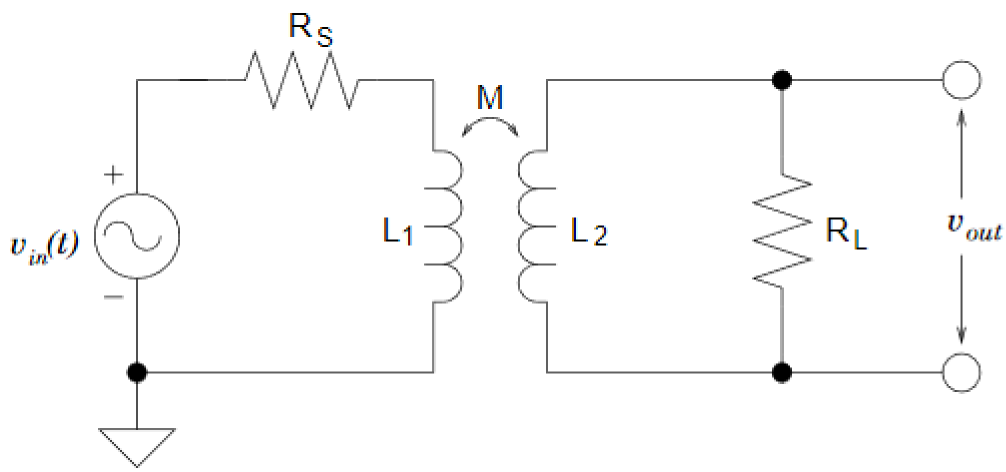
Physics 53600 – Assignment #2 – Due February 11, 2020

1. Consider the following transformer circuit:



(a) Show that when perfect coupling is assumed, so that $M = \sqrt{L_1 L_2}$ the output voltage is given by

$$V_{out} = \frac{j\omega M V_{in} R_L}{R_S R_L + j\omega(L_1 R_L + L_2 R_S)}$$

(b) Assuming that $L_1 \propto n_1^2$ and $L_2 \propto n_2^2$ and in the limit $R_S R_L \ll \omega(L_1 R_L + L_2 R_S)$, show that

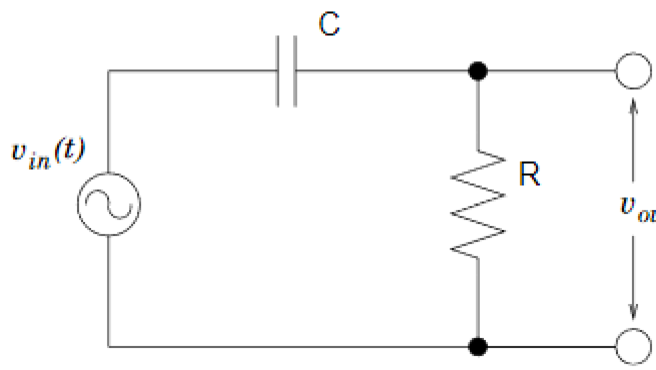
$$V_{out} = V\frac{n_2}{n_1}$$

where n_2/n_1 is the turns-ratio of the transformer.

(c) Show that the turns-ratio that maximizes the power dissipated by R_L is

$$\frac{n_2}{n_1} = \sqrt{\frac{R_L}{R_S}}$$

2. Consider the high-pass RC filter circuit:

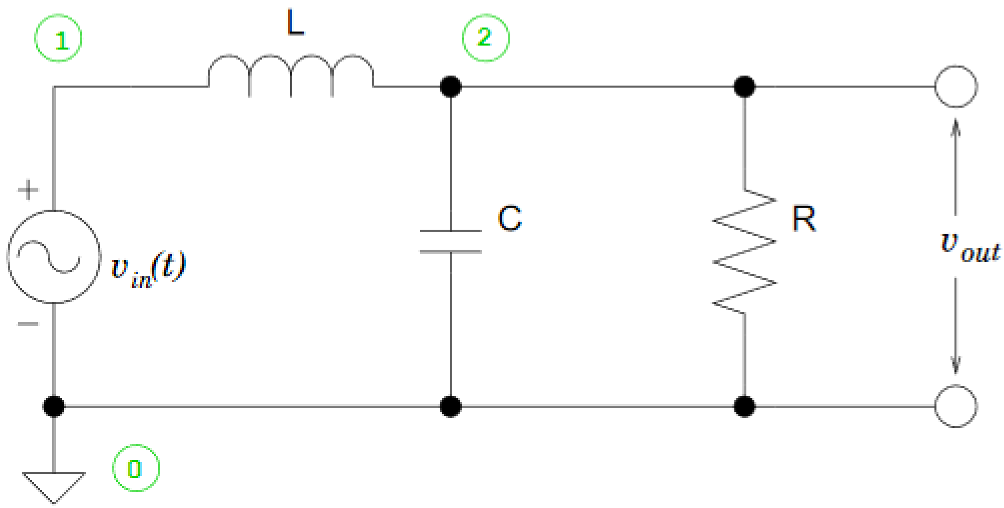


When $v_{in}(t) = V_{in}e^{j\omega t}$, the Thevenin equivalent circuit would consist of an ideal voltage source V_{Th} in series with an impedance Z_{Th}, both of which depend on the frequency, ω.

(a) With no additional load across the resistor, calculate V_{out}. How does V_{out} behave in the low-frequency ($\omega \ll 1/RC$) and high-frequency ($\omega \gg 1/RC$) limits?

(b) Calculalte the Thevenin equivalent impedance Z_{Th} for the circuit where the impedance of the resistor is R and the impedance of the capacitor is $j/\omega C$. How does Z_{Th} behave in the low-frequency and high-frequency limits?

3. Consider the following low-pass filter circuit:



(a) Assuming $v_{in}(t) = V_{in}e^{j\omega t}$ and $v_{out}(t) = V_{out}e^{j\omega t}$, solve for the magnitude of V_{out} in terms of V_{in}, R, L, and C.

(b) In principle, this circuit could have a resonance if R is too large. Assuming that R is large enough that it can be ignored, estimate the resonant frequency, ω_0.

(c) If this circuit is intended to form a low-pass filter, what value of R would be needed to provide a gain of -3 db at the frequency calculated in part (b)?

(d) At high frequencies, how many db per decade of frequency does this circuit attenuate? Compare this with the result for a first-order RC or RL low-pass filter.

(e) Select component values such that $f_0 = 2\pi\omega_0 = 1\ MHz$ and simulate the circuit using LtSPICE, plotting the gain as a function of frequency up to at least 100 MHz.