Measurement of the strong coupling constant

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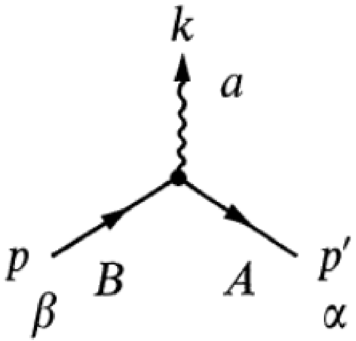
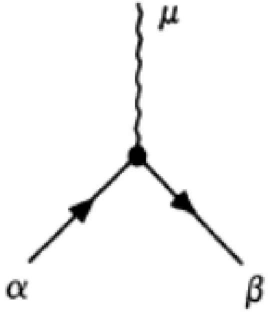
- The strong coupling constant α_s describes the strength of the strong interaction just like fine structure constant α describes the strength of the electromagnetic interaction.

- Right and left graphs are the Feynman rule for two quarks and one gluon as well as for two electrons and one photon, respectively [1], where $\alpha_s = g^2 / 4\pi$ and $\alpha = e^2 / 4\pi$ (Natural units).



$$g(\gamma_\mu)_{\alpha\beta} T^{a}_{AB} \times (2\pi)^4 \delta^4(p - p' - k)$$



$$-ie(\gamma_\mu)_{\beta\alpha}(2\pi)^4 \delta^4\left(\sum \text{incoming momenta}\right)$$

[1] Itzykson, C., and J.-B. Zuber, 1980, Quantum Field Theory (McGraw-Hill, New York).

Running coupling constant

According to Quantum Field Theory, α_s depends on the energy level that the scattering process involved with. The relation between the α_s in different energy level on the basis of one-loop approximation is [2]

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\beta_0 \ln \frac{Q^2}{\mu^2}}$$

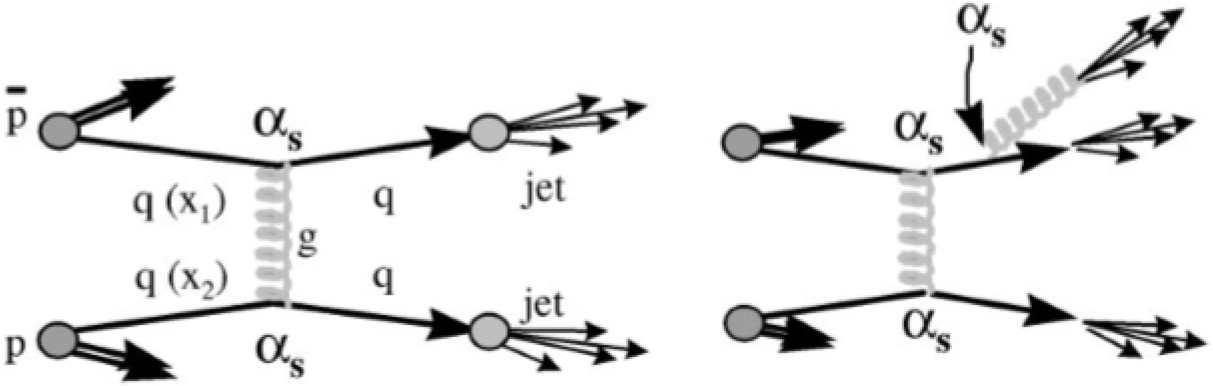
Commonly, the mass of the Z^0 boson is used as reference scale of α_s . The world average value is α_s（M_Z） $=0.1187 \pm 0.0007$ [3].

[2] S. Bethke, J. Phys. G 26 (2000) R27 [arXiv:hep-ex/0004021].

[3] Eur. Phys. J. C73(2013), no. 10 2604, [arXiv:1304.7498].

Determination of the strong coupling constant by measuring the ratio of the inclusive 3-jet cross section to the inclusive 2-jet cross section [3]

- The jet is a narrow cone of hadrons and other particles produced by the hadronization of a quark or gluon.
- The strong coupling constant is proportional to the ratio of the inclusive 3-jet cross section to the inclusive 2-jet cross section. The parton distribution function (PDF) is also need to determinate the strong coupling constant.



High energy quarks and gluons would excite quark-antiquark pair and other gluons and form a shower [4].

[4] G.C. Blazey and B.L. Flaugher, hep-ex/9903058

Determination of the strong coupling constant by measuring the ratio of the inclusive 3-jet cross section to the inclusive 2-jet cross section [3]

- The data sample was collected during 2011 at a proton-proton scattering with centre-of-mass energy of 7 TeV with the Compact Muon Solenoid (CMS) detector at the LHC.

- Jets are reconstructed using the infrared-and collinear-safe anti-k_T clustering algorithm.

- The theoretical predictions is given by using parton-level generator NLOJET++ with four PDF sets–NNPDF2.1, ABM11, MSTW2008, and CT10.

- The value of α_s（M_Z）is determined by minimizing the difference between the experimental measurement and the theoretical predictions.

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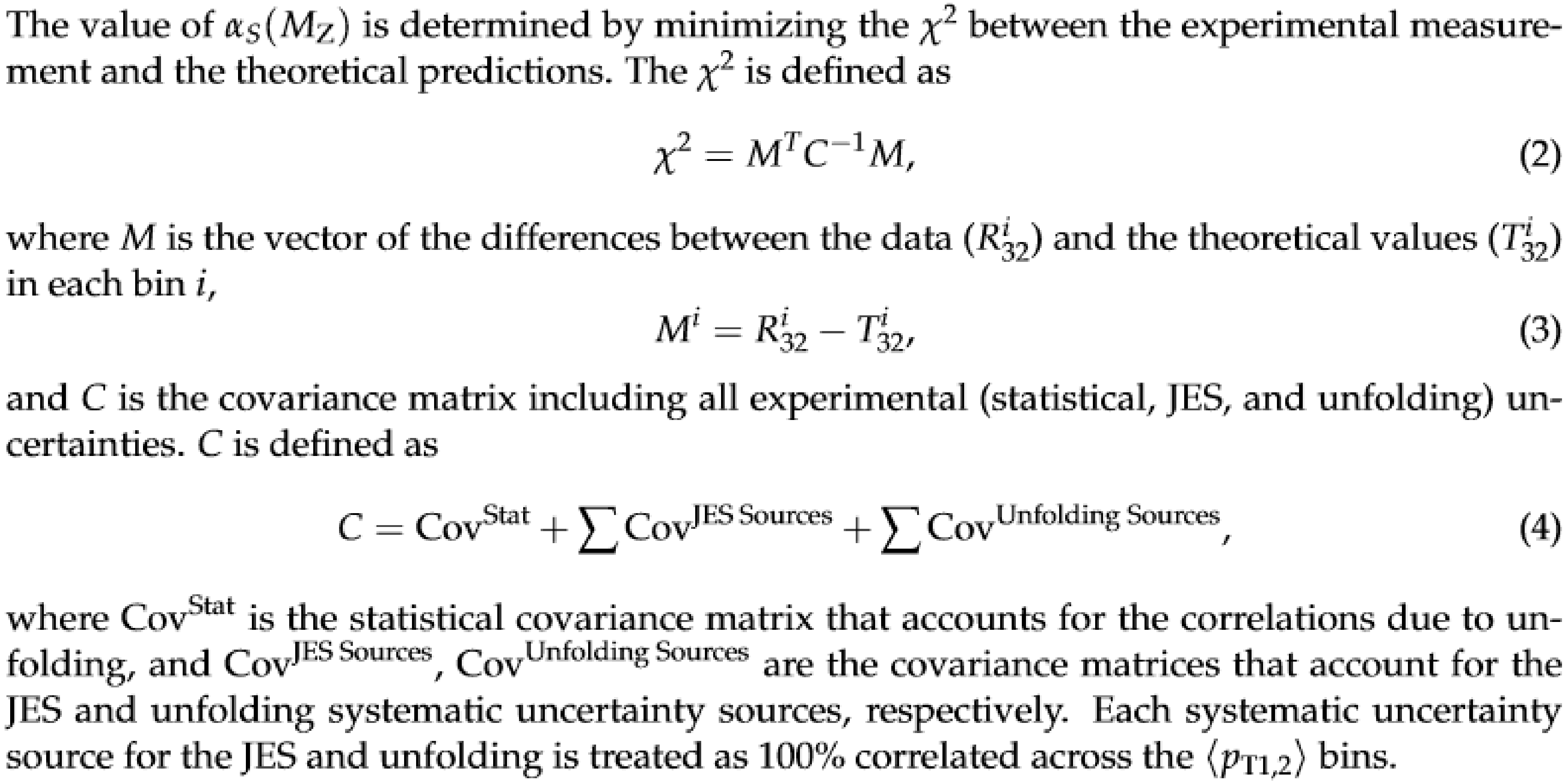

The value of $\alpha_S(M_Z)$ is determined by minimizing the χ^2 between the experimental measurement and the theoretical predictions. The χ^2 is defined as

$$\chi^2 = M^T C^{-1} M, \tag{2}$$

where M is the vector of the differences between the data (R^i_{32}) and the theoretical values (T^i_{32}) in each bin i,

$$M^i = R^i_{32} - T^i_{32}, \tag{3}$$

and C is the covariance matrix including all experimental (statistical, JES, and unfolding) uncertainties. C is defined as

$$C = \mathrm{Cov}^{\mathrm{Stat}} + \sum \mathrm{Cov}^{\mathrm{JES\ Sources}} + \sum \mathrm{Cov}^{\mathrm{Unfolding\ Sources}}, \tag{4}$$

where $\mathrm{Cov}^{\mathrm{Stat}}$ is the statistical covariance matrix that accounts for the correlations due to unfolding, and $\mathrm{Cov}^{\mathrm{JES\ Sources}}$, $\mathrm{Cov}^{\mathrm{Unfolding\ Sources}}$ are the covariance matrices that account for the JES and unfolding systematic uncertainty sources, respectively. Each systematic uncertainty source for the JES and unfolding is treated as 100% correlated across the $\langle p_{\mathrm{T}1,2}\rangle$ bins.

- The comparison between the data from experiment and the best matching theoretical predictions come from four different PDF sets is shown in right graph.

- The final values of the $\alpha_s(M_Z)$ after cut off the events which have energy lower than 420 GeV are

NNPDF2.1: $\alpha_s(M_Z) = 0.1148 \pm 0.0014$

MSTW2008: $\alpha_s(M_Z) = 0.1141 \pm 0.0022$

CT10: $\alpha_s(M_Z) = 0.1135 \pm 0.0019$

ABM11: $\alpha_s(M_Z) = 0.1214 \pm 0.0018$