PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 53600

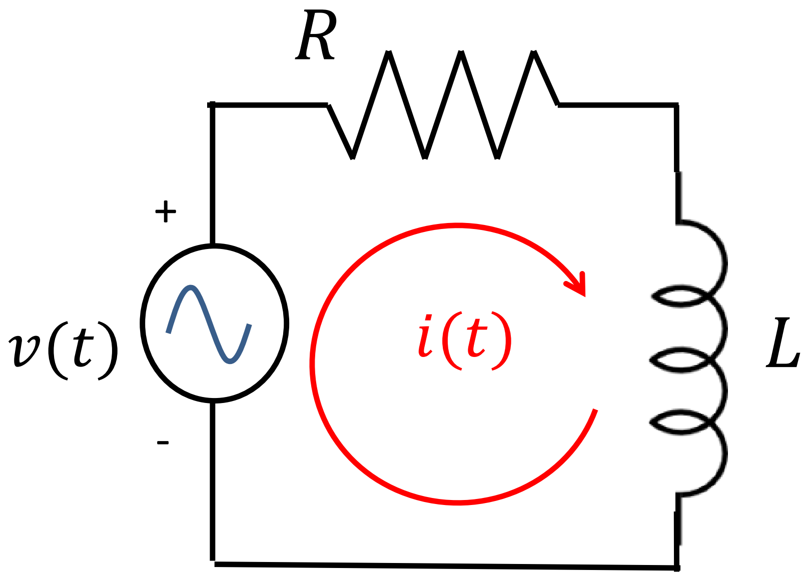
Electronics Techniques for Research

Now in PowerPoint!

Spring 2020 Semester

Prof. Matthew Jones

AC Frequency Analysis



$$v(t) - i(t)R - L\frac{di}{dt}$$

$$V - IR - jI\omega L = 0$$

$$I = \frac{V}{R + j\omega L}$$

- Magnitude of current: $|I| = \frac{|V|}{\sqrt{R^2+\omega^2 L^2}}$
- Phase with respect to V: $\tan\psi = -\frac{\omega L}{R}$

AC Frequency Analysis

- Limiting behavior:
  - Small ω: $\omega L \ll R$

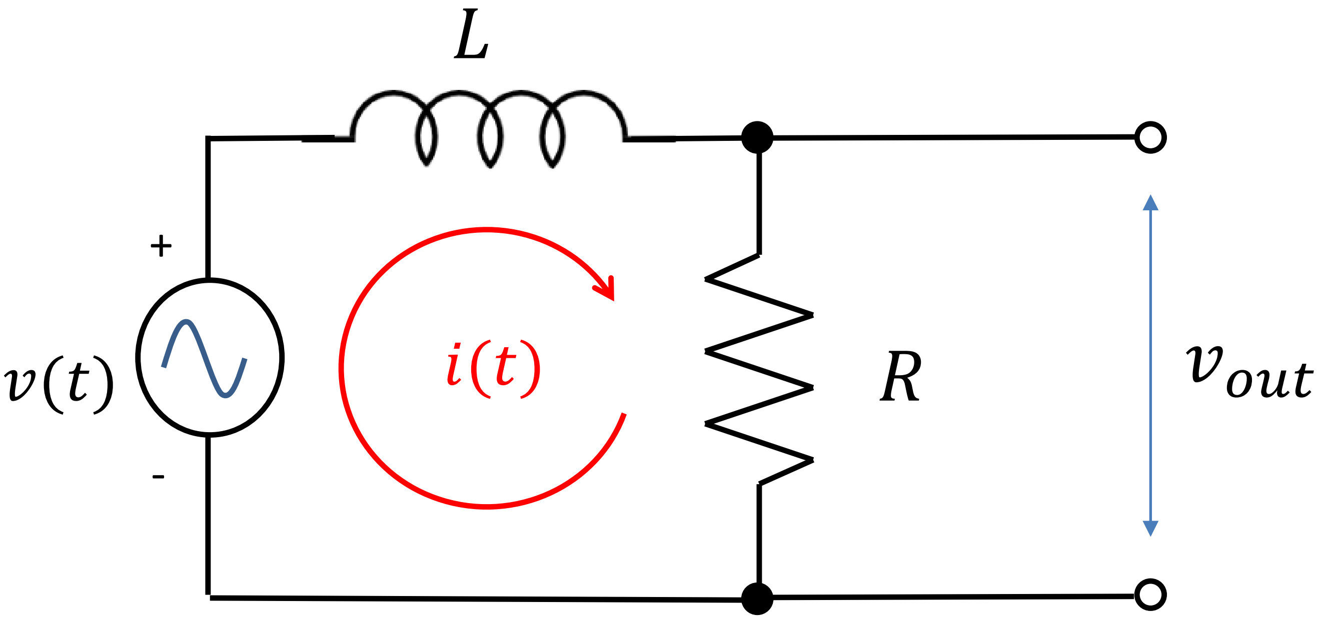
$$I \to V/R \qquad\qquad \psi \to 0$$

  - Large ω: $\omega L \gg R$

$$I \to V/\omega L \qquad\qquad \psi \to -\pi/2$$

- The circuit acts as a filter that prefers to pass the low-frequencies ($\omega \ll R/L$)

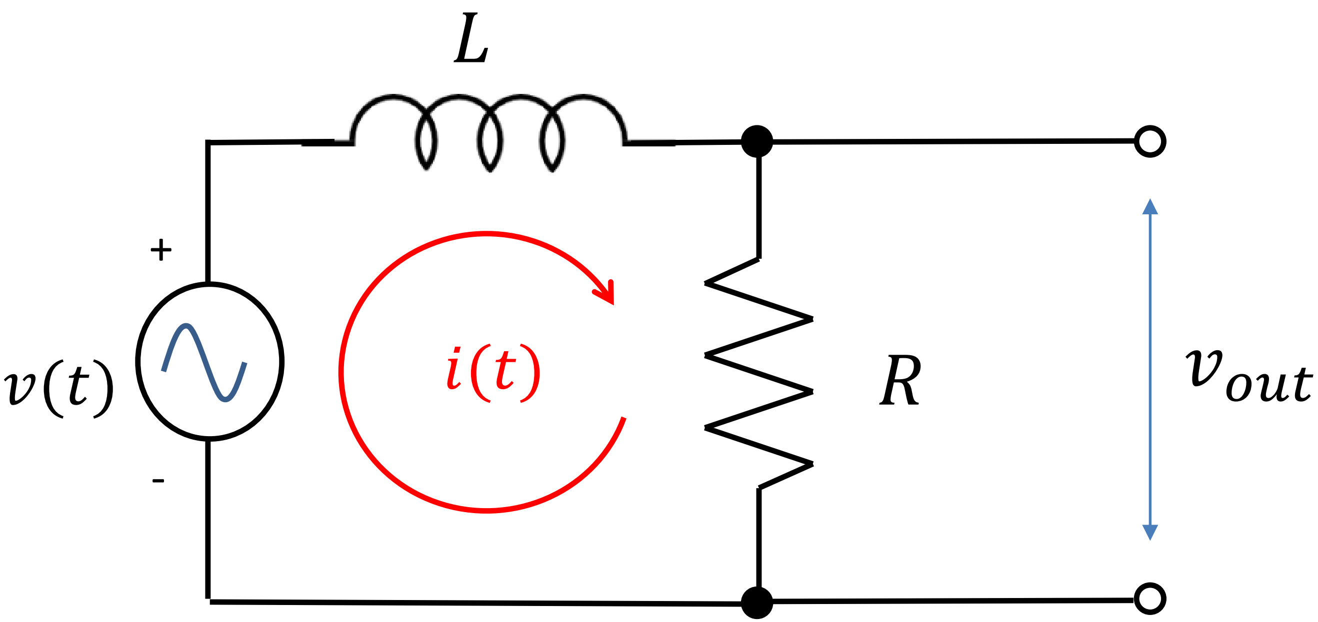
Frequency Response



$$I = \frac{V}{R + j\omega L} = \frac{V(R - j\omega L)}{R^2 + \omega^2 L^2}$$

$$V_{out} = \frac{VR(R - j\omega L)}{R^2 + \omega^2 L^2}$$

Frequency Response



Voltage gain: $A = \frac{|V_{out}|}{|V_{in}|} = \frac{R}{\sqrt{R^2+\omega^2L^2}}$

At low frequencies, $A \to 1$

At high frequencies, $A = R/\omega L$

Frequency Response

- Gain is often expressed in decibels (db):

$$G = 20\log_{10} A$$

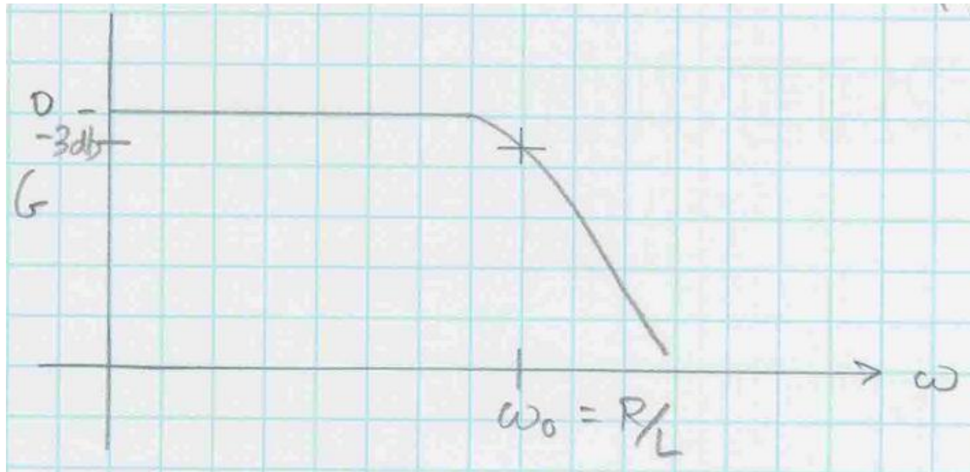
- For the low-pass filter:

$$G = 20\log_{10}\left(\frac{R}{\sqrt{R^2 + \omega^2 L^2}}\right)$$

- At low frequencies, $G \to 0$
- At high frequencies,

$$G \to 20\log_{10}\left(\frac{R}{L}\right) - 20\log_{10}\omega$$

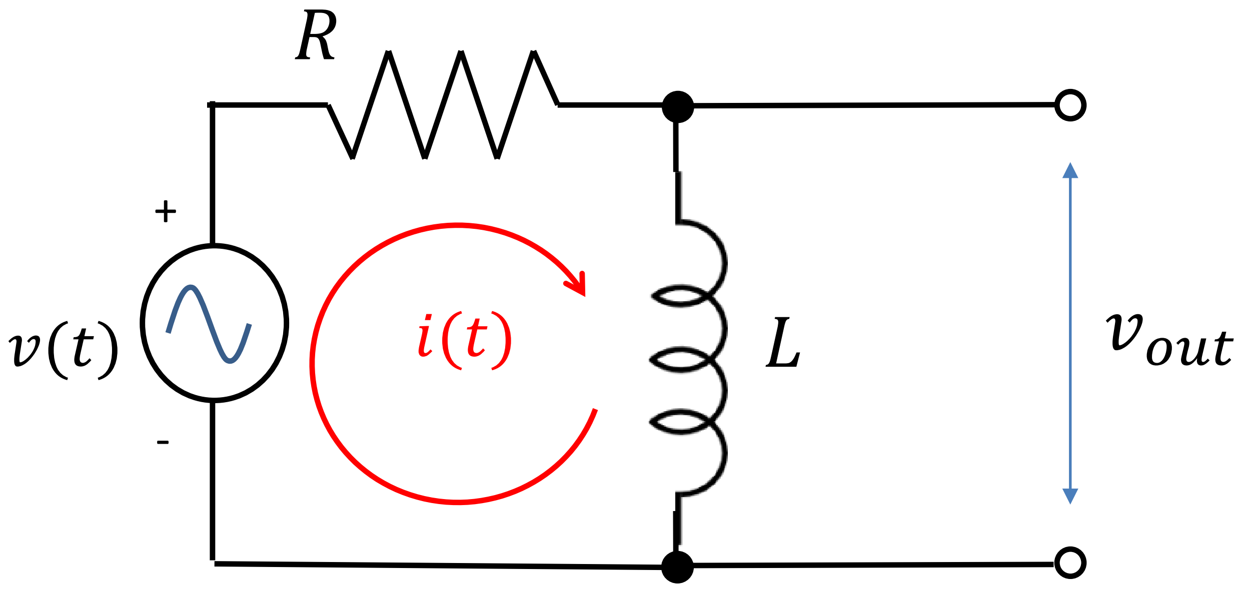
Frequency Response



- Where does $G(\omega)$ start to fall off?
- One useful definition is to define ω_0 to be the point where $A = 1/\sqrt{2}$.
- In this case,

$$G = 20\ \log_{10} 1/\sqrt{2} = -3.0103\ \approx -3\text{ db}$$
$$\omega_0 = R/L$$

Other Filter Configurations

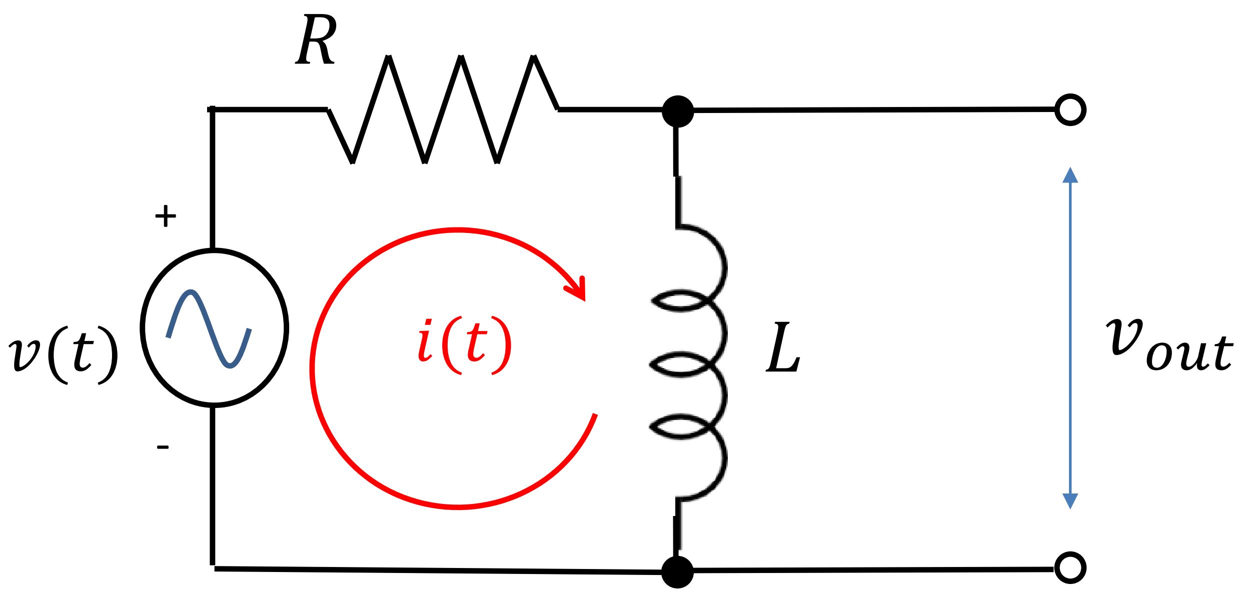


$$I = \frac{V}{R + j\omega L}$$

$$V_{out} = j\omega I L$$

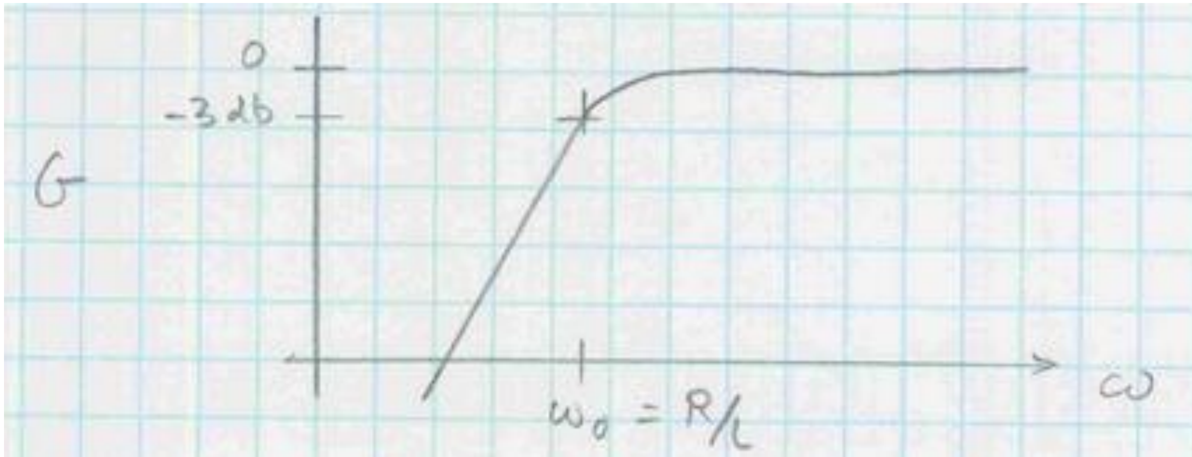
- Voltage gain: $A = \frac{|V_{out}|}{|V_{in}|} = \frac{\omega L}{\sqrt{R^2+\omega^2 L^2}} = \frac{1}{\sqrt{\frac{R^2}{\omega^2 L^2}+1}}$

Other Filter Configurations



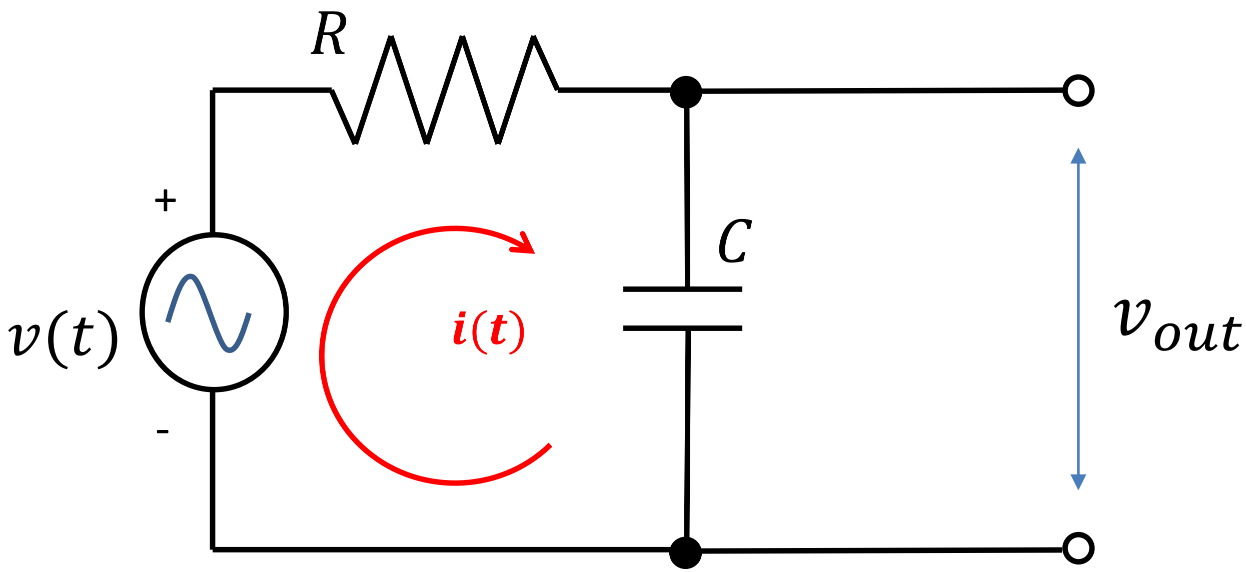
- At high frequencies, $\omega^2 L^2 \gg R^2$, $A \rightarrow 1$
- At low frequencies, $A \rightarrow \omega L/R$
- This is a high-pass filter: the inductor "shorts out" low frequency components.

Frequency Response



$$G = 20\ \log_{10} 1/\sqrt{2} = -3.0103\ \approx -3\ \mathrm{db}$$
$$\omega_0 = R/L$$

R-C Filters

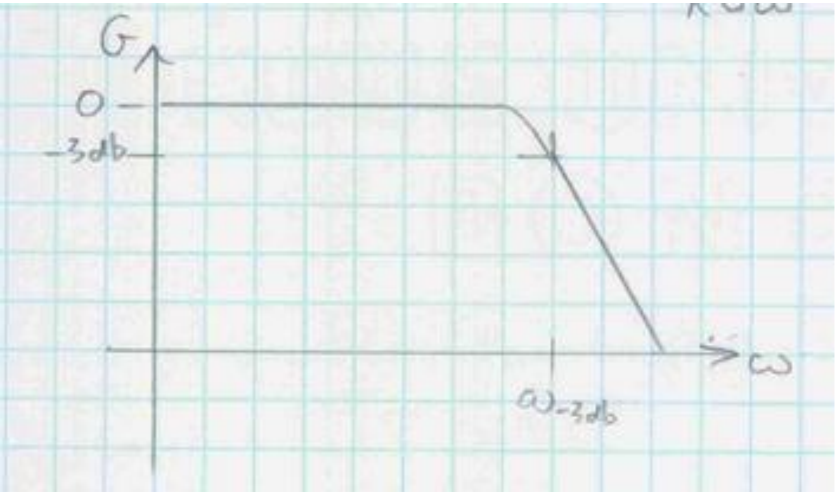


$$I = \frac{V}{R - j/\omega C}$$

$$V_{out} = \frac{V(-j/\omega C)}{R - j/\omega C}$$

$$A = \frac{|V_{out}|}{|V_{in}|} = \frac{1}{\sqrt{1 + R^2C^2\omega^2}}$$

$$\tan\psi = -\omega RC$$



Fourier Analysis

- What if a voltage source has two frequency components?

$$v(t) = V_{\omega_1} e^{j\omega_1 t} + V_{\omega_2} e^{j\omega_2 t}$$

- Then we expect that

$$i(t) = I_{\omega_1} e^{j\omega_1 t} + I_{\omega_2} e^{j\omega_2 t}$$

where

$$I_{\omega_1} = Z_{\omega_1}^{-1} V_{\omega_1}$$
$$I_{\omega_2} = Z_{\omega_2}^{-1} V_{\omega_2}$$

- In general, because the system of equations is linear,

$$i(t) = \sum_n Z_{\omega_n}^{-1} V_{\omega_n} e^{j\omega_n t}$$

- Now we can calculate the response due to an arbitrary periodic voltage source.

Fourier Analysis

- If $f(t)$ is periodic with period T, that is,

$$f(t+T) = f(t)$$

then $f(t)$ can be expressed:

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + b_n \sin(n\omega t)$$

- Fourier coefficients:

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(n\omega t)\,dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(n\omega t)\,dt$$

$$\omega = \frac{2\pi}{T}$$

Fourier Analysis

- Or, we can write,

$$f(t) = \sum_{n=0}^{\infty} c_n e^{j\omega t}$$

where

$$c_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)e^{-jn\omega t}dt$$

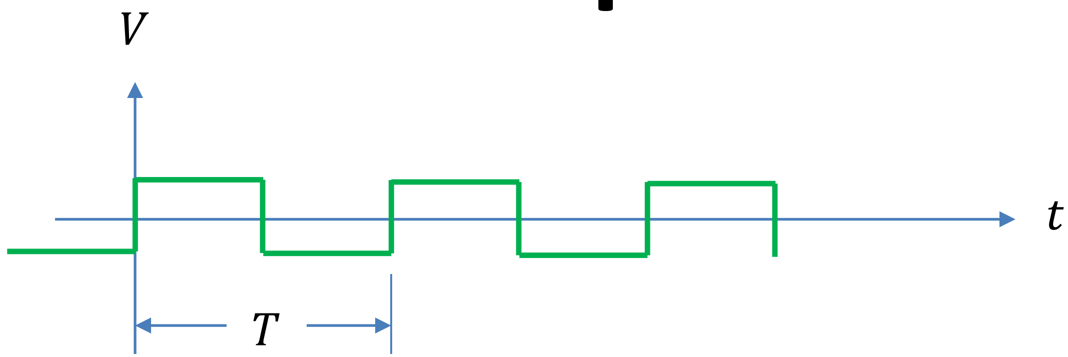
- Then,

$$i(t) = \sum_{n=0}^{\infty} d_n e^{jn\omega t}$$

where

$$d_n = Z_{n\omega}^{-1} c_n$$

Example



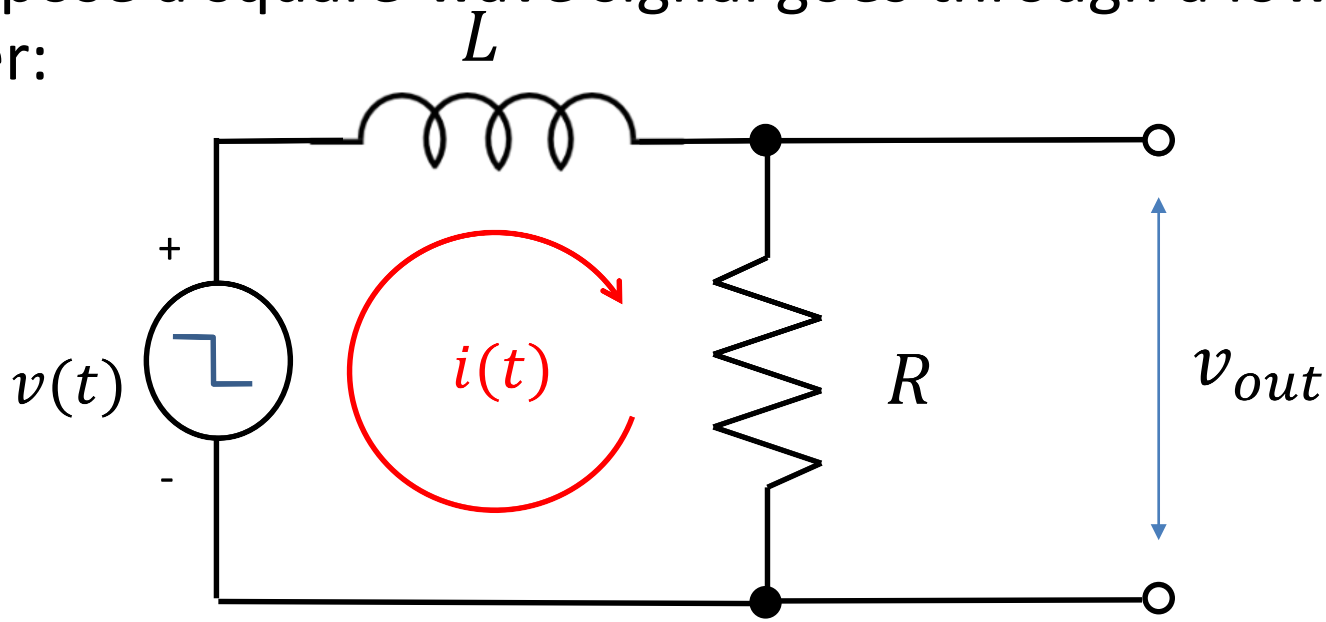
- In this case, $a_0 = 0$
- In fact, all $a_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\cos(n\omega t)\,dt = 0$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} f(t)\sin(n\omega t)\,dt$$

$$= \begin{cases} \dfrac{4}{n\pi} & \text{when } n \text{ is odd} \\ 0 & \text{when } n \text{ is even} \end{cases}$$

Low-Pass Filter

- Suppose a square-wave signal goes through a low-pass filter:



$$I_\omega = \frac{V_\omega}{R + j\omega L}$$

$$V_{out}(\omega) = \frac{V_\omega R}{R + j\omega L}$$

Low-Pass Filter

$$|V_{out}(\omega)| = |V_\omega| \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\tan \psi_\omega = -\frac{\omega L}{R}$$

- So, if

$$v(t) = \sum_{n=0}^{\infty} b_n \sin\left(\frac{2\pi n t}{T}\right)$$

- Then,

$$v_{out}(t) = \sum_{n=0}^{\infty} \frac{b_n R}{\sqrt{R^2 + \left(\frac{2\pi n}{T}\right)^2 L^2}} \sin\left(\frac{2\pi n t}{T} + \psi_{2\pi n}/T\right)$$

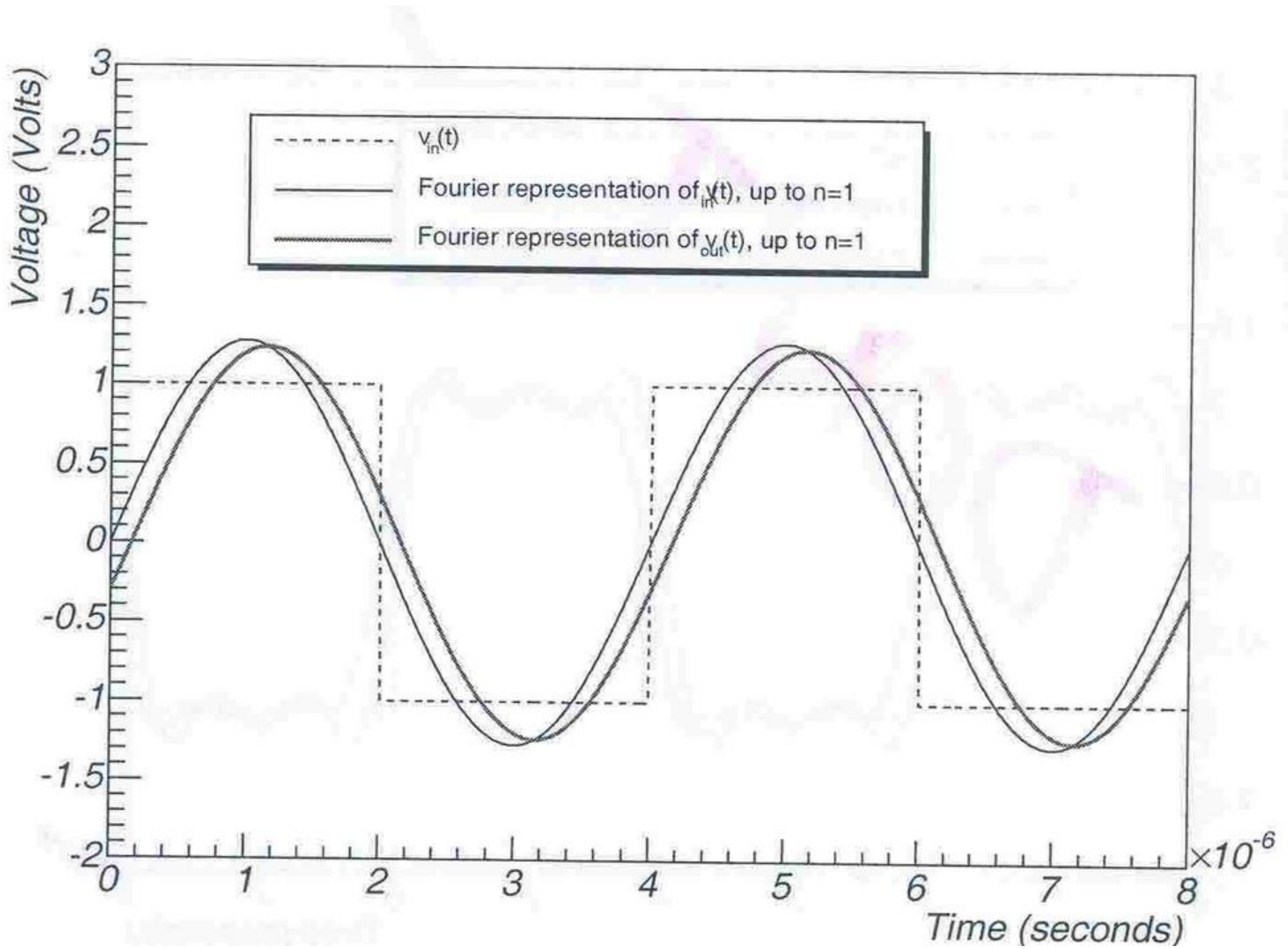
- Two effects:
  - Higher frequencies are attenuated
  - Higher frequencies are phase shifted

Low-Pass Filter

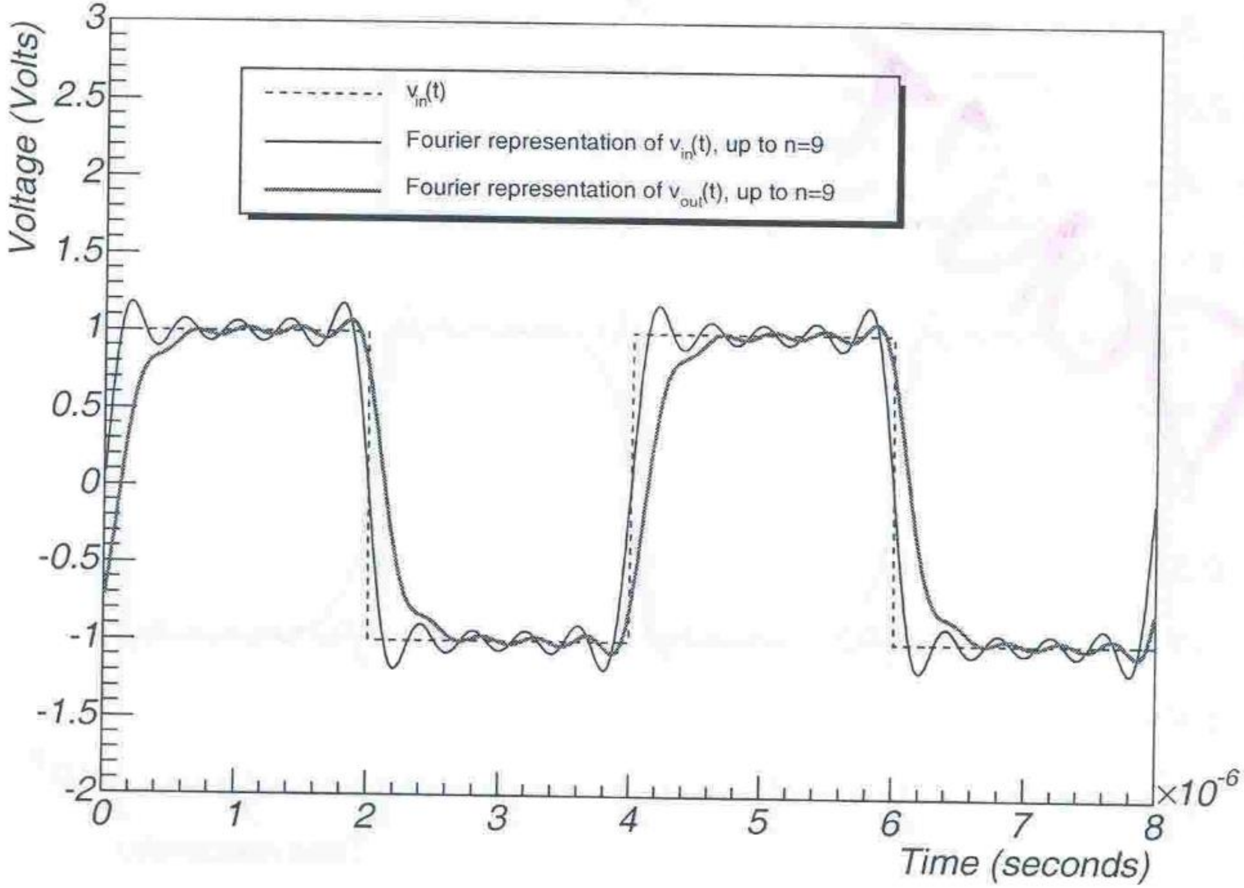
- Suppose that $T = 4\ \mu s \quad (f = 250\ kHz)$
- Recall that $\omega_{-3\ db} = R/L = 2\pi\ f_{-3\ db}$
- If $f_{-3\ db} = 1\ MHz$ then $R/L = 6.28 \times 10^6\ s^{-1}$
- Suppose $L = 10\ \mu H$ and $R = 63\ \Omega$

n	A_n	ϕ_n
1	0.97	-14 °
3	0.80	-36 °
5	0.63	-51 °
7	0.50	-60 °
9	0.41	-66 °

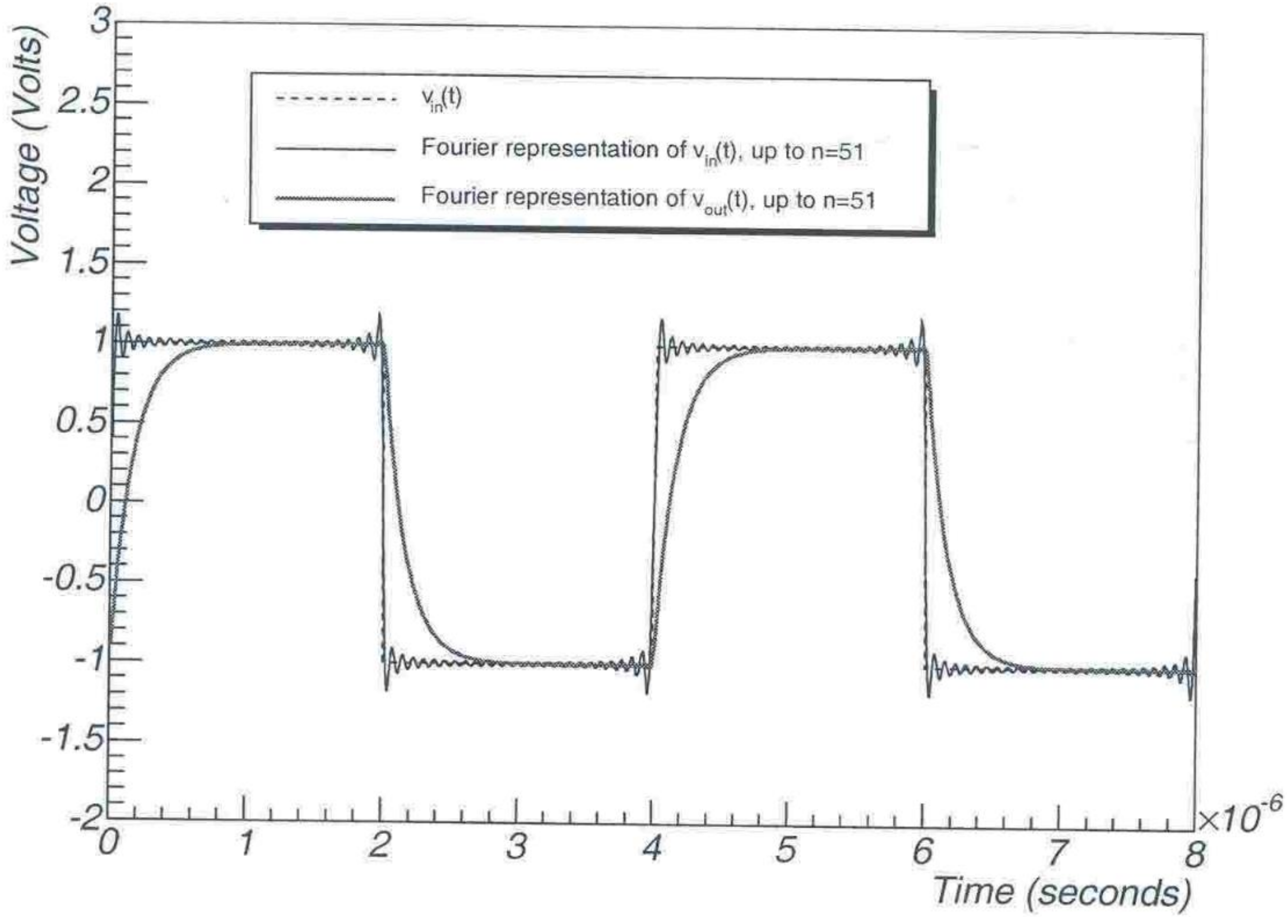
Fourier Analysis Example



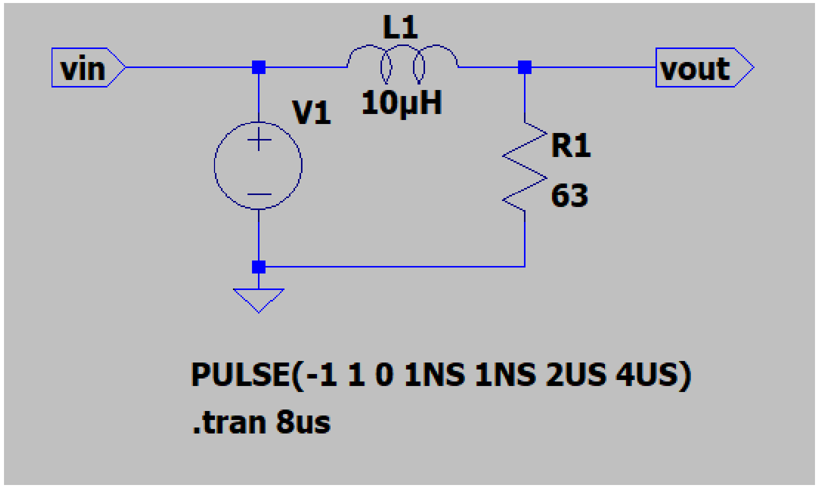
Fourier Analysis Example



Fourier Analysis Example



Analysis of Low-Pass Filter using SPICE



SPICE Netlist: C:\Users\mjones\Documents\Phys536\Spice\Lecture7.net

```
* C:\Users\mjones\Documents\Phys536\Spice\Lecture7.asc
R1 vout 0 63
L1 vin vout 10μH
V1 vin 0 PULSE(-1 1 0 1NS 1NS 2US 4US)
.tran 8us
.backanno
.end
```