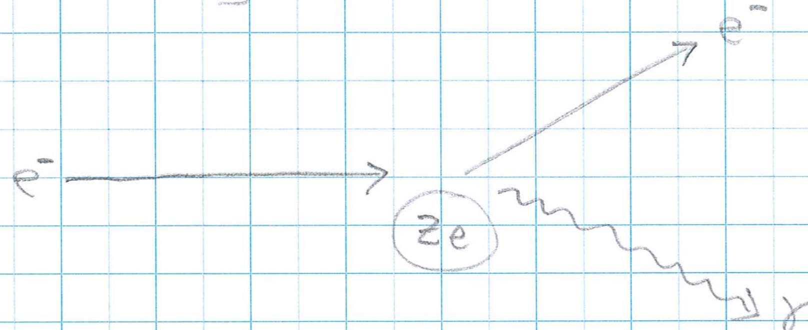
Application : Radiotherapy.

The dose is minimal on the surface. The depth of the Bragg peak can be adjusted by changing the incident beam energy.

Interaction of electrons with matter.

Accelerated charges radiate electromagnetic energy (even classically)

Bremsstrahlung :



Let $y = E_\gamma / E_{e^-}$ be the fraction of the incident electron's energy that is transferred to the photon.

Then $$\frac{d\sigma}{dE_\gamma} \approx \frac{A}{X_0 N_A E_\gamma}\left(\frac{4}{3} - \frac{4}{3}y + y^2\right)$$

This is fairly accurate except near $y=0$ or $y = 1$.

Synchrotron radiation.

When the acceleration is perpendicular to the motion (eg Lorentz force)

$$P_\perp = \frac{c}{6\pi \varepsilon_0} (ze)^2 \frac{(\beta\gamma)^4}{R^2}$$

R = radius of curvature.

Compare a 100 GeV proton and a 100 GeV electron.

$$\beta\gamma = \frac{p}{m} = \frac{100 \text{ GeV}}{1 \text{ GeV}} \sim 100 \text{ (proton)}$$

$$\beta\gamma = \frac{p}{m} = \frac{100 \text{ GeV}}{0.5 \text{ MeV}} \sim 200000 \text{ (electron)}$$

Electrons radiate 2000 times as much power as a proton.

General cases:

Photons lose energy by
(a) Ionization
(b) Compton scattering
(c) Pair production

Electrons lose energy by
(a) Ionization
(b) Bremsstrahlung

Other charged particles lose energy by
(a) Ionization
(b) Nuclear interactions (but not always)

One other mechanism: Čerenkov radiation.

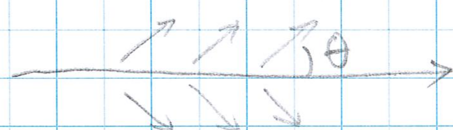
If a charged particle moves with a speed greater than the speed of light, ie,

$$\beta > c/n$$

n = index of refraction

then it will radiate light. Light is emitted at an angle θ given by

$$\cos\theta_c = \frac{1}{n\beta}$$



$$\frac{d^2N}{dx\,d\lambda} = \frac{4\pi^2 z^2 e^2}{hc\lambda^2}\left(1 - \frac{1}{n^2\beta^2}\right)$$

In the visible range, $\lambda = 400 - 700$ nm

$$\frac{dN_\gamma}{dx} \sim 490\, z^2 \sin^2\theta_c \quad \text{photons/cm}.$$

Eg, an electron with energy $E = 2$ MeV in water ($n = 1.33$).

$$\beta = \frac{p}{E} = \frac{\sqrt{E^2 - m^2}}{E} = \sqrt{1 - \frac{m^2}{E^2}} = 0.968$$

$$\cos\theta_c = \frac{1}{(1.33)(.968)} = 0.777 \Rightarrow \theta_c = 39^\circ$$

Then $\sin^2\theta_c = 0.463$ and $\frac{dN}{dx} = (490\ cm^{-1})(.463)$
$= 227\ cm^{-1}$.

Energy loss: $E = \frac{hc}{\lambda} \sim \frac{1240\ eV \cdot nm}{650\ nm} = 1.9\ eV$

The electron loses 431 eV per cm.

Not a significant source of energy loss but a useful way to detect charged particles.

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