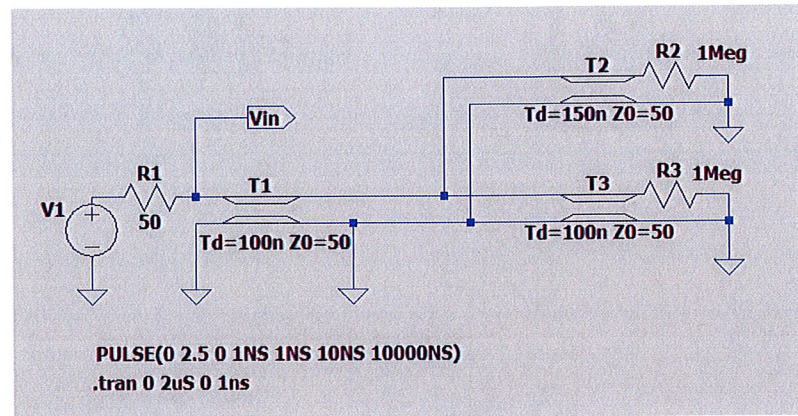


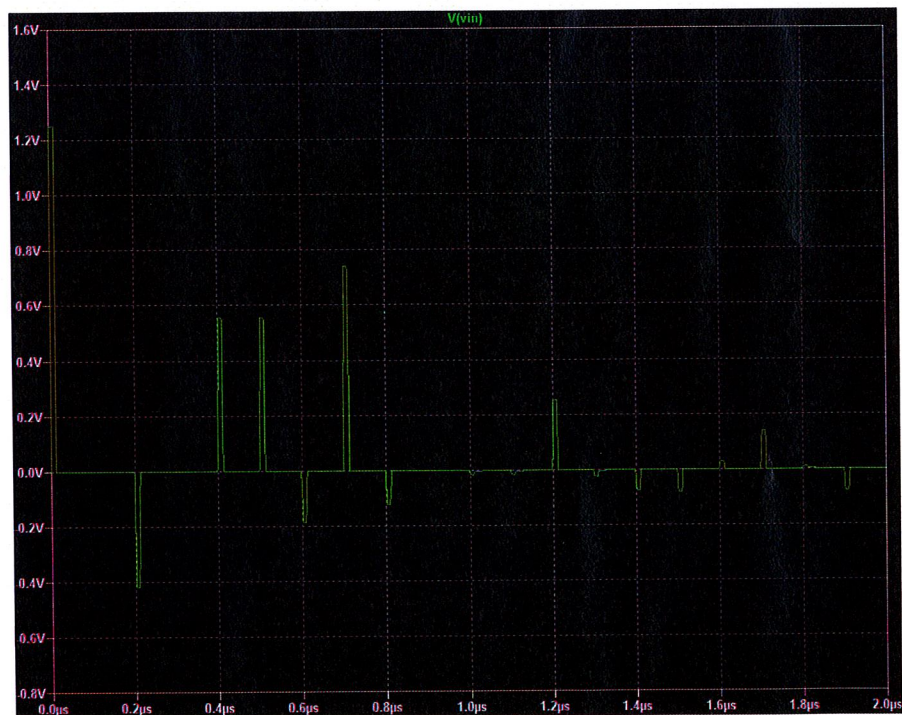
1

Physics 53600 – Assignment #3 Solutions

1. (a) This is the circuit consisting of lossless transmission lines, simulated with LTspice:

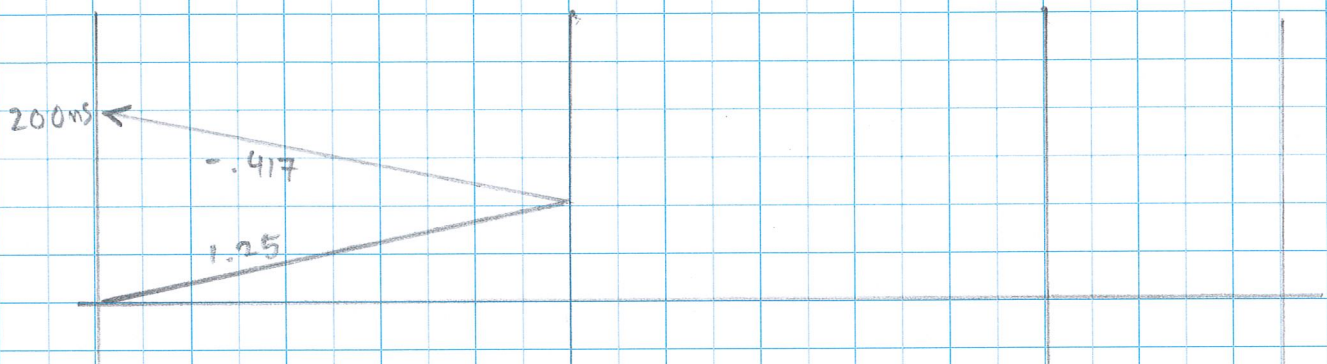


and this is the plot of the resulting waveform, probing the circuit at v_{in} :

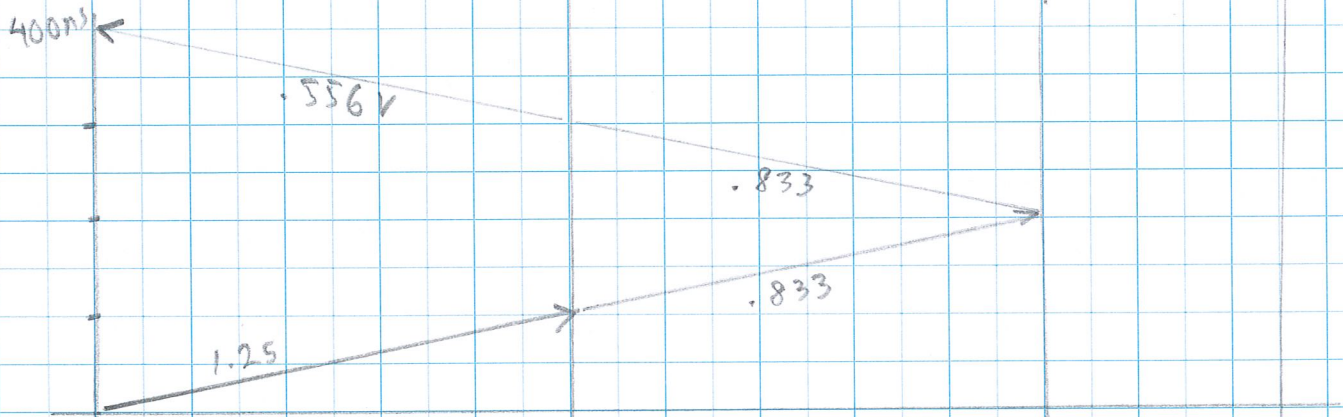


The interpretation of the different reflected pulses can be made by recognizing that the junction of the three transmission lines, the impedance is $25\ \Omega$ (two $50\ \Omega$ transmission lines in parallel) so there will be a negative reflection coefficient at this point in the network. The reflection coefficients at the ends of the two transmission lines on the right are $+1$ and the reflection coefficient at the end of the transmission line on the left is 0 because the source impedance ($50\ \Omega$) matches the impedance of the transmission line.

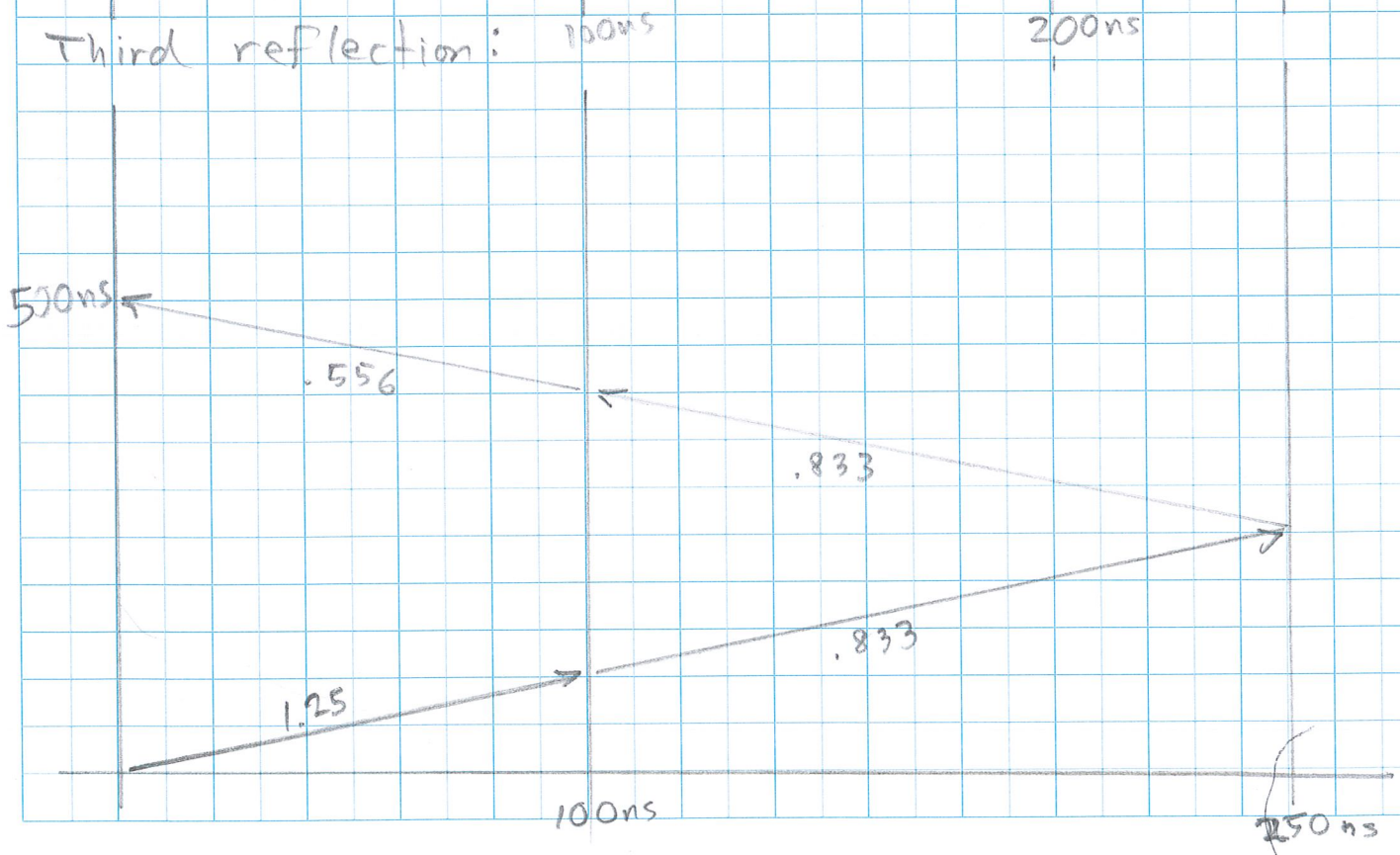
First reflection:



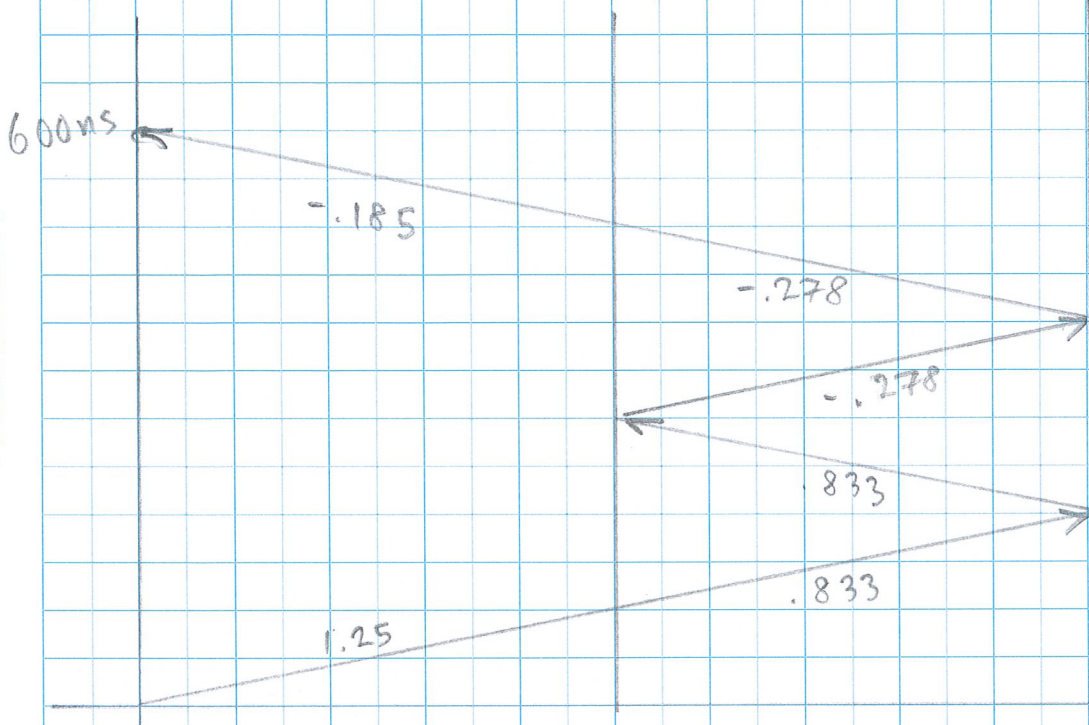
Second reflection:



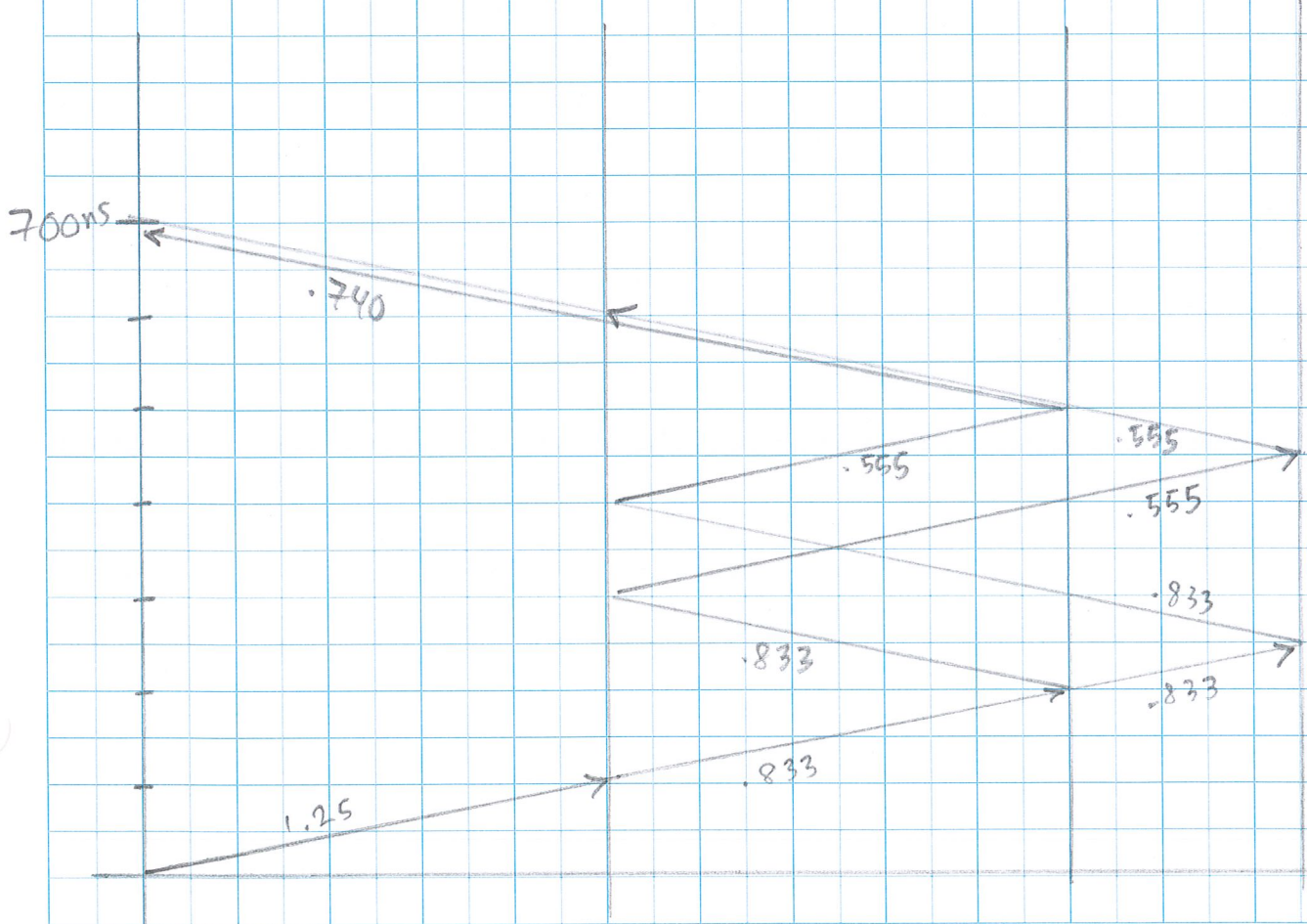
Third reflection:

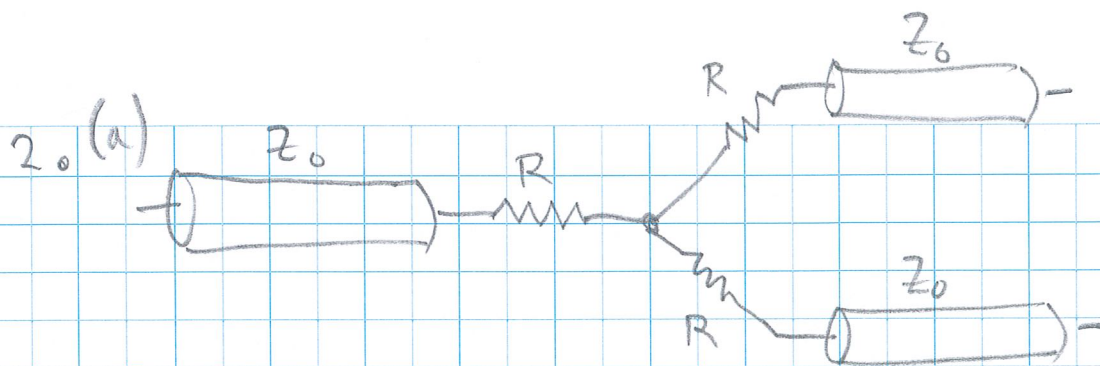


Fourth reflection:



Fifth reflection:



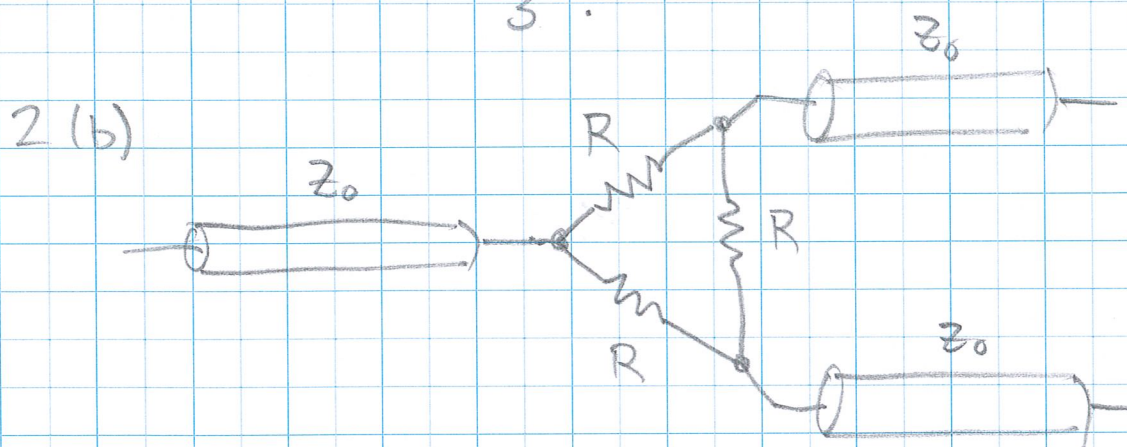


We want to select R such that

$$R + \frac{1}{2}(R + Z_0) = Z_0.$$

Thus, $\frac{3R}{2} = \frac{Z_0}{2}$

$$R = \frac{Z_0}{3} = 16.67 \Omega$$



Now we want R such that

$$\frac{1}{2}(R + Z_0) = Z_0$$

$$\frac{1}{2}R = \frac{1}{2}Z_0$$

$$R = Z_0 = 50 \Omega$$

(6)

2. The incident pulse has an amplitude of $V_0 = 1.0 \text{ V}$.

The first reflected pulse has an amplitude of $V_1 = 0.2 \text{ V}$.

$$V_1 = \Gamma V_0$$

$$\text{where } \Gamma = \frac{Z'_0 - Z_0}{Z'_0 + Z_0} = 0.2$$

$$\begin{aligned} \text{So } Z'_0 &= Z_0 \left(\frac{1 + \Gamma}{1 - \Gamma} \right) \\ &= (50 \Omega) \left(\frac{1.2}{0.8} \right) = 75 \Omega. \end{aligned}$$

The reflection coefficient at the end of the transmission line is then

$$\Gamma' = \frac{50 \Omega - 75 \Omega}{50 \Omega + 75 \Omega} = -0.2.$$

The second reflected pulse arrives 500 ns after the incident pulse so the length of the unknown transmission line is

$$\Delta T = \frac{1}{2} (500 \text{ ns} - 200 \text{ ns}) = 150 \text{ ns}.$$