Purdue University Department of Physics

Physics 53600

Electronics Techniques for Research

Now in PowerPoint!

Spring 2020 Semester

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Inductors

- Just like energy can be stored in an electric field (ie, a capacitor) it can also be stored in a magnetic field.
- Charge carriers lose energy when they increase the magnetic field and they gain energy from the magnetic field when it decays.
- Faraday's law of induction:

$$\oint_{\boldsymbol{C}} \boldsymbol{E} \cdot d\boldsymbol{l} = -\frac{d\phi_m}{dt}$$

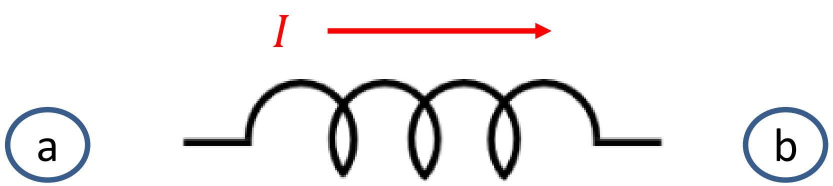
Inductors

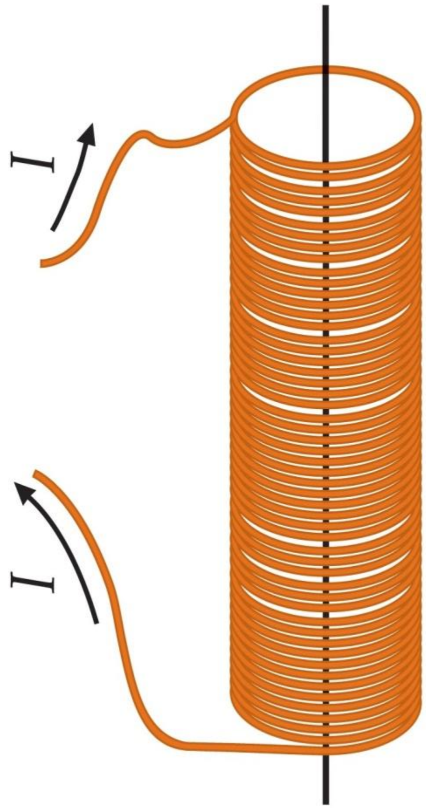
- Consider a solenoid: $B = \mu_0 n I$
  - Turns per unit length: $n = N/\ell$
- Magnetic flux: $\phi_m = BA$
- Potential difference:

$$\Delta V = -\frac{\mu_0 N^2 A}{\ell}\frac{dI}{dt}$$

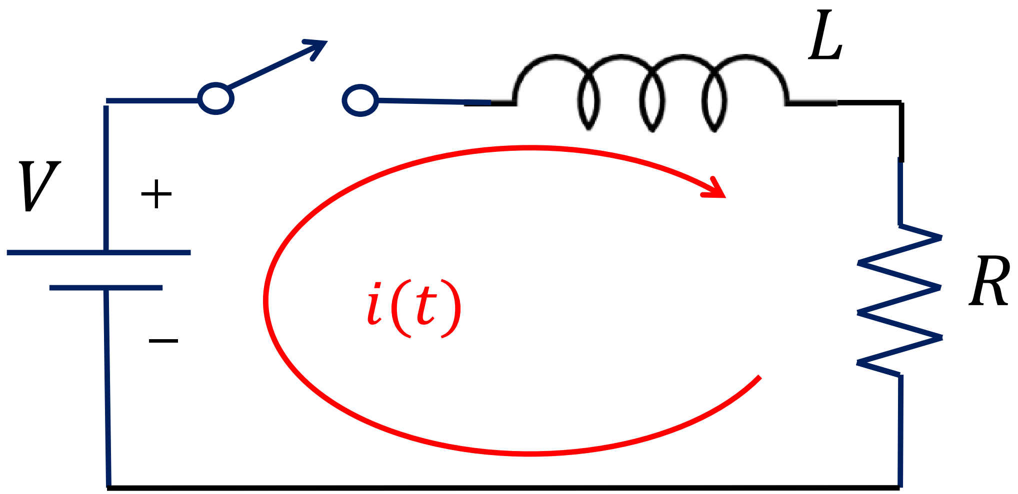
- In general,

$$V_b - V_a = -L\frac{dI}{dt}$$





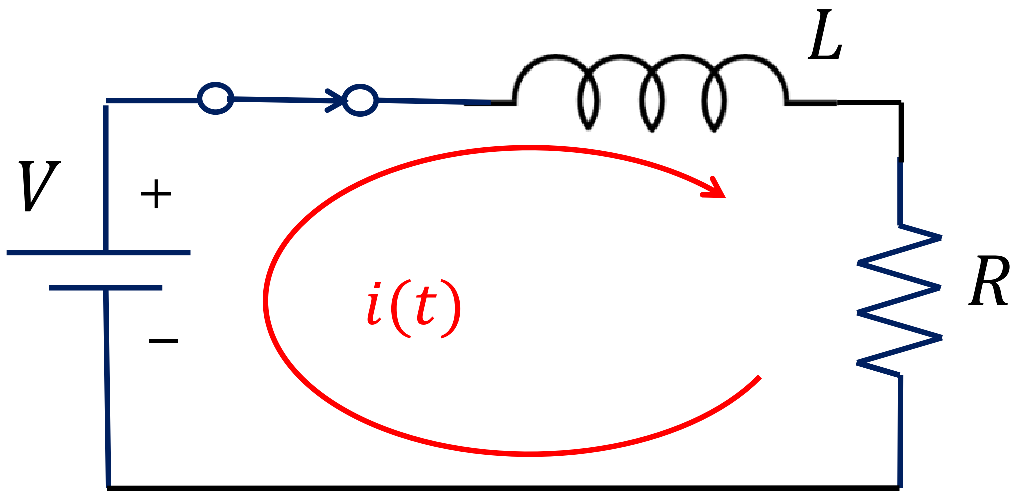
One Loop



$$V - L\frac{di}{dt} - iR = 0$$

Initial condition: $i(0) = 0$

One Loop



$$V - L\frac{di}{dt} - iR = 0$$

$$\frac{di}{dt} + \frac{R}{L}i = \frac{V}{L}$$

One Loop

- Homogeneous equation:

$$\frac{di}{dt} + \frac{R}{L} i = 0$$

- Solution to the homogeneous equation:

$$i(t) = I_0 e^{-(R/L)t}$$

- Particular solution:

$$i_0 = \frac{V}{R}$$

- Complete solution:

$$i(t) = I_0 e^{-(R/L)t} + \frac{V}{R}$$

One Loop

- Complete solution:

$$i(t) = I_0 e^{-(R/L)t} + \frac{V}{R}$$

- Initial condition:

$$i(0) = I_0 + \frac{V}{R} = 0$$

$$I_0 = -\frac{V}{R}$$

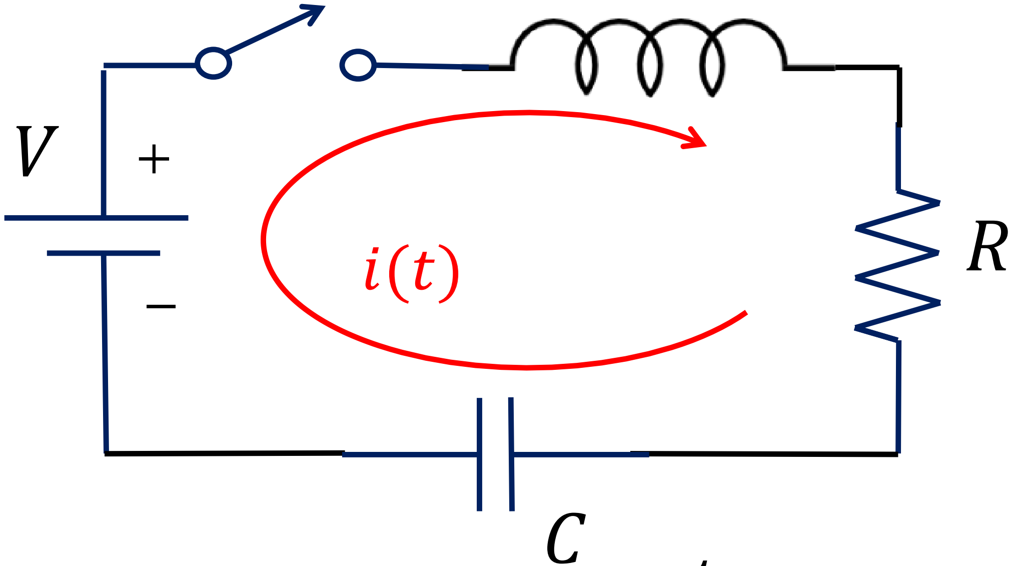
- Solution:

$$i(t) = \frac{V}{R}\left(1 - e^{-(R/L)t}\right)$$

Inductors in Circuits

- Initially, an inductor acts like an open circuit
  - No current is flowing through the inductor
  - All the electrical potential energy is used up in creating the magnetic field
- As $t \rightarrow \infty$, an inductor acts like a wire
  - No potential difference across the inductor
  - The magnetic field is constant ($d\phi_m/dt = 0$)

RLC Circuits



$$V - L\frac{di}{dt} - iR - \frac{1}{C}\int_0^t i(\tau)d\tau = 0$$

Differentiate…

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

RLC Circuits

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{i}{LC} = 0$$

- Let $\omega_0^2 = 1/LC$ and $\gamma = R/2L$

$$\frac{d^2i}{dt^2} + 2\gamma\frac{di}{dt} + \omega_0^2 i = 0$$

- Suppose $i(t)$ is of the form $i(t) = c\ e^{\alpha t}$
- Then $\alpha^2 + 2\gamma\alpha + \ \omega_0^2 = 0$

$$\alpha = -\gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

RLC Circuits

- If $\gamma > \omega_0$ then the roots are real:

$$i(t) = Ae^{-\gamma_+ t} + Be^{-\gamma_- t}$$

$$\gamma_\pm = \gamma \pm \sqrt{\gamma^2 - \omega_0^2}$$

- If $\gamma < \omega_0$ then the roots are complex:

$$i(t) = Ae^{-\gamma t}\sin\omega t + Be^{-\gamma t}\cos\omega t$$

$$\omega = \sqrt{\omega_0^2 - \gamma^2}$$

RLC Circuits

- Suppose the roots are complex and we have a solution that oscillates
- At time $t = 0$, no current is flowing because of the inductor.
  - Therefore, $B = 0$
- Original equation at time $t = 0$:

$$V - L\frac{di}{dt} = 0$$

$$\frac{di}{dt}|_{t=0} = A\omega \Rightarrow A = V/\omega L$$

RLC Circuits

$$i(t) = \frac{V}{\omega L} e^{-\gamma t} \sin \omega t$$

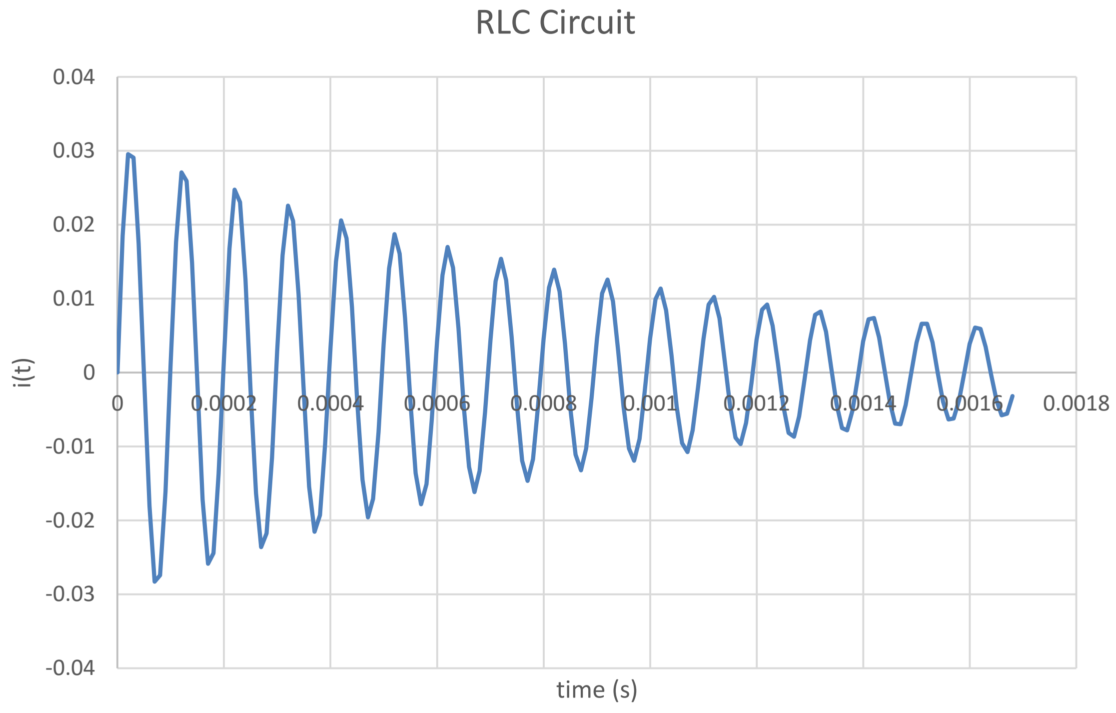
- Example:

$V = 10\ \text{V}$

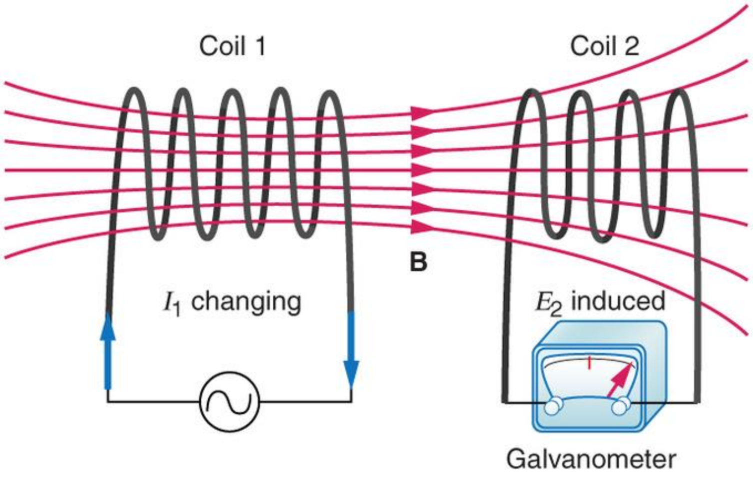
$R = 10\ \Omega$

$L = 5\ \text{mH}$

$C = 0.05\ \mu\text{F}$



Mutual Inductance



- The magnetic flux is generated by the current in Coil 1
- The changing magnetic flux induces a potential difference across Coil 2

$$\Delta V_2 = -N_2 \frac{d\phi_{12}}{dt} = -M_{12} \frac{dI_1}{dt}$$

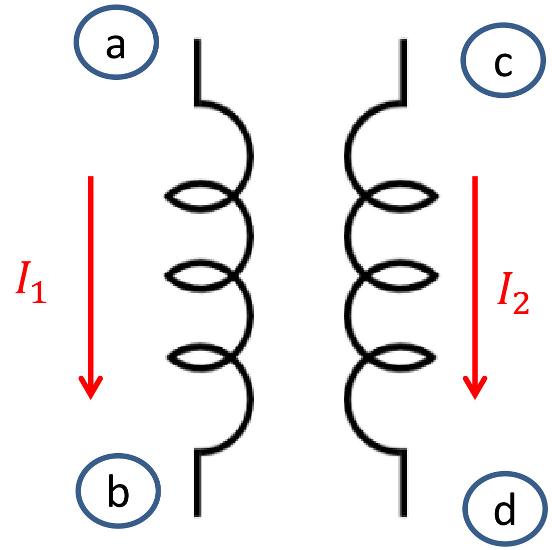
- Likewise, a magnetic flux can be generated by current in Coil 2 which induces a potential difference across Coil 1:

$$\Delta V_1 = -N_1 \frac{d\phi_{21}}{dt} = -M_{21} \frac{dI_2}{dt}$$

Mutual Inductance

- The reciprocity theorem can be used to show that
$$M_{12} = M_{21} \equiv M$$
- Mutual inductance can be expressed in terms of self-inductance of each coil:
$$M = \sqrt{L_1 L_2}$$
- This assumes optimal coupling between the two coils which is not always the case
$$M = k\sqrt{L_1 L_2}$$

Mutual Inductance



Remember Lenz's Law!

Notice the sign convention of the currents used in this definition!

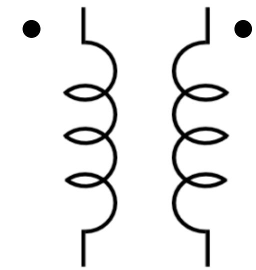
- Kirchhoff's rules:

$$V_b = V_a - L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt}$$

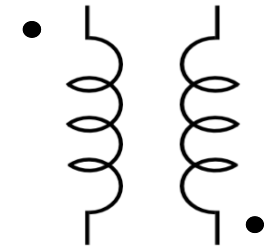
$$V_d = V_c - L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt}$$

Coupled Inductors

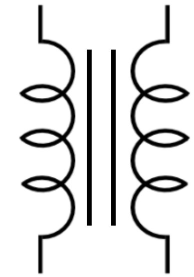
- This represents an arbitrary mutual inductance as we have been discussing:



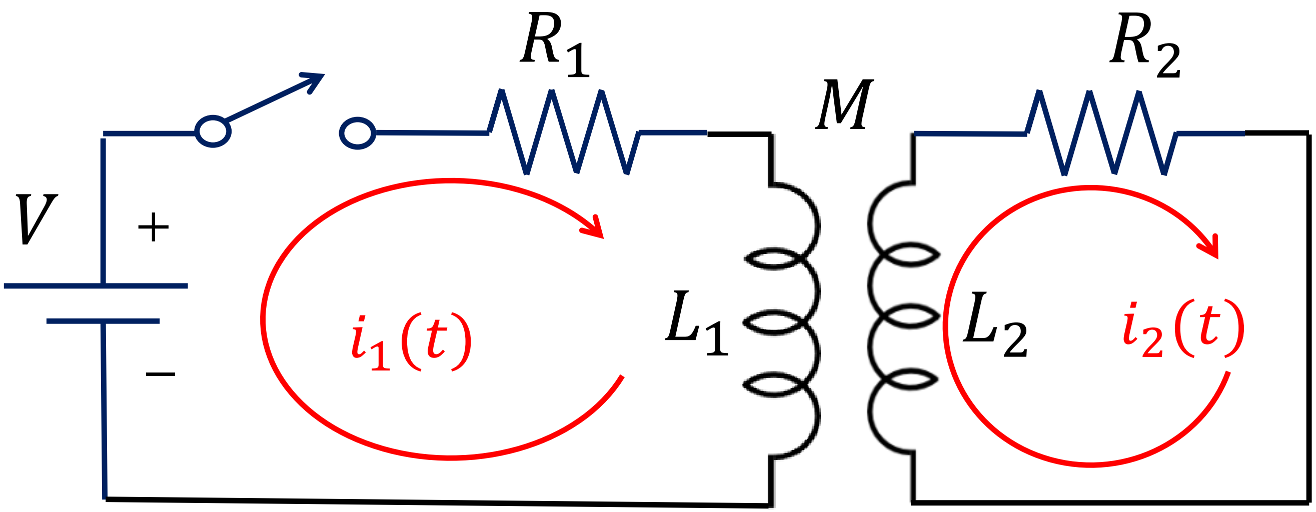
- When the secondary coil is connected backwards we need to flip the sign of the induced potential:



- This just indicates that the flux is contained in a core material (like iron or ferrite):



A Circuit with Mutual Inductance



$$V - i_1 R_1 - L_1 \frac{di_1}{dt} + M \frac{di_2}{dt} = 0$$

$$-i_2 R_2 - L_2 \frac{di_2}{dt} + M \frac{di_1}{dt} = 0$$

A Circuit with Mutual Inductance

- Assume solutions might be of the form:

$$\boldsymbol{i}(t) = \boldsymbol{I}\, e^{-\alpha t}$$

$$\begin{pmatrix} R_1 - \alpha L_1 & \alpha M \\ \alpha M & R_2 - \alpha L_2 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} V \\ 0 \end{pmatrix}$$

- First, solve the homogeneous equation:

$$\begin{pmatrix} R_1 - \alpha L_1 & \alpha M \\ \alpha M & R_2 - \alpha L_2 \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\alpha^2(L_1 L_2 - M^2) - \alpha(R_1 L_2 + R_2 L_1) + R_1 R_2 = 0$$

- When the coupling is perfect, $M^2 = L_1 L_2$ and

$$\alpha = \frac{R_1 R_2}{R_1 L_2 + R_2 L_1}$$

A Circuit with Mutual Inductance

- Eigenvectors:

$$\begin{pmatrix} R_1 - \alpha L_1 & \alpha M \\ \alpha M & R_2 - \alpha L_2 \end{pmatrix} \begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$B = \frac{\alpha L_1 - R_1}{\alpha M} A$$

- Add in the particular solution:

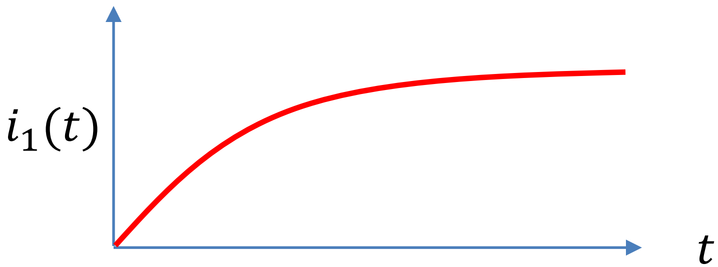
$$i_1(t) = A\, e^{-\alpha t} + \frac{V}{R_1}$$

$$i_2(t) = A \left(\frac{\alpha L_1 - R_1}{\alpha M} \right) e^{-\alpha t}$$

- Initial condition: $i_1(0) = 0 \Rightarrow A = -V/R_1$

A Circuit with Mutual Inductance

- The current in the first loop behaves as before:



- The current in the second loop:

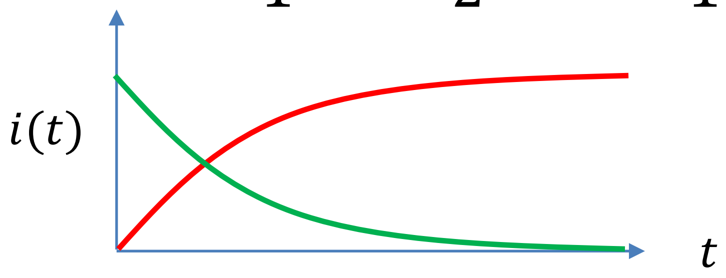
$$i_2(t) = -\frac{V}{R_1}\left(\frac{L_1 - R_1/\alpha}{M}\right)e^{-\alpha t}$$

$$= \frac{V}{R_1}\left(\frac{R_1 L_2}{M R_2}\right)e^{-\alpha t} = \frac{V}{R_2}\sqrt{\frac{L_2}{L_1}}e^{-\alpha t}$$

(Assuming perfect coupling)

A Circuit with Mutual Inductance

- Special case when $R_1 = R_2$ and $L_1 = L_2$:



- Recall that for an ideal solenoid,

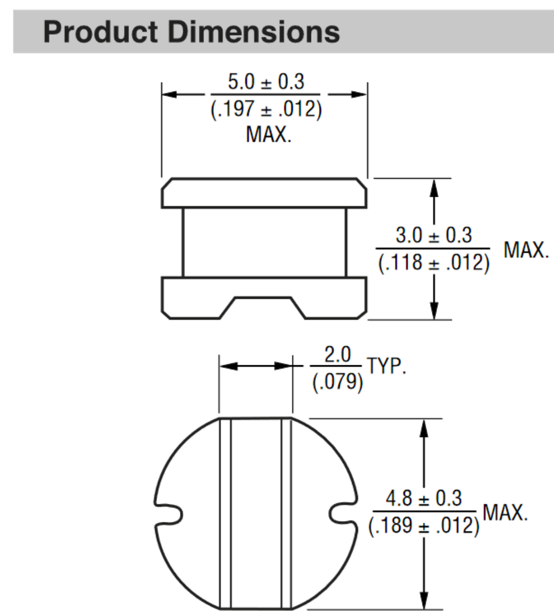
$$L = \frac{\mu_0 N^2 A}{\ell}$$

$$\sqrt{\frac{L_2}{L_1}} = \frac{N_2}{N_1}$$

(sometimes called the “turns ratio”)

Real Inductors

- In the earlier example we used $L = 5\ \mathrm{mH}$
- What does a 5 mH inductor look like?
- Example: Bournes SDR0503-502JL



Real Inductors

Features

- Available in E12 series
- Low profile of only 3.3 mm
- High inductance of 15 mH
- RoHS compliant*

Applications

- Input/output of DC/DC converters
- Power supplies for:
  - Portable communications equipment
  - Camcorders
  - LCD TVs

SDR0503 Series - SMD Power Inductors

Electrical Specifications

Bourns Part No.	Inductance 1kHz (µH)	Inductance 1kHz Tol. %	Q Ref.	Test Frequency (MHz)	SRF Min. (MHz)	RDC Max. (Ω)	I rms Max. (A)	I sat Typ. (A)
SDR0503-100ML	10	± 20	10	2.52	30.0	0.13	1.300	1.600
SDR0503-120ML	12	± 20	20	2.52	29.0	0.16	1.200	1.450
SDR0503-150ML	15	± 20	20	2.52	27.0	0.19	1.050	1.260
SDR0503-180ML	18	± 20	20	2.52	24.0	0.21	0.950	1.300
SDR0503-220ML	22	± 20	20	2.52	22.0	0.28	0.900	1.060
SDR0503-270ML	27	± 20	20	2.52	20.0	0.32	0.800	1.000
SDR0503-330KL	33	± 10	15	2.52	18.0	0.38	0.700	0.850
SDR0503-390KL	39	± 10	15	2.52	17.0	0.42	0.650	0.800
SDR0503-470KL	47	± 10	20	2.52	14.0	0.60	0.600	0.750
SDR0503-560KL	56	± 10	20	2.52	13.0	0.71	0.500	0.700
SDR0503-680KL	68	± 10	20	2.52	12.0	0.76	0.450	0.600
SDR0503-820KL	82	± 10	15	2.52	10.0	0.88	0.420	0.520
SDR0503-101KL	100	± 10	40	0.796	9.0	1.60	0.400	0.480

General Specifications

Test Voltage .... 1 V
Reflow Soldering .... 230 °C, 50 sec. max.
Operating Temperature ...-40 °C to +125 °C (Temperature rise included)
Storage Temperature... -40 °C to +125 °C
Resistance to Soldering Heat ....260 °C for 5 sec.
Moisture Sensitivity Level.......................1
ESD Classification (HBM)..................N/A

Materials

Core.. Ferrite DR
Wire ....Enameled copper
Terminal....................................Ag/Ni/Sn
Rated Current..Ind. drop 10 % typ. at Isat
Temp. Rise.........40 °C max. at rated Irms
Packaging...................... 500 pcs. per reel

SDR0503-502JL	5000	± 5	40	0.252	1.0	60.00	0.039	0.050

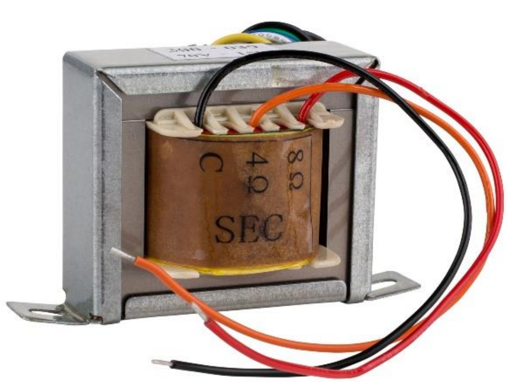
Real Inductors

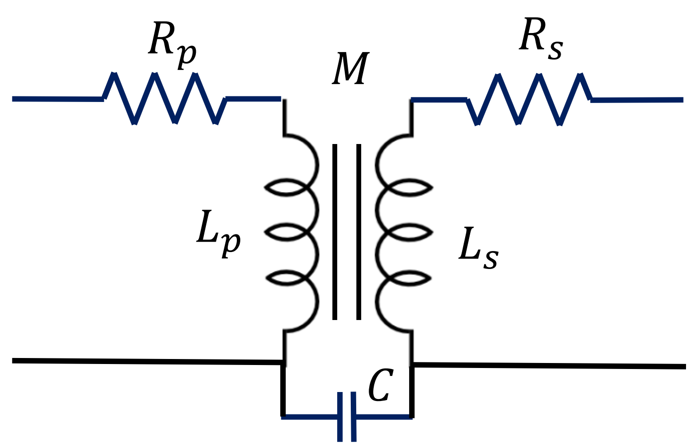
- Real inductors dissipate energy in the core material.
  - A "high-Q" inductor dissipates less energy than a "low-Q" inductor
- Real inductors are made with a coil if wire that is usually very thin and has its own resistance.
  - RDC is the DC resistance of the inductor
  - In this example it is 60 Ω.
- If you put too much current through the inductor it will dissipate too much heat and start to smoke…
  - In this example the maximum current is 39 mA
- The core material will saturate with a maximum magnetic field strength
  - In this case the core saturates at 50 mA

Real Inductors

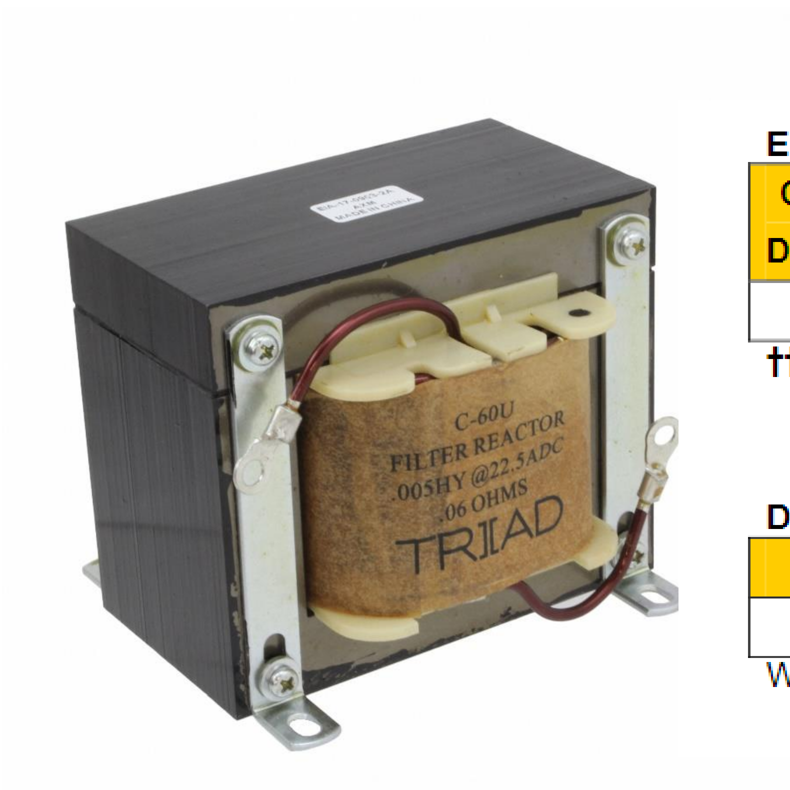
- A model for a real inductor can be constructed from ideal circuit elements:

- A model for real coupled inductors:





Real Inductors



Electrical Specifications (@25C):

Current	Inductance ††	Resistance
DC Amps	Henries	Ohms
22.5	0.005	0.06

†† = Inductance tolerance -20%, + 50 %

Dimensions: Unit: In inches

A	B	C	D	E
3.750	4.50	4.187	3.750	3.50

Weight: 12.75 lbs.

Real Inductors

This equipment might need to handle 500 kV and thousands of amperes.