1. Current through voltage divider is

$$I = \frac{(V_{CC} - V_{EE})}{R_1 + R_2}$$

$$V_{out} = V_{CC} - IR_1 = \frac{V_{CC}(R_1 + R_2) - V_{CC}R_1 + V_{EE}R_1}{R_1 + R_2}$$

$$= \frac{V_{CC}R_2 + V_{EE}R_1}{R_1 + R_2}$$

$$V_{out}(R_1 + R_2) = V_{CC}R_2 + V_{EE}R_1$$

$$(V_{CC} - V_{out})R_2 = (V_{out} - V_{EE})R_1$$

The output impedance is the parallel combination of R_1 and R_2:

$$Z = \frac{R_1 R_2}{R_1 + R_2}$$

$$\frac{1}{Z} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_1} = \left(\frac{V_{out} - V_{EE}}{V_{CC} - V_{out}}\right)\frac{1}{R_2} = \frac{1}{Z} - \frac{1}{R_2}$$

So $$\frac{1}{Z} = \left(\frac{V_{out} - V_{EE} + V_{CC} - V_{out}}{V_{CC} - V_{out}}\right)\frac{1}{R_2}$$

$$Z = \left(\frac{V_{CC} - V_{out}}{V_{CC} - V_{EE}}\right)R_2$$

$$R_2 = Z\left(\frac{V_{CC} - V_{EE}}{V_{CC} - V_{out}}\right)$$

$$R_1 = \left(\frac{V_{CC} - V_{out}}{V_{out} - V_{EE}}\right)R_2 = Z\left(\frac{V_{CC} - V_{EE}}{V_{out} - V_{EE}}\right)$$

FORM C
APPROVED FOR USE IN
PURDUE UNIVERSITY

Thus, $R_1 = (1k\Omega)\left(\frac{5V - (-5V)}{0.5V - (-5V)}\right)$

$= (1k\Omega)\left(\frac{10V}{5.5V}\right)$

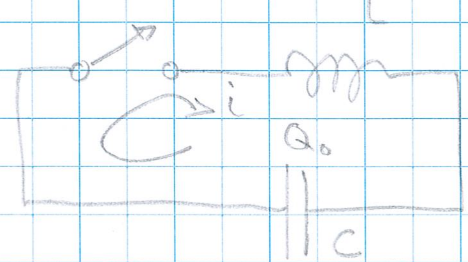
$= 1.82\ k\Omega$

$R_2 = (1k\Omega)\left(\frac{5V - (-5V)}{5V - 0.5V}\right)$

$= (1k\Omega)\left(\frac{10V}{4.5V}\right)$

$= 2.22\ k\Omega$

2.



$$-L\frac{di}{dt} - \frac{1}{C}\left(Q_0 + \int_0^t i(t)\,dt\right) = 0$$

$$\frac{d^2i}{dt^2} + \frac{1}{LC}\,i(t) = 0$$

$$\frac{d^2i}{dt^2} + \omega^2 i(t) = 0 \qquad \omega = \sqrt{\frac{1}{LC}}$$

$$i(t) = A\sin\omega t + B\cos\omega t.$$

At $t=0$, $i(t)=0$ so $B=0$.

Also, at $t=0$, $\frac{di}{dt} = A\omega\cos\omega t$.

$$-A\omega - \frac{Q_0}{LC} = 0.$$

So $A = -\frac{Q_0}{LC\omega} = -\frac{Q_0}{\sqrt{LC}}$

$$i(t) = -\frac{Q_0}{\sqrt{LC}}\sin\omega t$$

(a) Its a low pass filter.

3.

$$V_{in} - I_1 R - (I_1 - I_2) Z_c = 0$$

$$- I_2 R - I_2 Z_c - (I_2 - I_1) Z_c = 0$$

$$\begin{pmatrix} R + Z_c & -Z_c \\ -Z_c & R + 2Z_c \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} V_{in} \\ 0 \end{pmatrix}$$

$$I_2 = \frac{\begin{vmatrix} R + Z_c & V_{in} \\ -Z_c & 0 \end{vmatrix}}{\begin{vmatrix} R + Z_c & -Z_c \\ -Z_c & R + 2Z_c \end{vmatrix}}$$

$$= \frac{V_{in} Z_c}{(R + Z_c)(R + 2Z_c) - Z_c^2}$$

$$V_{out} = I_2 Z_c = \frac{V_{in} Z_c^2}{R^2 + 3RZ_c + Z_c^2}$$

$$Z_c = \frac{-i}{\omega C}$$

$$V_{out} = V_{in} \left(\frac{1}{1 - \omega^2 R^2 C^2 + 3i\omega RC} \right)$$

$$\left| \frac{V_{out}}{V_{in}} \right| = \frac{1}{\left((1 + \omega^2 R^2 C^2)^2 + 9\omega^2 R^2 C^2 \right)^{1/2}}$$

-3db at

$$\left| \frac{V_{out}}{V_{in}} \right| = 1/\sqrt{2}$$

$$\omega^4 R^4 C^4 + 11 \omega^2 R^2 C^2 + 1 = 2$$

$$\omega^2 = \frac{-11R^2C^2 \pm \sqrt{(11R^2C^2)^2 + R^4C^4}}{2R^4C^4} = \frac{1}{R^2C^2}\left(\frac{\sqrt{122} - 11}{2} \right)$$

As $\omega \to \infty$,

$$A = \left|\frac{V_{out}}{V_{in}}\right| \to \frac{1}{\omega^2 R^2 C^2}$$

$$G \to 20 \log_{10} A = -40 + \text{const.}$$

Gain falls off at 40 dB per decade.

4.

$$R + \frac{Z_0 + R}{3} = Z_0$$

$$3R + Z_0 + R = 3Z_0$$

$$4R = 2Z_0$$

$$R = \frac{1}{2} Z_0 .$$

In general, $R + \dfrac{Z_0 + R}{n} = Z_0$

$$nR + Z_0 + R = nZ_0$$

$$(n+1)R = (n-1)Z_0$$

$$R = Z_0 \left(\frac{n-1}{n+1} \right)$$

when $n = 2$, $R = \dfrac{Z_0}{3}$

when $n = 3$, $R = \dfrac{Z_0}{2}$.

$n = 4$, $R = \dfrac{3Z_0}{5}$