PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 53600

Electronics Techniques for Research

Now in PowerPoint!

Spring 2020 Semester

Prof. Matthew Jones

More Information

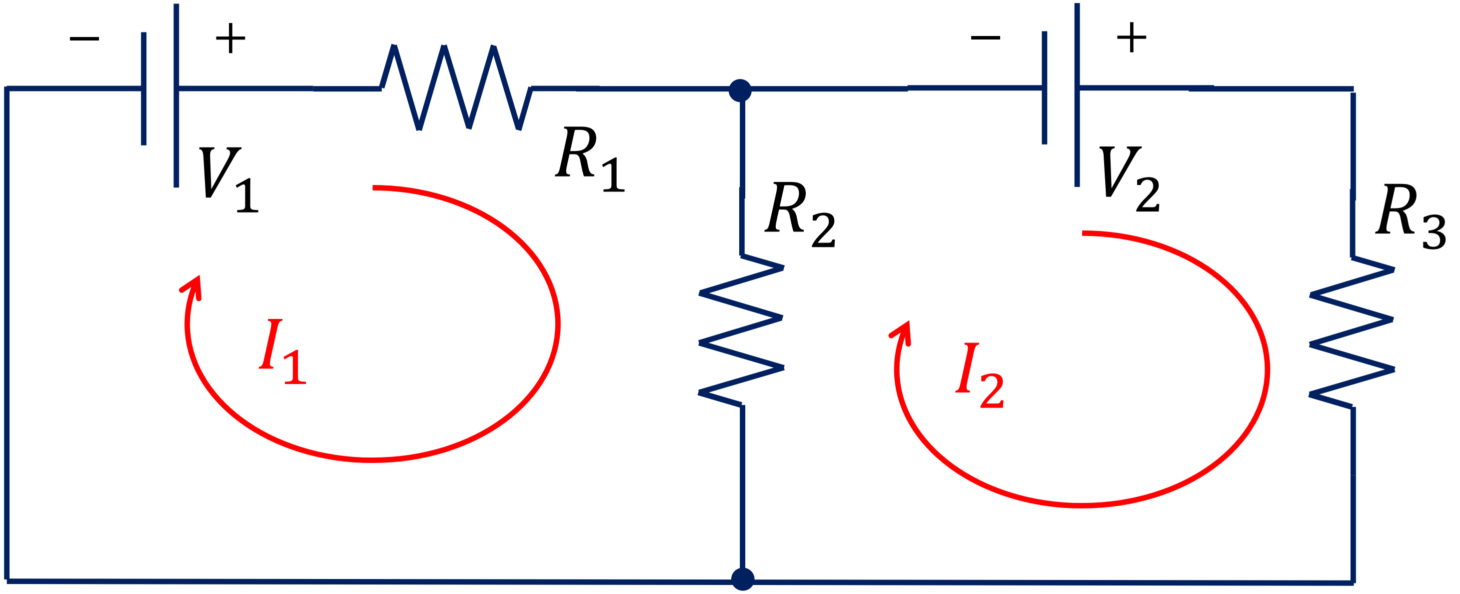
- Course web page, whereat can be found the syllabus:
  http://www.physics.purdue.edu/~mjones/phys53600_Spring2020
- Information about the lab is posted at the URL:

http://www.physics.purdue.edu/~mjones/phys53600_Spring2020/lab

- Please look at the introductory material before the lab and in the future, review the instructions before your lab period.
- This week will be a bit informal since I'm still figuring out how this all will work…

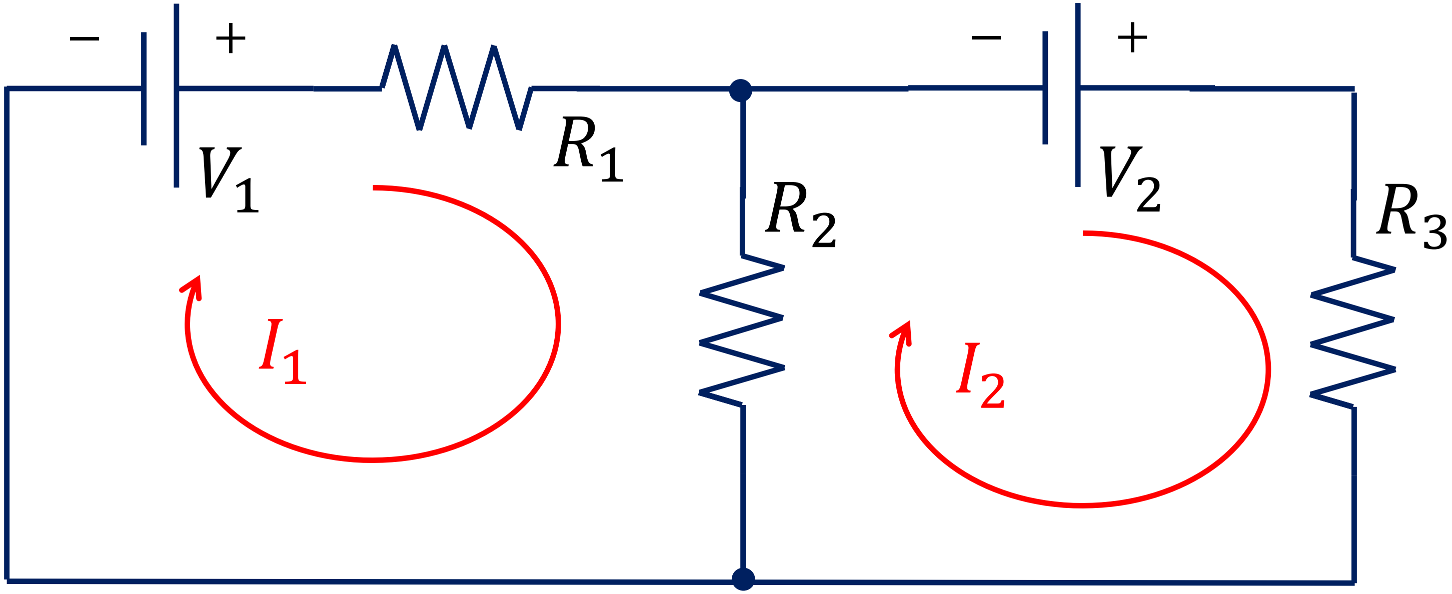
Two Loops

- Step 1: Assign currents to each loop



Two Loops

- Step 2: Apply Kirchhoff's Loop rule



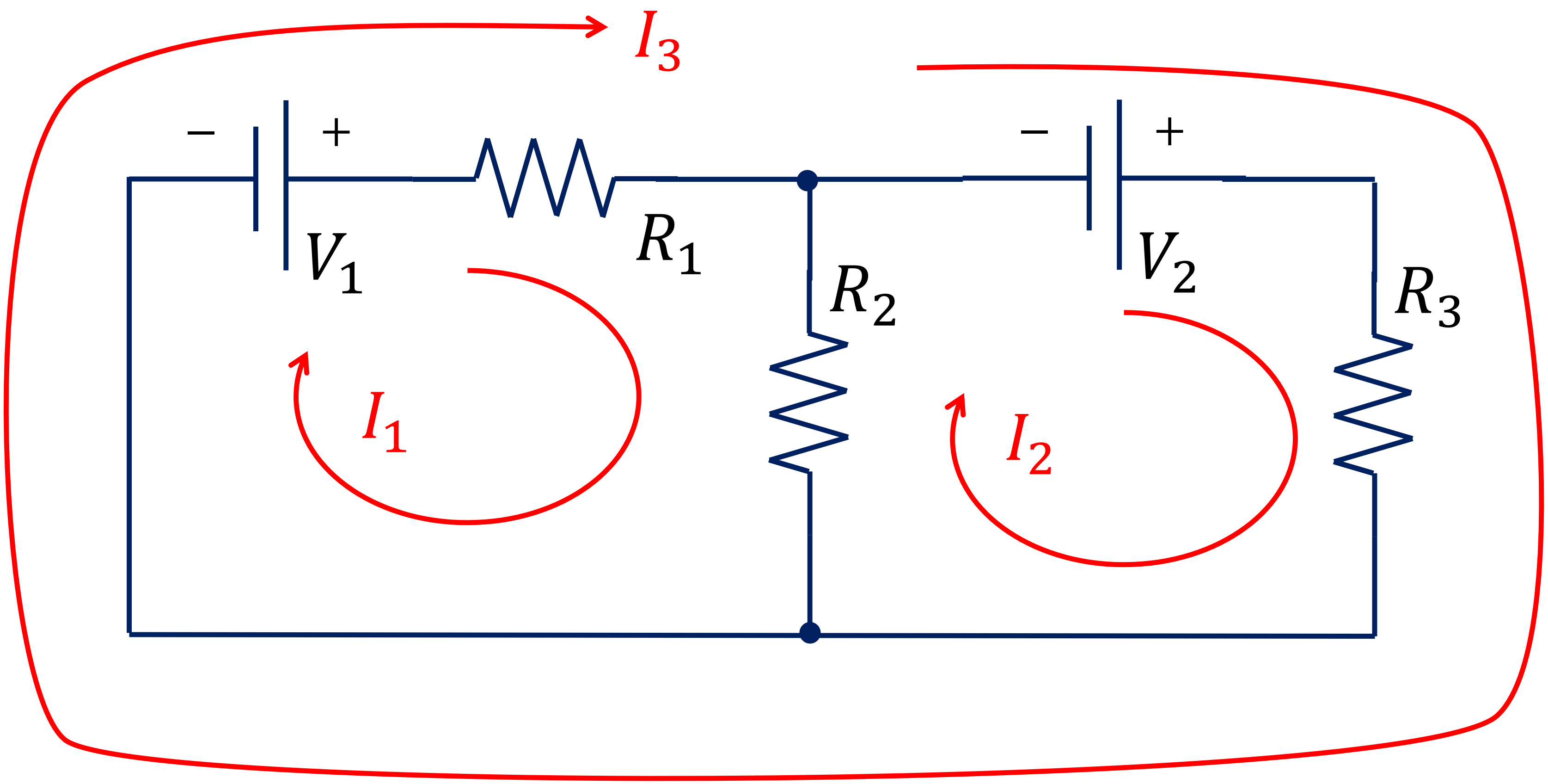
$$V_1 - I_1 R_1 - (I_1 - I_2) R_2 = 0$$
$$V_2 - I_2 R_3 - (I_2 - I_1) R_2 = 0$$

Two Loops

- Interesting questions:
  - Is there only one way to draw the loops?
  - What if we draw them differently?
  - Do we get the same answer?
  - What if we draw more than two loops?
- This provides a good example for applying Kirchoff’s rules

Three Loops?

- Step 1: Assign currents to each loop



Three Loops?

- Step 2: Apply Kirchhoff's Loop rule

$$V_1 - (I_1 + I_3)R_1 - (I_1 - I_2)R_2 = 0$$
$$V_2 - (I_2 + I_3)R_3 - (I_2 - I_1)R_2 = 0$$
$$V_1 - (I_1 + I_3)R_1 + V_2 - (I_2 + I_3)R_3 = 0$$

- Notice that these three equations are not linearly independent.
  - The last equation is the sum of the first two equations.

Three Loops?

- Write this as a matrix equation:

$$\begin{pmatrix} R_1 + R_2 & -R_2 & R_1 \\ -R_2 & R_2 + R_3 & R_3 \\ R_1 & R_3 & R_1 + R_3 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \\ V_1 + V_2 \end{pmatrix}$$

- Determinant:

$$(R_1 + R_2)\Big((R_2 + R_3)(R_1 + R_3) - R_3^2\Big)$$
$$- R_2(R_2(R_1 + R_3) + R_1 R_3)$$
$$- R_1\big(R_2 R_3 + R_1(R_2 + R_3)\big) = 0$$

- I_3 can be expressed as a function of I_1 and I_2

Three Loops?

- Change of variables in the first two equations:

$$I_1' = I_1 + I_3$$
$$I_2' = I_2 + I_3$$

- Then,

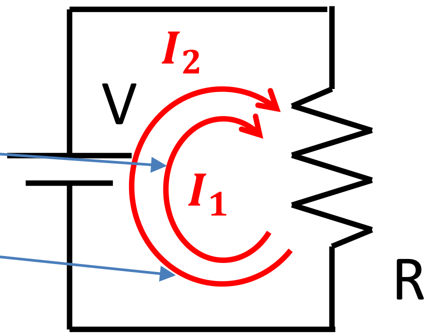
$$V_1 - I_1'R_1 - (I_1' - I_2')R_2 = 0$$
$$V_2 - I_2'R_3 - (I_2' - I_1')R_2 = 0$$

- The point is that we need to generate a complete set of linearly independent equations

Very Simple Example

- Back to one loop:
- We could draw the same loop twice
- Kirchhoff's rules:

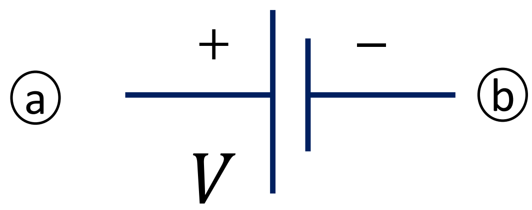
$$V - (I_1 + I_2)R = 0$$
$$V - (I_1 + I_2)R = 0$$



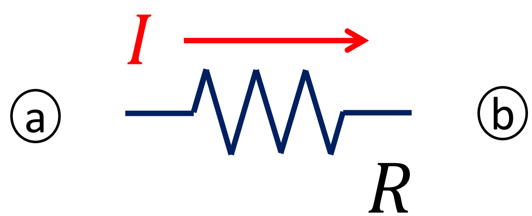
- There is not enough information to solve for I_1 and I_2 independently.
- Too many unknowns, not enough linearly independent equations.

Resistors and Capacitors

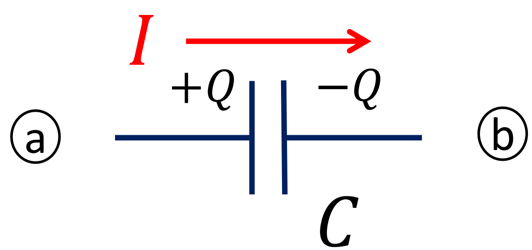
- Potential differences:



$$V_b = V_a - V$$



$$V_b = V_a - IR$$



$$V_b = V_a - \frac{Q}{C}$$

The electric potential at (a) is greater than the electric potential at (b).

Charge on Capacitors

- The charge on a capacitor is the integrated current:

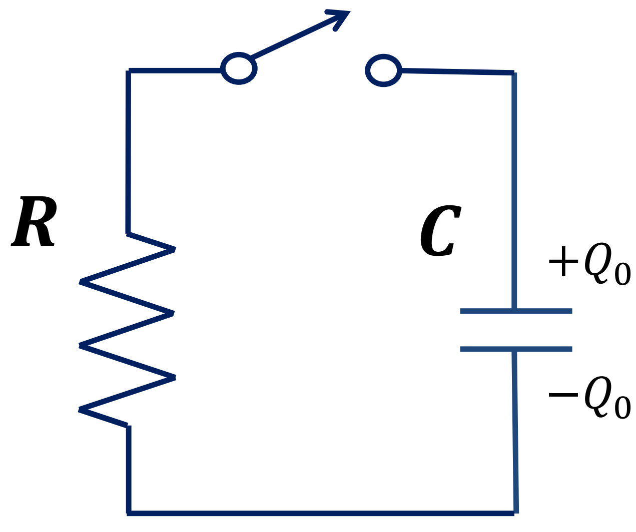
$$Q(t) = Q_0 + \int_0^t I(\tau)d\tau$$

$$I(t) = \frac{dQ}{dt}$$

- We still need to solve for $I(t)$ but now it is the solution to an integral equation.
- It will be convenient to transform it into a differential equation
- But don't forget about the initial conditions!

Very Simple Circuit

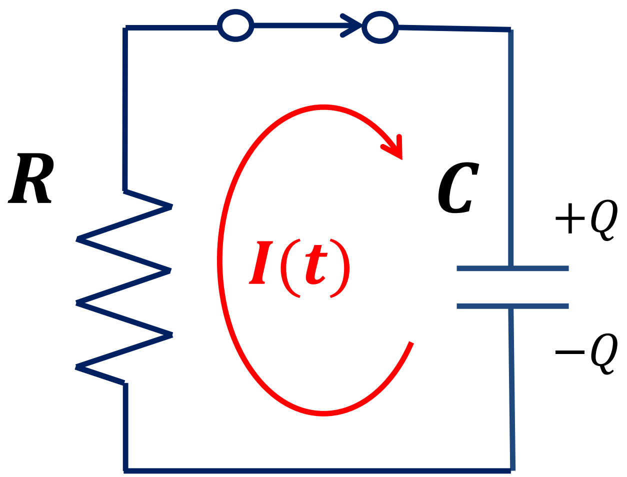
- The switch is closed at $t = 0$ allowing the current to flow in the circuit.



RC Circuits

- Kirchhoff's Loop Rule:

$$-I(t)\,R \;-\; \frac{Q(t)}{C} = 0$$



- Differentiate…

$$R\frac{dI}{dt} + \frac{1}{C}I(t) = 0$$

$$\frac{dI}{dt} = -\frac{I(t)}{RC}$$

This reminds you that I is not constant… it is a function of t.

- This is a differential equation…

RC Circuits

- You should know how to solve this differential equation.

$$\frac{dI}{I} = -\frac{dt}{RC}$$

$$\log\left(\frac{I(t)}{I_0}\right) = -\frac{t}{RC}$$

$$I(t) = I_0 e^{-t/RC}$$

- Now we must apply the initial conditions.
- At $t = 0$, Kirchhoff's rules give:

$$-I_0\, R\ - \frac{Q_0}{C} = 0$$

$$I_0 = -\frac{Q_0}{RC}$$

Notice that this is negative!

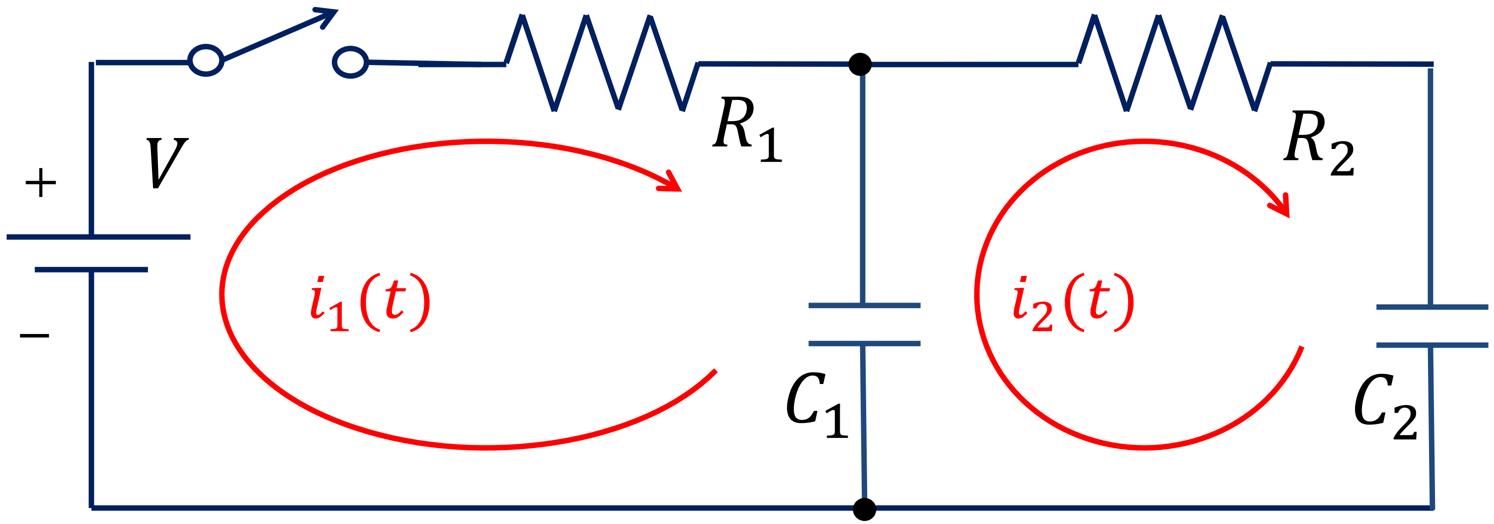
RC Circuits

- Complete solution:

$$I(t) = -\frac{Q_0}{RC} e^{-t/RC}$$

- The current flows opposite the assumed direction
  – We drew a clockwise loop of current
  – The solution is always negative
  – The current actually will flow anti-clockwise

Another Example



$$V - i_1 R_1 - \frac{1}{C_1}\int_0^t \big(i_1(\tau) - i_2(\tau)\big)d\tau = 0$$

$$-i_2 R_2 - \frac{1}{C_2}\int_0^t i_2(\tau)d\tau - \frac{1}{C_1}\int_0^t \big(i_2(\tau) - i_1(\tau)\big)d\tau = 0$$

Initial conditions:

Assume both capacitors are discharged at t=0.

Another Example

- Differentiate the system of equations:

$$-R_1 \frac{di_1}{dt} - \frac{1}{C_1}(i_1 - i_2) = 0$$

$$-R_2 \frac{di_2}{dt} - \frac{1}{C_2} i_2 - \frac{1}{C_1}(i_2 - i_1) = 0$$

- Assume that the solutions are of the form

$$i_1(t) = I_1 e^{-\gamma t}$$

$$i_2(t) = I_2 e^{-\gamma t}$$

Another Example

$$\gamma i_1(t) - \frac{i_1(t)}{C_1R_1} + \frac{i_2(t)}{C_1R_1} = 0$$

$$\gamma i_2(t) - \frac{i_2(t)}{C_2R_2} - \frac{i_2(t)}{C_1R_2} + \frac{i_1(t)}{C_1R_2} = 0$$

$$\begin{pmatrix} \gamma - \dfrac{1}{C_1R_1} & \dfrac{1}{C_1R_1} \\ \dfrac{1}{C_1R_2} & \gamma - \dfrac{1}{C_2R_2} - \dfrac{1}{C_1R_2} \end{pmatrix} \begin{pmatrix} i_1(t) \\ i_2(t) \end{pmatrix} = 0$$

Another Example

- Non-trivial solutions exist when

$$\det\begin{pmatrix} \gamma - \frac{1}{C_1R_1} & \frac{1}{C_1R_1} \\ \frac{1}{C_1R_2} & \gamma - \frac{1}{C_2R_2} - \frac{1}{C_1R_2} \end{pmatrix} = 0$$

$$\gamma^2 - \gamma\left(\frac{1}{C_1R_2} + \frac{1}{C_1R_1} + \frac{1}{C_2R_2}\right) - \left(\frac{1}{C_1R_1}\right)\left(\frac{1}{C_2R_2}\right) = 0$$

- The roots to this quadratic equation are real and we label them γ_1 and γ_2.
- Eigenvectors are labeled $\boldsymbol{I}_1$ and $\boldsymbol{I}_2$.

Another Example

- The general solution can be written
$$\boldsymbol{i}(t) = A\boldsymbol{I}_1 e^{-\gamma_1 t} + B\boldsymbol{I}_2 e^{-\gamma_2 t}$$
- At time $t = 0$,
$$i_1(0) = \frac{V}{R_1}$$
$$i_2(0) = 0$$
- This is sufficient information to solve the system of differential equations.
- But the algebra gets tedious…

Numerical Methods

- If we assigned numerical values to each component, then we could solve this numerically:

$$\Delta i_1 = -\frac{i_1 - i_2}{R_1 C_1}\Delta t$$

$$\Delta i_2 = \left(-\frac{i_2}{R_2 C_2} - \frac{i_2 - i_1}{R_2 C_1}\right)\Delta t$$

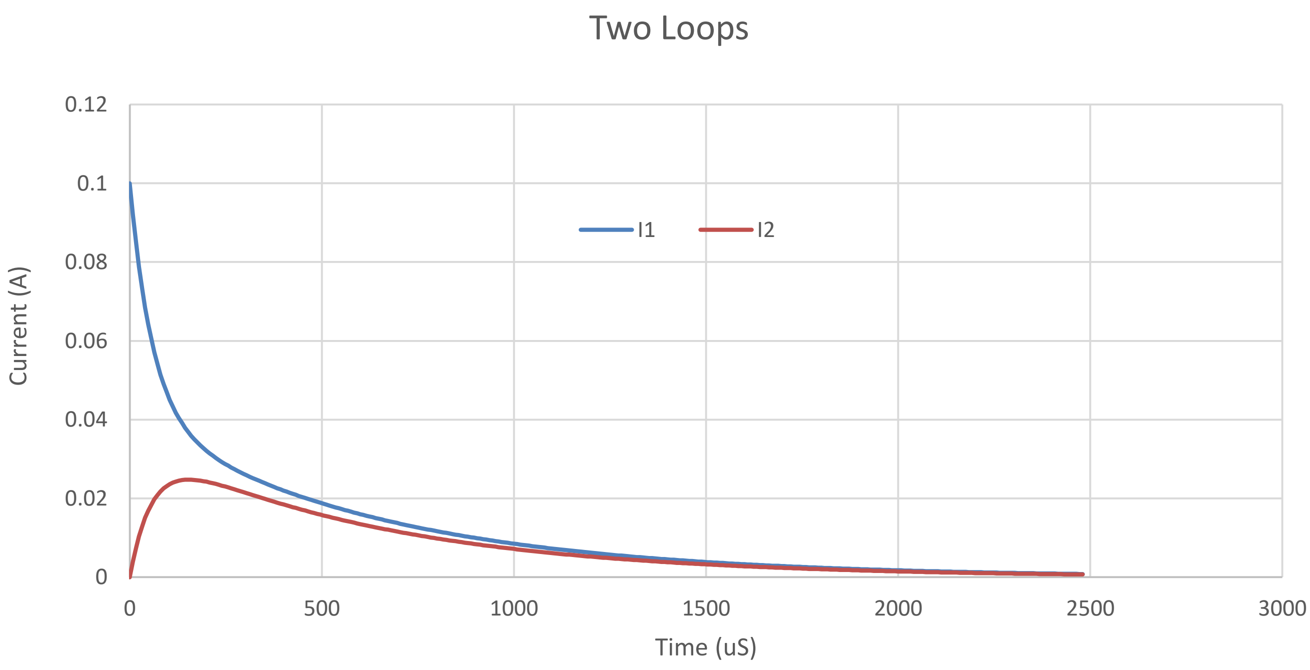
- Initial conditions:

$$i_1(0) = \frac{V}{R_1}$$

$$i_2(0) = 0$$

Numerical Methods

$$V = 10\ V$$
$$R_1 = 100\ \Omega$$
$$R_2 = 200\ \Omega$$
$$C_1 = 1\ \mu F$$
$$C_2 = 2\ \mu F$$

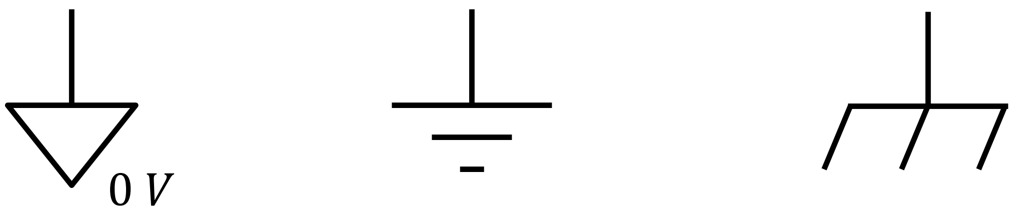


Ground

- So far we have calculated potential differences across circuit elements
  - The absolute potential of the circuit is irrelevant
- Nevertheless, it is extremely convenient to assign the potential at ONE point in the circuit
  - Usually to a potential of zero volts
- All potential differences are then relative to this fixed reference potential
- We can refer to these potential differences as the voltage at any point in the circuit

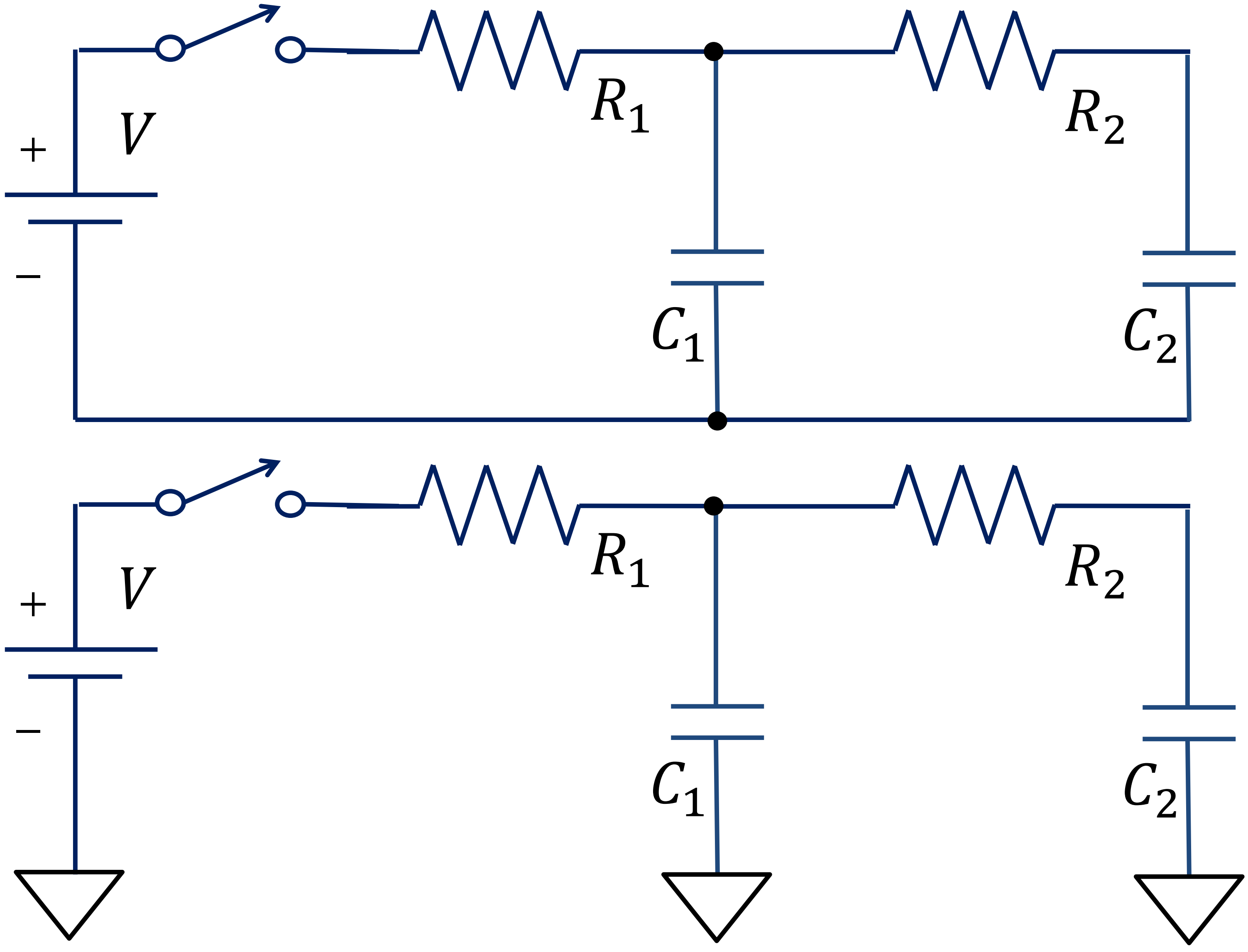
Ground

- We label the point in the circuit that is at ground potential (zero volts) with a special symbol:



- Sometimes we also label the ground potential
- Everything attached to a ground symbol is at the same electrical potential (typically zero volts)
- There is no need to join everything at zero volts by a single wire

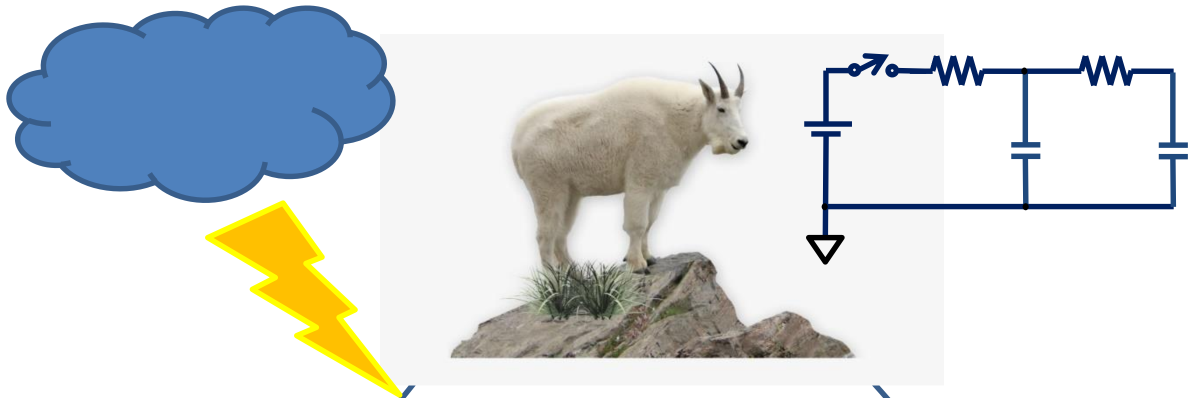
Ground

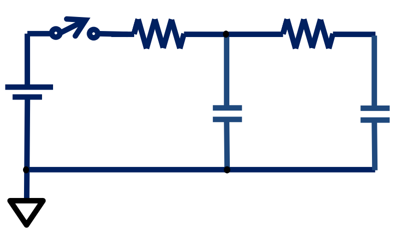


Ground

- Most numerical techniques store voltages at each node in the circuit relative to ground.
- Each circuit must define a ground reference somewhere.
- But be careful! Ground potential in one circuit might not be the same electrical potential as the ground potential in another circuit.
  - If the two grounds were connected by a wire, would a current flow?

Ground





- The points that are labeled as “ground” in each circuit are probably not at the same electrical potential.
- Each circuit might operate as expected in isolation but connecting them by a wire might be extremely unsafe!