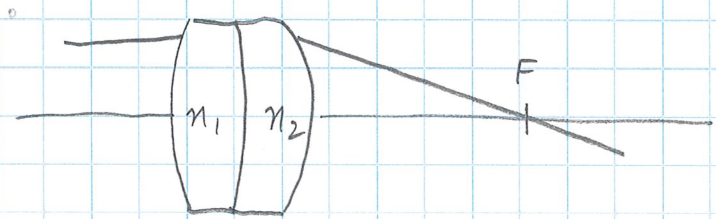
1.



The object and image distances for a single refracting surface satisfy

$$\frac{n_1}{s_o} + \frac{n_2}{s_i} = \frac{n_2 - n_1}{R}$$

where n_1 and n_2 are the indices of refraction on the object and image sides, and R is the radius of curvature.

With air on the left, an object at $s_o \to \infty$ forms an image located at s_{i_1}:

$$\frac{n_1}{s_{i_1}} = \frac{n_1 - 1}{R_1}$$

This image is treated as a virtual object which is imaged by the second refracting surface:

$$\frac{n_1}{s_{i_1}} + \frac{n_2}{s_{i_2}} = \frac{n_2 - n_1}{R_2}$$

$$\frac{n_2}{s_{i_2}} = \frac{n_2 - n_1}{R_2} - \frac{n_1}{s_{i_1}} = \frac{n_2 - n_1}{R_2} - \frac{n_1 - 1}{R_1}$$

Likewise, the final image is formed at the focal point

$$\frac{n_2}{s_{i_2}} + \frac{1}{f} = \frac{1 - n_2}{R_3} \Rightarrow \frac{1}{f} = \frac{n_1 - 1}{R_1} + \frac{n_2 - n_1}{R_2} + \frac{1 - n_2}{R_3}.$$

FORM C
APPROVED FOR USE IN
PURDUE UNIVERSITY

$$\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\frac{1}{f_2} = (n_2 - 1)\left(\frac{1}{R_2} - \frac{1}{R_3}\right)$$

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{(n_1 - 1)}{R_1} - \frac{(n_1 - 1)}{R_2} + \frac{(n_2 - 1)}{R_2} - \frac{(n_2 - 1)}{R_3}$$

$$= \frac{n_1 - 1}{R_1} + \frac{n_2 - n_1}{R_2} + \frac{1 - n_2}{R_3}$$

This is a different way to analyze the problem, but it gives the same result.

2. It is easier to calculate the final image location using a series of numerical steps rather than derive one formula that represents the final focal length.

For the parameters

$$n_1 = 1.5$$
$$n_2 = 2.0$$
$$R_1 = 10 \text{ cm}$$
$$R_2 = -20 \text{ cm}$$
$$R_3 = -10 \text{ cm}$$
$$d_1 = 1 \text{ cm}$$
$$d_2 = 1 \text{ cm}$$

an object located at ∞ will form an image located at s_{i_1} due to the first refracting surface:

$$\frac{n_1}{s_{i_1}} = \frac{n_1 - 1}{R_1} \Rightarrow s_{i_1} = \frac{n_1}{n_1 - 1} R_1 = \frac{(1.5)}{(1.5-1)}(10 \text{ cm})$$

$$= 30 \text{ cm}$$

This image can be treated as an object to be imaged by the second refracting surface but it is located at $s_{o_1} = d_1 - s_{i_1}$:

$$\frac{n_2}{s_{i_2}} + \frac{n_1}{s_{o_1}} = \frac{n_2}{s_{i_2}} + \frac{n_1}{d_1 - s_{i_1}} = \frac{n_2 - n_1}{R_2}$$

$$s_{i_2} = n_2 \left(\frac{n_2 - n_1}{R_2} - \frac{n_1}{d_1 - s_{i_1}} \right)^{-1}$$

$$= 2.0 \left(\frac{2.0 - 1.5}{-20 \text{ cm}} - \frac{1.5}{-29 \text{ cm}} \right)^{-1} = 74.8387 \text{ cm}$$

The final image is formed at the focal point when measured from the position of the third refracting surface:

$$\frac{1}{f} + \frac{n_2}{s_{o_2}} = \frac{1}{f} + \frac{n_2}{d_2 - s_{i_2}} = \frac{1-n_2}{R_3}$$

$$f = \left(\frac{1-n_2}{R_3} - \frac{n_2}{d_2 - s_{i_2}} \right)^{-1}$$

$$= \left(\frac{1-2.0}{-10\text{ cm}} - \frac{2.0}{-73.8387\text{ cm}} \right)^{-1}$$

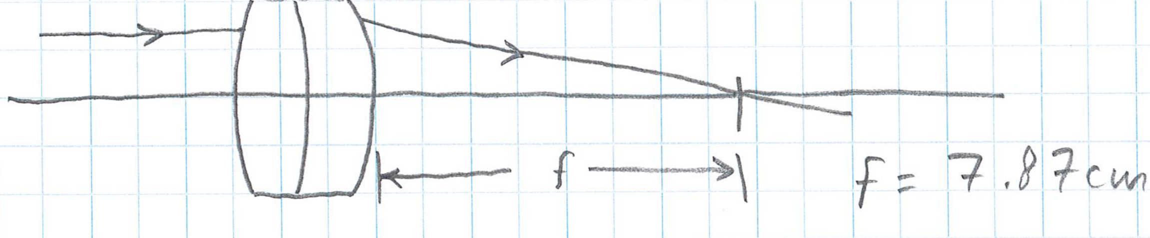
$$= 7.87 \text{ cm}.$$

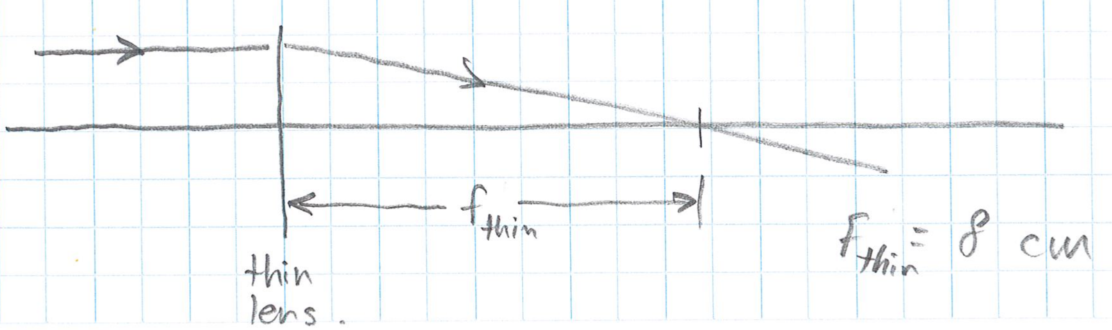
For the thin lens,

$$f_{thin} = \left(\frac{n_1 - 1}{R_1} + \frac{n_2 - n_1}{R_2} + \frac{1-n_2}{R_3} \right)^{-1}$$

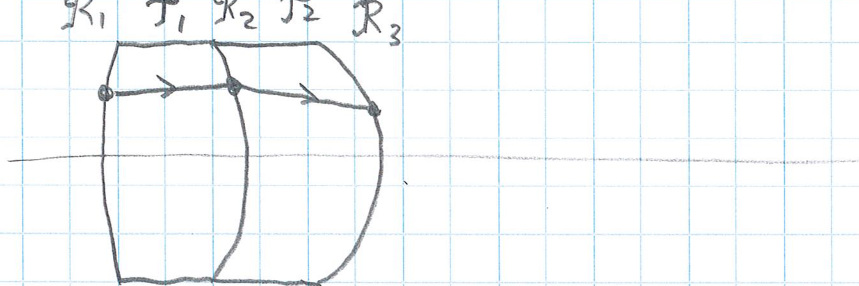
$$= \left(\frac{1.5-1}{10\text{ cm}} + \frac{2-1.5}{-20\text{ cm}} + \frac{1-1.5}{-10\text{ cm}} \right)^{-1}$$

$$= 8 \text{ cm}.$$





3. The system matrix consists of the following refractions and translations:



Then, $S = \mathcal{R}_3 \mathcal{T}_2 \mathcal{R}_2 \mathcal{T}_1 \mathcal{R}_1$

where $\mathcal{R} = \begin{pmatrix} 1 & -D \\ 0 & 1 \end{pmatrix}$; $D = \dfrac{n' - n}{R}$

and $\mathcal{T} = \begin{pmatrix} 1 & 0 \\ d/n & 1 \end{pmatrix}$

Thus, $D_1 = \dfrac{n_1 - 1}{R_1} = \dfrac{1.5 - 1}{10\,\text{cm}} = 0.05\ \text{cm}^{-1}$.

$$\mathcal{R}_1 = \begin{pmatrix} 1 & -0.05\ \text{cm}^{-1} \\ 0 & 1 \end{pmatrix}$$

$$\mathcal{T}_1 = \begin{pmatrix} 1 & 0 \\ 1\,\text{cm}/1.5 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0.6667\,\text{cm} & 1 \end{pmatrix}$$

$$D_2 = \frac{n_2 - n_1}{R_2} = \frac{2.0 - 1.5}{-20\ \text{cm}} = \frac{0.5}{-20\,\text{cm}} = -.025\ \text{cm}^{-1}$$

$$\mathcal{R}_2 = \begin{pmatrix} 1 & 0.025\ \text{cm}^{-1} \\ 0 & 1 \end{pmatrix}$$

$$\mathcal{T}_2 = \begin{pmatrix} 1 & 0 \\ 1\,\text{cm}/2.0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0.5\,\text{cm} & 1 \end{pmatrix}$$

$$D_3 = \frac{1 - n_2}{R_3} = \frac{1 - 2}{-10\text{ cm}} = 0.1\text{ cm}^{-1}$$

$$\mathcal{R}_3 = \begin{pmatrix} 1 & -0.1\text{ cm}^{-1} \\ 0 & 1 \end{pmatrix}$$

Then,

$$S = \begin{pmatrix} 1 & -.1\text{cm}^{-1} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0.5\text{cm} & 1 \end{pmatrix}\begin{pmatrix} 1 & 0.025\text{cm}^{-1} \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0.6667\text{cm} & 1 \end{pmatrix}\begin{pmatrix} 1 & -0.05\text{cm}^{-1} \\ 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 0.899164\text{ cm} & -0.121208\text{ cm}^{-1} \\ 1.17503\text{ cm} & 0.953748\text{ cm}^{-1} \end{pmatrix}$$

(b) If a ray parallel to the optical axis is incident on the lens then at the point where the final ray crosses the optical axis is

$$\begin{pmatrix} \alpha' \\ 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ f & 1 \end{pmatrix}\begin{pmatrix} S \end{pmatrix}\begin{pmatrix} 0 \\ y \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ f & 1 \end{pmatrix}\begin{pmatrix} (-0.121208)\cdot y \\ (0.953748)\cdot y \end{pmatrix}$$

Hence, $f\cdot(-0.121208) + (0.953748) = 0$

$$f = \frac{0.953748}{0.121208} = 7.869\text{ cm}.$$

I think this is correct because when I repeated the calculation with $R_1 = 100$ cm, $R_2 = -200$ cm and $R_3 = -100$ cm I got $f = 79.9$ cm, which is very close to $f = 80$ cm, calculated using the thin lens approximation.