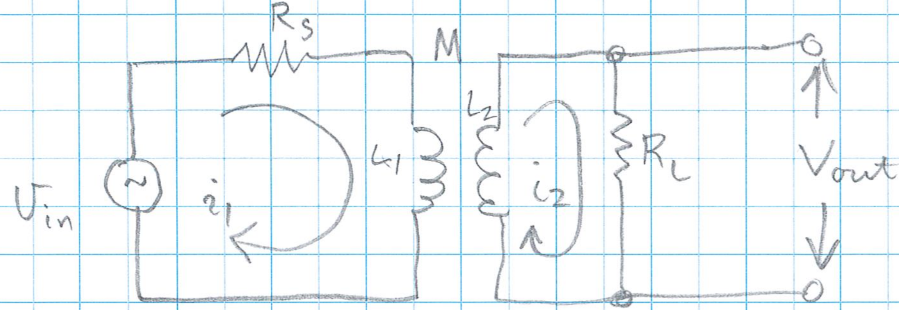
Assignment # 2



Kircchoff's rules:

$$V_{in} - i_1 R_S - Z_{L_1} i_1 + Z_M i_2 = 0$$

$$-i_2 R_L - Z_{L_2} i_2 + Z_M i_1 = 0$$

$$\begin{pmatrix} R_S + Z_{L_1} & -Z_M \\ -Z_M & R_L + Z_{L_2} \end{pmatrix} \begin{pmatrix} i_1 \\ i_2 \end{pmatrix} = \begin{pmatrix} V_{in} \\ 0 \end{pmatrix}$$

$$i_2 = \frac{V_{in} Z_M}{(R_S + Z_{L_1})(R_L + Z_{L_2}) - Z_M^2}$$

$$V_{out} = \frac{V_{in} R_L Z_M}{R_S R_L + R_S Z_{L_2} + R_L Z_{L_1} + Z_{L_1} Z_{L_2} - Z_M^2}$$

(a) With perfect coupling, $Z_M^2 = Z_{L_1} Z_{L_2}$

So $$V_{out} = \frac{V_{in} R_L Z_M}{R_S R_L + R_S Z_{L_2} + R_L Z_{L_1}}$$

$$= \frac{j\omega M V_{in} R_L}{R_S R_L + j\omega (L_1 R_L + L_2 R_S)}$$

(b) Assuming that $L_1 \propto n_1^2$ and $L_2 \propto n_2^2$ and in the limit where $R_S R_L \ll \omega(L_1 R_L + L_2 R_S)$,

$$V_{out} = \frac{\sqrt{L_1 L_2}\; V_{in} R_L}{L_1 R_L + L_2 R_S}$$

$$= \frac{n_1 n_2 V_{in} R_L}{R_L n_1^2 + R_S n_2^2}$$

If $R_S \ll R_L$ then

$$V_{out} = V_{in} \frac{n_2}{n_1}$$

(c) The power dissipated in the load is

$$P = I_2 V_2 = I_2^2 R_L = V_{in}^2 R_L \left(\frac{n_1 n_2}{n_1^2 R_L + n_2^2 R_S} \right)^2$$

$$= V_{in}^2 R_L \left(\frac{n_2/n_1}{R_L + R_S\, n_2^2/n_1^2} \right)^2$$

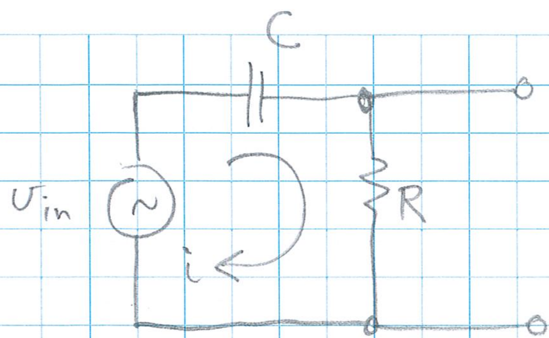
If we write $r = n_2/n_1$ then P is maximized when

$$\frac{dP}{dr} = \left(\frac{2V_{in}^2 R_L r}{R_L + r^2 R_S} \right) \left(\frac{1}{R_S r^2 + R_L} - \frac{2R_S r^2}{(R_S r^2 + R_L)^2} \right) = 0$$

$$\Rightarrow R_S r^2 + R_L - 2R_S r^2 = 0$$

$$r^2 = \frac{R_L}{R_S} \Rightarrow r = \frac{n_2}{n_1} = \sqrt{\frac{R_L}{R_S}}$$

2.



Kircchoff's rules:

$$v_{in} - i Z_c - iR = 0$$

$$i = \frac{v_{in}}{R + Z_c}$$

$$V_{out} = iR = \frac{R v_{in}}{R + Z_c}$$

$$= \frac{R V_{in}}{R - j/\omega C} = \frac{V_{in}}{1 - j/\omega RC}$$

$$= \frac{j\omega RC\, V_{in}}{j\omega RC + 1}$$

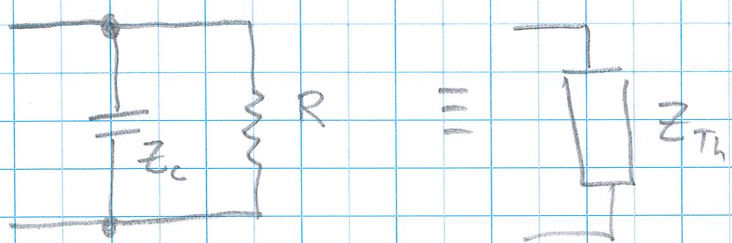
(a) In the high frequency limit, $\omega \gg 1/RC$,

$\frac{1}{\omega RC} \ll 1$ and $V_{out} \longrightarrow V_{in}$.

In the low frequency limit, $\omega \ll 1/RC$

$V_{out} \longrightarrow 0$.

(b) The Thevenin equivalent impedance is calculated by considering the R and C in parallel:



$$Z_{Th} = \left(\frac{1}{R} + \frac{1}{Z_c} \right)^{-1} = \left(\frac{1}{R} + j\omega C \right)^{-1}$$

$$= \frac{R}{1 + j\omega RC}$$

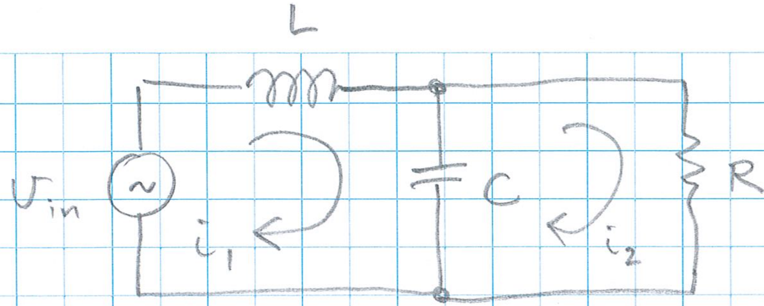
In the low frequency limit, $\omega \ll 1/RC$,

$$Z_{Th} \rightarrow R$$

In the high frequency limit, $\omega \gg 1/RC$

$$Z_{Th} \rightarrow \frac{1}{j\omega C}$$

3.



(a) $$V_{in} - I_1 Z_L - (I_1 - I_2) Z_C = 0$$

$$-I_2 R - (I_2 - I_1) Z_C = 0$$

$$\begin{pmatrix} Z_L + Z_C & -Z_C \\ -Z_C & R + Z_C \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} V_{in} \\ 0 \end{pmatrix}$$

$$I_2 = \frac{\begin{vmatrix} Z_L + Z_C & V_{in} \\ -Z_C & 0 \end{vmatrix}}{\begin{vmatrix} Z_L + Z_C & -Z_C \\ -Z_C & R + Z_C \end{vmatrix}}$$

$$= \frac{V_{in} Z_C}{(Z_L + Z_C)(R + Z_C) - Z_C^2}$$

$$= \frac{V_{in} Z_C}{R Z_L + Z_C (R + Z_L)}$$

$$V_{out} = I_2 R = \frac{V_{in} R Z_C}{R(Z_L + Z_C) + Z_L Z_C}$$

$$= \frac{V_{in} R \left(-j/\omega C\right)}{R\left(j\omega L - j/\omega C\right) + L/C}$$

$$= \frac{-V_{in} R}{R\omega^2 LC - R - j\omega L}$$

$$= \frac{V_{in}}{(1 - \omega^2 LC) - j\omega L/R}$$

The magnitude of V_{out} is

$$|V_{out}| = \frac{|V_{in}|}{\sqrt{(1-\omega^2 LC)^2 + \omega^2 L^2/R^2}}$$

(b) When R is large, the resonant frequency is ω_0, given by

$$1 - \omega_0^2 LC = 0$$

$$\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

(c) When $\omega = \omega_0$ the amplitude of V_{out} will be

$$|V_{out}| = \frac{|V_{in}|}{\sqrt{L^2/R^2 LC}}$$

$$= R|V_{in}|\sqrt{\frac{C}{L}}$$

When the gain is -3 db,

$$\frac{|V_{out}|}{|V_{in}|} = R\sqrt{\frac{C}{L}} = \frac{1}{\sqrt{2}}$$

Hence, $R = \sqrt{\frac{L}{2C}}$

(d) At high frequencies, the gain approaches the following form:

$$A \rightarrow \frac{1}{\omega^2 LC}$$

If ω increases by a factor of 10 then A will decrease by a factor of 100.

Thus, $G = 20 \log_{10} 10^{-2}$

$= -40$ db per decade.

For a first-order low-pass filter the high frequency response is -20 db/decade.

(e) Component values that give $f_0 = \dfrac{\omega_0}{2\pi} = 1\,\text{MHz}$ must satisfy

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

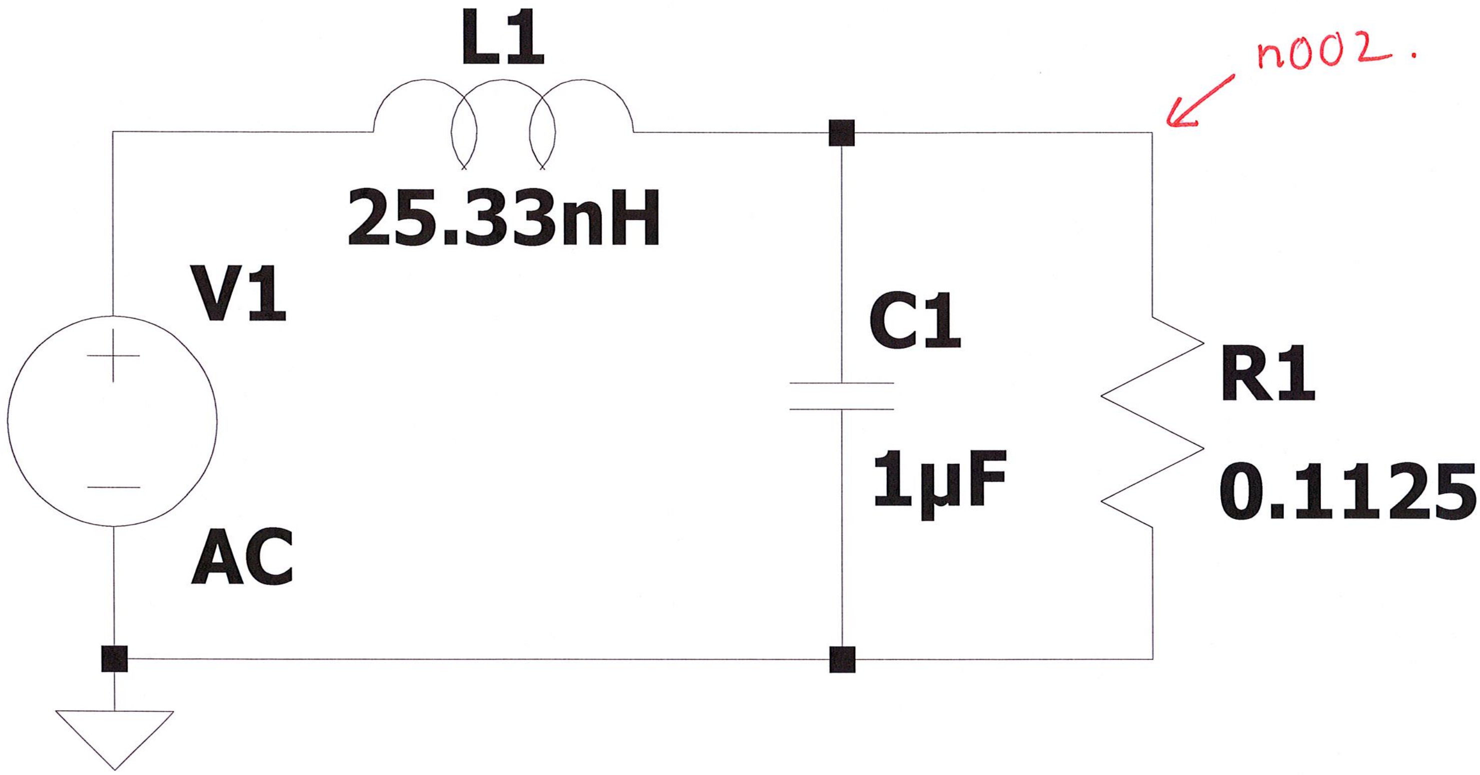
If we pick C then L is given by

$$L = \left(\frac{1}{2\pi f_0}\right)^2 \cdot \frac{1}{C}$$

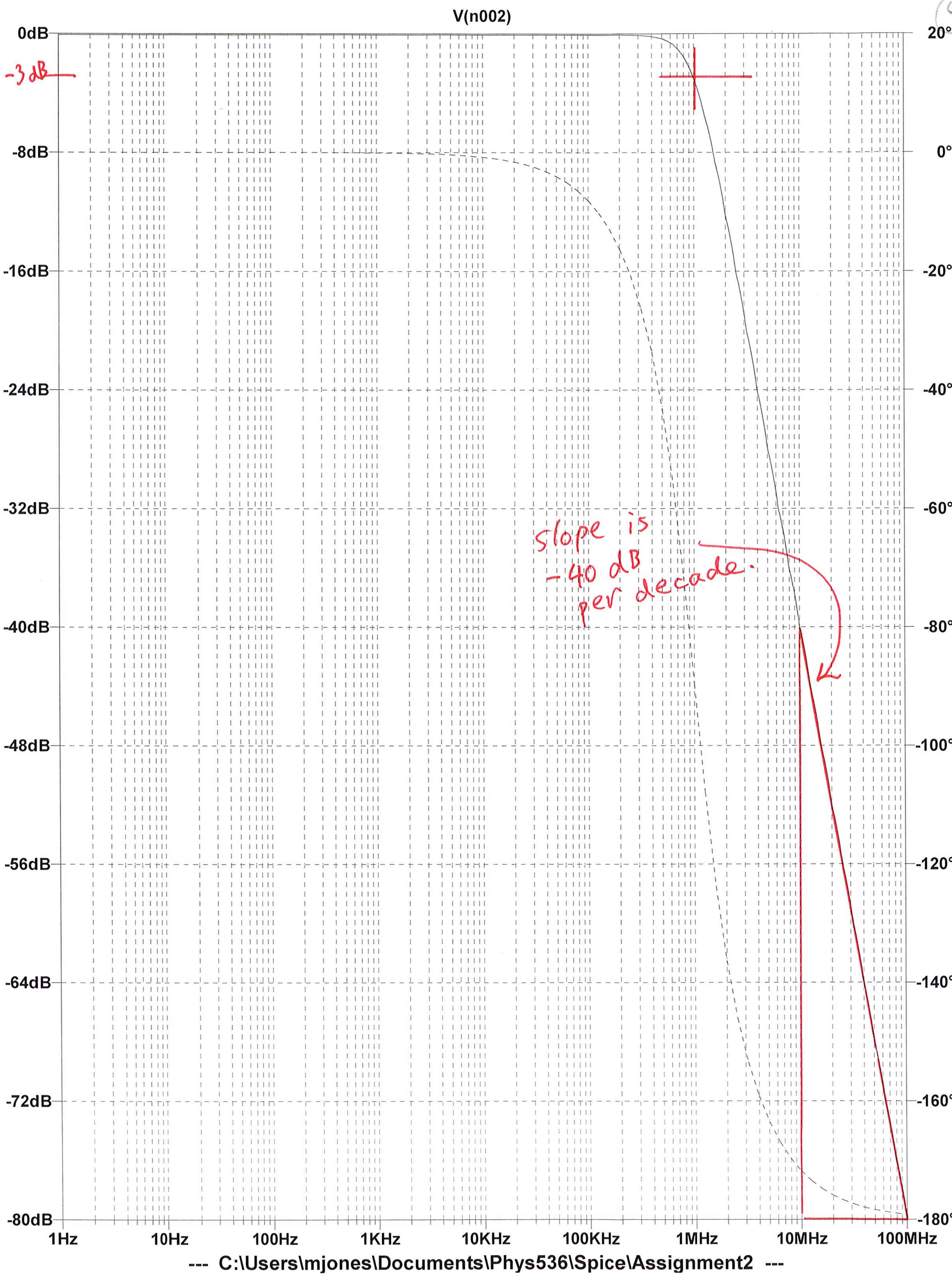
With $C = 1\,\mu F$, $L = 25.33$ nH

Then we could pick $R = \sqrt{\dfrac{L}{2C}} = \sqrt{\dfrac{25.33\,nH}{2 \cdot 1\,\mu F}}$

$= 0.1125\ \Omega$



.ac dec 10 1 1E8


V(n002)
0dB
-3 dB
-8dB
-16dB
-24dB
-32dB
-40dB
-48dB
-56dB
-64dB
-72dB
-80dB
20°
0°
-20°
-40°
-60°
-80°
-100°
-120°
-140°
-160°
-180°
1Hz
10Hz
100Hz
1KHz
10KHz
100KHz
1MHz
10MHz
100MHz
slope is -40 dB per decade.
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