Frequency of Response of RC Circuits

A. Introduction

The response of an electronic circuit depends on the frequency of the input signal. Although there are many variations, two circuits are primarily responsible for amplitude change as a function of signal frequency, one for high frequencies and another for low frequencies. The filter circuit that attenuates low frequencies is called a High Pass Filter and the circuit that attenuates high frequencies is called a Los Pass Filter. The frequency response (gain) and phase as a function of frequency, or ω, can be viewed graphically in a Bode plot. A typical Bode plot plots gain on the ordinate and frequency on the abscissa. Both axes are generally logarithmic. Likewise the phase angle can be plotted on the y-axis (linear) and the frequency on the x-axis.

1. *Low-frequency attenuation - High Pass RC filters:* The attenuation is determined by the following frequency dependent function.

$$Gain = v_0 / v_i = F_L = \left[1 + (f_b / f)^2 \right]^{-1/2} \qquad (1.1)$$

$F_L = f / f_b$ when $f \ll f_b$

$F_L = .707$ when $f = f_b$

$F_L = 1$ when $f \gg f_b$

The break frequency, f_b, is given by

$$f_b = 1/(2\pi RC) \qquad (1.2)$$

The output voltage, v_o, leads the input voltage, v_i, by the phase angle ϕ

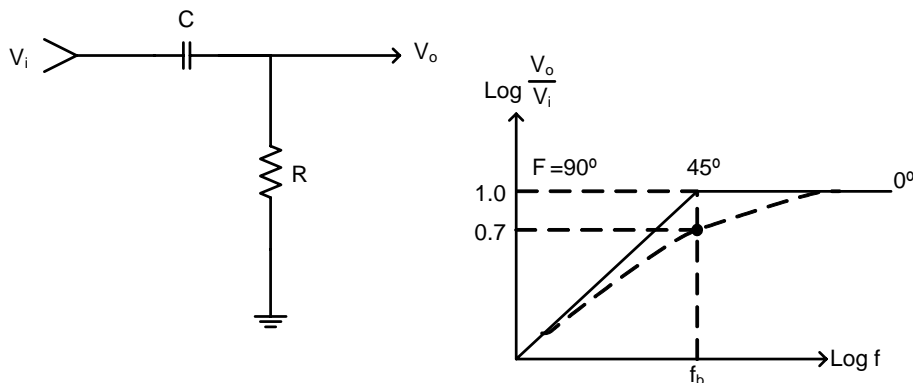$$\phi = \tan^{-1} (f_b / f) \qquad (1.3)$$

$\phi = +90°$ when $f \ll f_b$

$\phi = +45°$ when $f = f_b$

$\phi = +0°$ when $f \gg f_b$

Figure 1

2. *High-frequency attenuation - Low Pass RC filters*: High-frequency attenuation is similar to low-frequency attenuation, but f / f_b is inverted relative to the low-frequency function. The response can be derived in the similar way as for High Pass filters.

$$v_0 / v_i = F_H = \left[1 + (f / f_b)^2 \right]^{-1/2} \tag{1.4}$$

$F_H = 1$ when $f \ll f_b$
$F_H = 0.707$ when $f = f_b$
$F_H = f_b/f$ when $f \gg f_b$

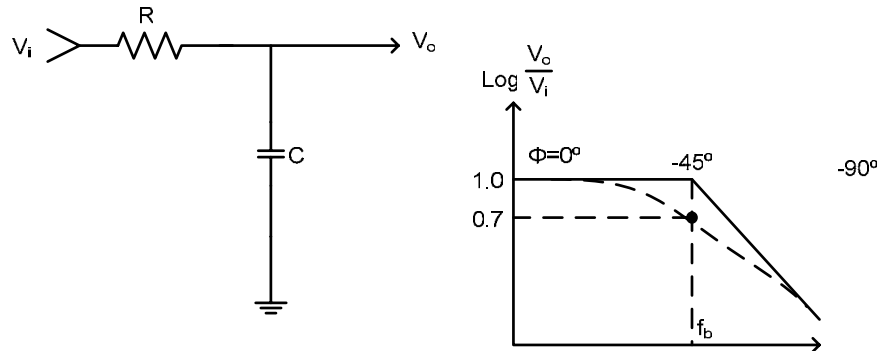The high break-frequency is defined in the same way as that for low frequency filters. The phase of v_o lags v_i.

$$\phi = -\tan^{-1}(f / f_b) \tag{1.5}$$

$\phi = 0°$ when $f \ll f_b$
$\phi = -45°$ when $f = f_b$
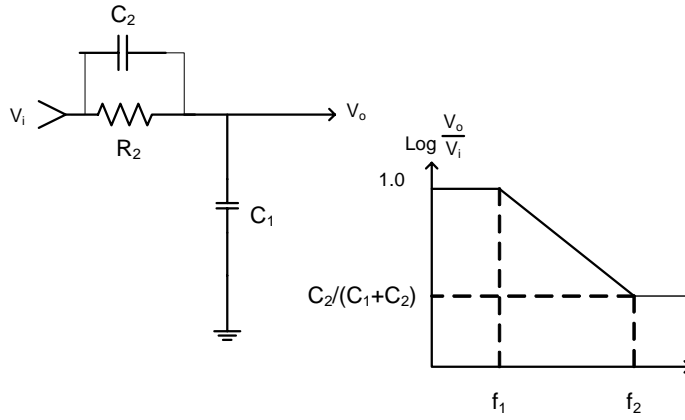$\phi = -90°$ when $f \gg f_b$

Figure 2

3. *Opposite Break Circuit*: This is an opposite break circuit, with two break frequencies. It attenuates at high frequency but not at low frequency. It is used to prevent oscillation in negative feedback circuits.

Figure 3

The break points are given by

$$f_1 = \frac{1}{2\pi R_2 (C_1 + C_2)} \tag{1.6}$$

and

$$f_2 = \frac{1}{2\pi R_2 C_2} \tag{1.7}$$

The signal is not affected when its frequency is below the first break, f_1. The attenuation above, f_2 is given by

$$\frac{v_o}{v_i} = \frac{C_2}{C_1 + C_2} \tag{1.8}$$

The attenuation in the transition region is given by

$$\frac{v_o}{v_1} = \frac{\sqrt{1 + (f / f_2)^2}}{\sqrt{1 + (f / f_1)^2}}. \tag{1.9}$$

Using eqn. (1.9) it can be shown that

$$v_o(f = f_1) = \frac{v_i}{\sqrt{2}} \tag{1.10}$$

and

$$v_o(f = f_2) = v_i \sqrt{2} \frac{C_2}{C_1 + C_2} \tag{1.11}$$

This circuit is used to reduce the amplitude of a signal at high frequency without causing high phase shift the maximum phase shift occurs at

$$f = \sqrt{f_1 f_2} \tag{1.12}$$

and is given by

$$\phi(\text{max}) = -\text{Arctan}^{-1}\sqrt{f_2 / f_1} + \tan^{-1}\sqrt{f_1 / f_2} \tag{1.13}$$

4. Clipping: The pulse response of many common electronic circuits can be understood using two principles:

a) The time dependent term in al *R-C* changing problems has the form $e^{\frac{-t}{RC}}$.

b) The voltage across a capacitor cannot change quickly when there is a resistor that restricts the flow of charge to the capacitor.

These two principles are helpful in understanding the circuit given below.

Figure 4
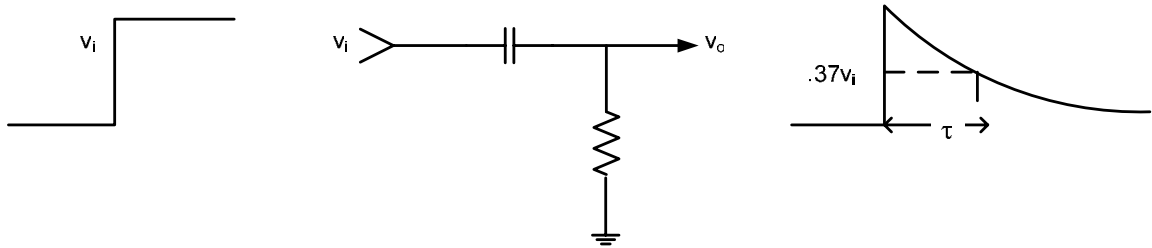
A step pulse is shortened by a clipping circuit.
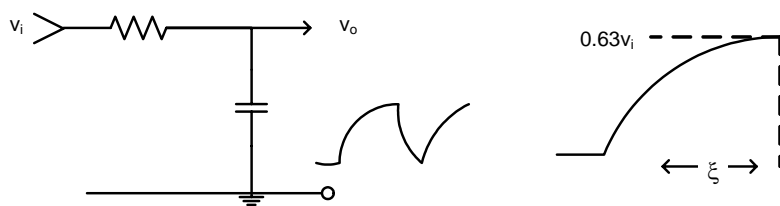
$$v_0 = v_i e^{-t/RC} \tag{1.14}$$

A time constant for this circuit is given by the product of *R* and *C*

$$\tau = RC \tag{1.15}$$

4. Integration: This circuit can be used to delay the peak of a square wave signal. The time required for a pulse to reach its maximum value is indicated by the time constant. This simple circuit is employed in a variety of different applications.
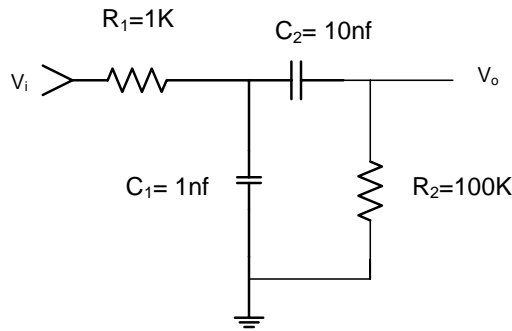
Figure 5

The output voltage is given by

$$v_0 = v_i(1 - e^{-t/RC}) \tag{1.16}$$

B. High-Break and Low-Break Bode Plot

Figure 6

Many practical circuits have a high-break and a low-break. The general pattern can be illustrated with the preceding circuit. The two time constants have been selected so that the breaks are far apart. Also, $R_1 \ll R_2$ so the attenuation between the breaks is minor.

$$f_b(\text{low}) = \left[2\pi(R_1 + R_2)C_2\right]^{-1}$$
$$f_b(\text{high}) = \left[2\pi(R_1, R_2)C_1\right]^{-1}$$

1) Calculate the two break frequencies using the given values. Measure the two break frequencies by finding the points where $v_0 / v_i = 0.7$. Section 5.6 of GIL provides further instructions if needed. Although any amplitude input signal can be used a good recommended signal is 10 V (p-p). The measured break points may be different than those calculated. Why? Measure the attenuation near the low-and high-frequency breaks at f / f_b =0.1, 0.5, 1,2, and 10. Record the values and then enter the data in a spread sheet program. Plot the values using logarithmic axes, *i.e.*make the Bode plot. Before you leave this section you could plot some values to verify that the measurements are correct. Add the expected attenuation. Use a dual trace to compare v_i and v_0, observing the phase shift. Do not record values for the phase shift but include a sentence or two in your lab report describing the phase shift

C. Common Variations

The attenuations in this section have a frequency <u>independent</u> term. Measure f_b as follows: adjust the input amplitude so that V_o has a convenient amplitude on the scope at a frequency far from the break frequency, *i.e.* where $F = 1$, then change the frequency until you find the point where v_0 drops by 30%. It is not necessary to measure the phase shift.

Figure 7

C = 10nf
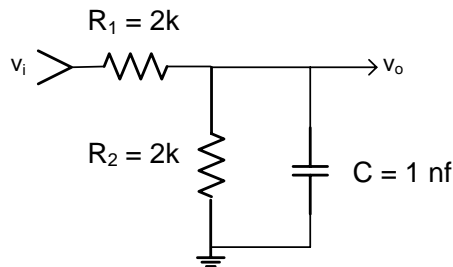

V_i

$R_1 = 2k$

$R_2 = 2k$

V_o
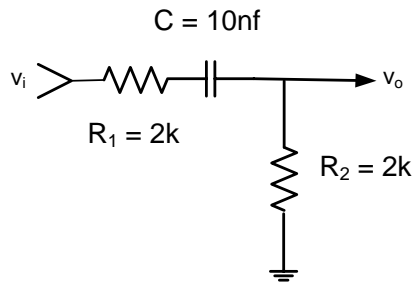
2) Calculate the break frequency for the circuit shown in Fig. 7. Show your work in the lab report. Measure the gain of this circuit for $f = 0.1\ f_b$ and $10\ f_b$. Describe the effect of this circuit on the input signal.

$$V_o / V_i = \frac{R_2}{R_1 + R_2} F_L$$

$$\tau = (R_1 + R_2)C$$

Figure 8

$R_1 = 2k$
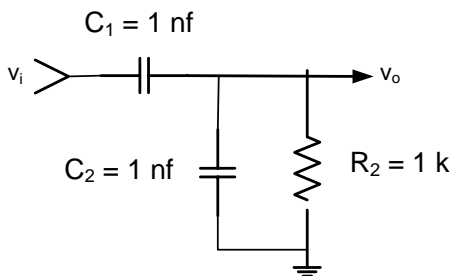

V_i

V_o

$R_2 = 2k$

$C = 1\ nf$

3) Calculate the break frequency for the circuit shown in Fig. 8. Show your work in the lab report. Measure the gain of this circuit for $f = 0.1\ f_b$ and $10\ f_b$. Describe the effect of this circuit on the input signal.

$$V_o / V_i = \frac{R_2}{R_1 + R_2} F_H$$

$$\tau = (R_1 \| R_2)C$$

Figure 9

$C_1 = 1\ nf$

V_i

V_o

$C_2 = 1\ nf$

$R_2 = 1\ k$

4) Calculate the break frequency for the circuit shown in Fig. 9. Show your work in the lab report. Measure the gain of this circuit for $f = 0.1\ f_b$ and $10\ f_b$. Describe the effect of this circuit on the input signal.
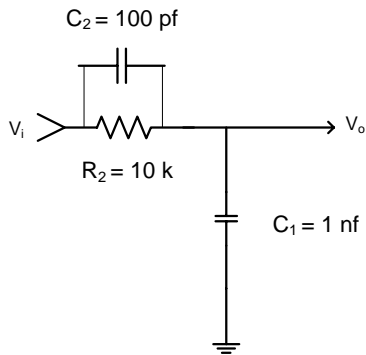
$$V_o / V_i = \frac{C_1}{C_1 + C_2} F_L$$

$$\tau = (C_1 + C_2)R$$

D. Opposite Break Circuit to Reduce Signal Amplitude at High Frequency
This circuit is used to attenuate a signal at high frequency without causing large phase shift.

Figure 10

$C_2 = 100$ pf
$R_2 = 10$ k
$C_1 = 1$ nf
V_i
V_o

5) Calculate the two break frequencies for the circuit shown in Fig. 10. Show your work in the lab report. Measure the gain of this circuit for $f = 0.1 f_1$, f_1, f_2 and $10 f_2$. Calculate the maximum phase shift and the frequency at which it occurs. Measure the same quantities using a 10 V (p-p) signal.

E. Basic Pulse Circuits

In this section the change in a square wave produced by clipping and integration will be investigated. Use a 100 kHz square wave.

6) Observe v_0 using the oscilloscope for each circuit shown in Fig. 11 and measure the time constant. Sketch the observed waveform and include this in your lab report.

Figure 11

$C = 1$ nf
V_i
V_o
$R_2 = 1$ k

$R_1 = 1$ k
V_i
V_o
$C = 1$ nf
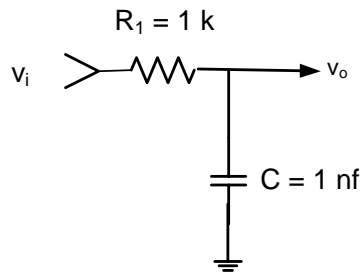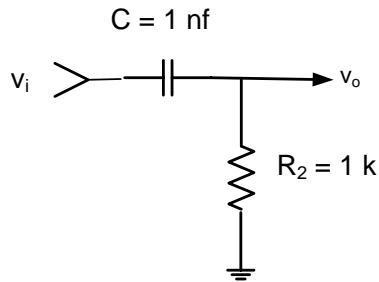
Components:
Initially get only the components that you need for the first part of the experiment, to avoid delays for other students.
Resistors: 100Ω, 1 K, 2 2 K, 24 K, 100K
Capacitors: 2 1nf, 10nf