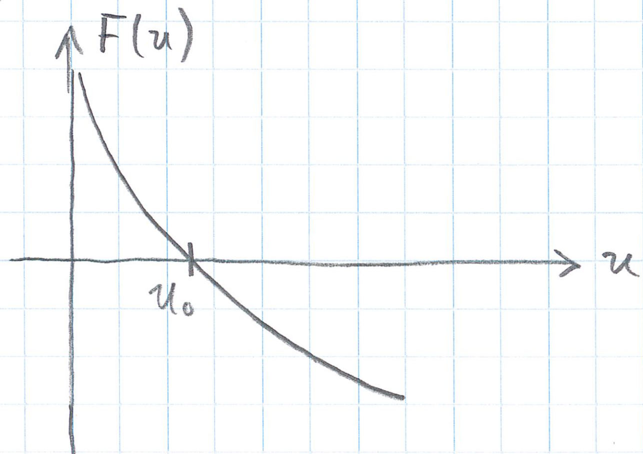
Assignment # 3

1. The force acting on the conical float is

$$F(u) = \rho g V_0 (1 - 3u + 3u^2 - u^3) - mg$$

where $u = \frac{y}{L}$ and V_0 is the total volume of the float.

If we graph $F(u)$ it will look like this:



where $F(u_0) = 0$ corresponds to the equilibrium position u_0.

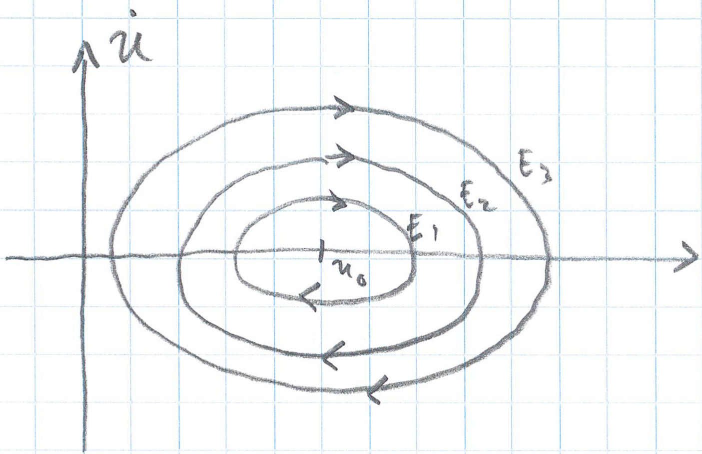
The slope of $F(u)$ at u_0 can be written as k in which case small oscillations Δu about u_0 will satisfy

$$m \frac{d^2 \Delta u}{dt^2} + k \Delta u = 0$$

and the total energy will be

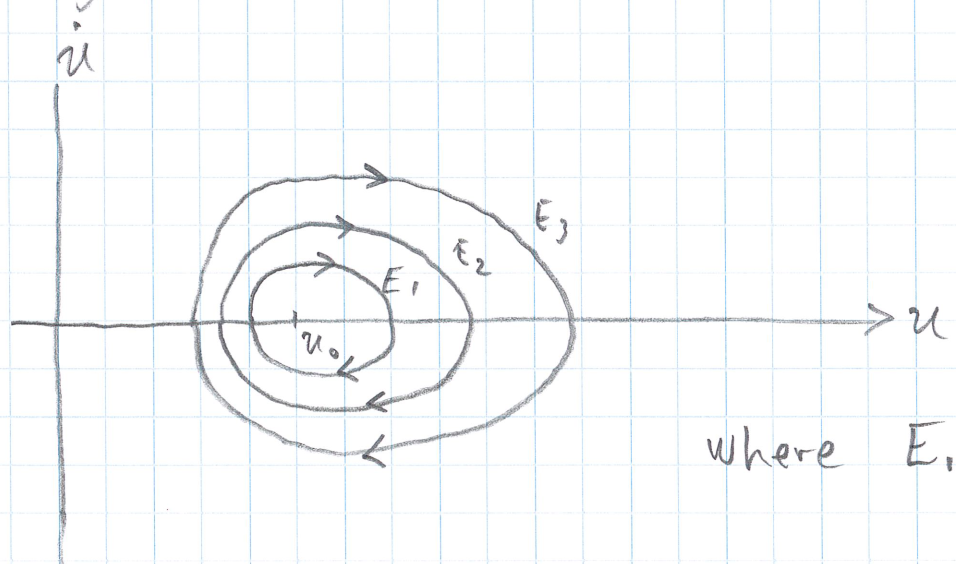
$$E = \frac{1}{2} m \Delta \dot{u}^2 + \frac{1}{2} k \Delta u^2 .$$

The phase diagram will look like ellipses in the $\dot{u}$ vs u plane, centered at u_0:



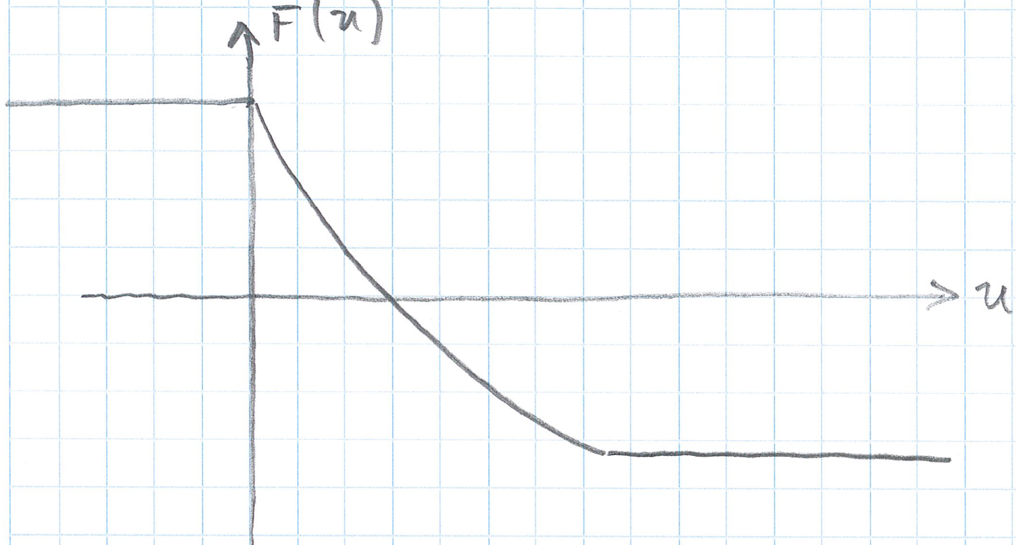
where $E_3 > E_2 > E_1$.

(b) For large oscillations the force is stronger when $u < u_0$ and weaker when $u > u_0$ so the phase diagram will look like this:

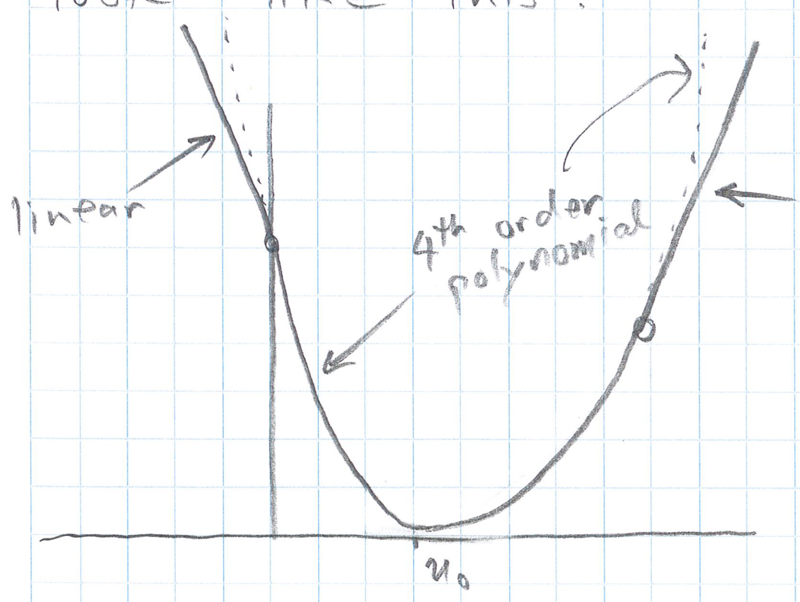


where $E_1 < E_2 < E_3$.

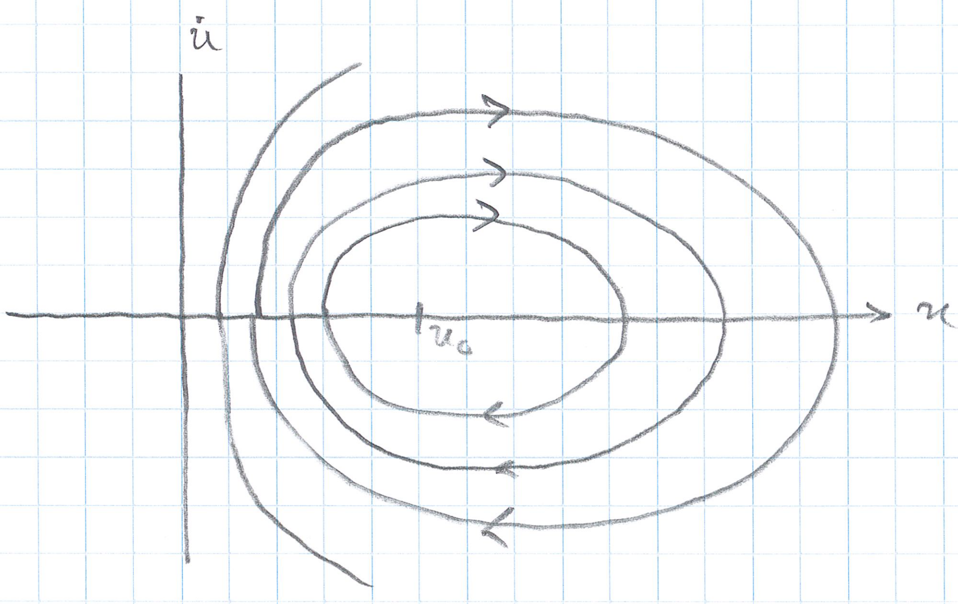
(c) If the float is entirely out of the water, or entirely submerged, then the force becomes constant.



The potential energy function will then look like this:



In this case, large amplitude oscillations will have larger amplitudes than would be observed when the force was just due to buoyancy.



The spacing of the curves as they cross the u-axis becomes linear with energy when the mass is entirely submerged or out of the water.

2. The damping force can be written

$$F_d = -b\dot{x}$$

and the spring force can be written

$$F_s = -kx \,.$$

When these are equal, we can ellimінate b:

$$bu = kd \longrightarrow b = \frac{kd}{u}\,.$$

(a) The differential equation can then be written

$$m\ddot{x} + b\dot{x} + kx = 0$$

or
$$\ddot{x} + \frac{kd}{mu}\dot{x} + \frac{k}{m}x = 0$$

or
$$\ddot{x} + \omega_0^2\frac{d}{u}\dot{x} + \omega_0^2 x = 0$$

where $\omega_0^2 = \frac{k}{m}$ is the frequency of free oscillations without damping.

(b) For the special case $\frac{u}{d} = \sqrt{\frac{k}{m}}$ the differential equation is

$$\ddot{x} + \omega_0 \dot{x} + \omega_0^2 x = 0 .$$

or $$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = 0$$

where $\gamma = \omega_0$.

The frequency of oscillation is

$$\omega = \sqrt{\omega_0^2 - \gamma^2/4}$$

$$= \sqrt{\omega_0^2 - \omega_0^2/4} = \omega_0 \frac{\sqrt{3}}{2}$$

the ratio of oscillation frequencies is then

$$\frac{\omega}{\omega_0} = \frac{\sqrt{3}}{2} .$$

(c) The Q-value is just

$$Q = \frac{\omega_0}{\gamma} = 1 .$$