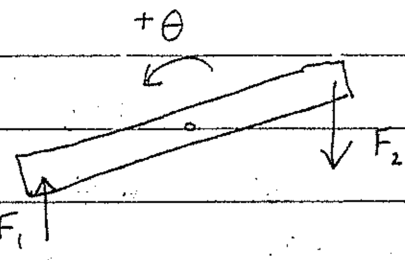
1.



$$F_1 = k\frac{L}{2}\theta \qquad F_2 = -k\frac{L}{2}\theta$$

$$N_1 = -\frac{kL^2\theta}{4} \qquad N_2 = -\frac{kL^2\theta}{4}$$

(a) $\sum N = -\frac{1}{2}kL^2\theta$

(b) $\sum N = I\ddot{\theta} = -\frac{1}{2}kL^2\theta$

(c) $I\ddot{\theta} + \frac{1}{2}kL^2\theta = 0$

$$\ddot{\theta} + \omega_0^2\theta = 0$$

where $\omega_0^2 = \frac{kL^2}{2I}$

Angular frequency is $\omega_0 = L\sqrt{\frac{k}{2I}}$

2

2. Period of oscillation is $T = 1$ second.
Equation describing motion is

$$y(t) = A e^{-\gamma t/2} \cos \omega t$$

where $\omega = \frac{2\pi}{T}$.

When $T' = 5$ seconds, $y(T') = \frac{1}{2} y(0)$.

So, $e^{-\gamma T'/2} = \frac{1}{2}$

$$-\frac{\gamma T'}{2} = -\log 2$$

$$\gamma = \frac{2 \log 2}{T'}$$

(a) $Q = \frac{\omega_0}{\gamma} = \frac{2\pi}{T} \cdot \frac{T'}{2 \log 2} = \frac{5\pi}{\log 2}$

(b) At resonance, the motion of the mass is Q times the driving motion. Therefore $A' = \frac{10\text{ cm}}{Q}$.

Therefore, the earth moves with an amplitude of $A' \doteq \frac{(10\text{ cm})\log 2}{5\pi} = 0.44\text{ cm}$.

3. $F_1 = -\frac{mg x_1}{\ell} - k(x_1 - x_2) = m\ddot{x}_1$

$F_2 = -\frac{mg x_2}{2\ell} - k(x_2 - x_1) = \frac{m}{2}\ddot{x}_2$

$\ddot{x}_1 + \frac{g x_1}{\ell} + \frac{k}{m}x_1 - \frac{k}{m}x_2 = 0$

$\ddot{x}_2 + \frac{g x_2}{\ell} + \frac{2k x_2}{m} - \frac{2k x_1}{m} = 0$

Let $\omega_0^2 = k/m$ $\quad \omega_g^2 = g/\ell$

Then if $x_1 = A\cos\omega t$, $x_2 = B\cos\omega t$

then,

$$\begin{pmatrix} -\omega^2 + \omega_0^2 + \omega_g^2 & -\omega_0^2 \\ -2\omega_0^2 & -\omega^2 + 2\omega_0^2 + \omega_g^2 \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = 0$$

(4)

Let $\lambda = \omega^2$ then it must be true that

$$\begin{vmatrix} -\lambda + \omega_0^2 + \omega_g^2 & -\omega_0^2 \\ -2\omega_0^2 & -\lambda + 2\omega_0^2 + \omega_g^2 \end{vmatrix} = 0$$

$$\Rightarrow \left(\lambda - (\omega_0^2 + \omega_g^2)\right)\left(\lambda - (2\omega_0^2 + \omega_g^2)\right) - 2\omega_0^4 = 0$$

$$\lambda^2 - \lambda(3\omega_0^2 + 2\omega_g^2) + \cancel{2\omega_0^4} + 3\omega_0^2\omega_g^2 + \omega_g^4 - \cancel{2\omega_0^4} = 0$$

Use the quadratic formula:

$$\lambda = \frac{1}{2}(2\omega_g^2 + 3\omega_0^2) \pm \frac{1}{2}\sqrt{9\omega_0^4 + \cancel{12\omega_0^2\omega_g^2} + \cancel{4\omega_g^4} - \cancel{12\omega_0^2\omega_g^2} - \cancel{4\omega_g^4}}$$

$$= \frac{1}{2}(2\omega_g^2 + 3\omega_0^2) \pm \frac{3}{2}\omega_0^2$$

So $\omega_1^2 = \omega_g$

and $\omega_2^2 = \omega_g + 3\omega_0^2$.

(5)

4. $m\ddot{y}_1 + 2ky_1 + k(y_1 - y_2) = 0$

$m\ddot{y}_2 + k(y_2 - y_1) = 0$

$$\ddot{y}_1 + \frac{2k}{m}y_1 + \frac{k}{m}y_1 - \frac{k}{m}y_2 = 0$$

$$\ddot{y}_2 + \frac{k}{m}y_2 - \frac{k}{m}y_1 = 0$$

(a) $\ddot{y}_1 + 3\omega_0^2 y_1 - \omega_0^2 y_2 = 0$

$\ddot{y}_2 + \omega_0^2 y_2 - \omega_0^2 y_1 = 0$

(b) $-\omega^2 y_1 + 3\omega_0^2 y_1 - \omega_0^2 y_2 = 0$

$-\omega^2 y_2 + \omega_0^2 y_2 - \omega_0^2 y_1 = 0$

$$\begin{pmatrix} -\lambda + 3\omega_0^2 & -\omega_0^2 \\ -\omega_0^2 & -\lambda + \omega_0^2 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = 0$$

$$(\lambda - 3\omega_0^2)(\lambda - \omega_0^2) - \omega_0^4 = 0$$

$$\lambda^2 - 4\omega_0^2\lambda + 3\omega_0^4 - \omega_0^4 = 0$$

$$\lambda^2 - 4\lambda\omega_0^2 + 2\omega_0^4 = 0$$

$$\lambda = 2\omega_0^2 \pm \sqrt{4\omega_0^4 - 2\omega_0^4}$$

$$= 2\omega_0^2 \pm \sqrt{2\omega_0^2}$$

$$= \omega_0^2\left(2 \pm \sqrt{2}\right)$$

$$\omega_1 = \omega_0\sqrt{2+\sqrt{2}}$$

$$\omega_2 = \omega_0\sqrt{2-\sqrt{2}}$$

⑦

5. $y_n(x,t) = \sin\left(\frac{n\pi x}{L}\right) \cos \omega_n t$

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$-\left(\frac{n\pi}{L}\right)^2 = -\frac{\omega_n^2}{v^2}$$

$$\omega_n = v\,\frac{n\pi}{L} \quad \text{where } v = \sqrt{T/\mu}$$

$$\omega_n = \frac{n\pi}{L}\sqrt{\frac{T}{\mu}}$$

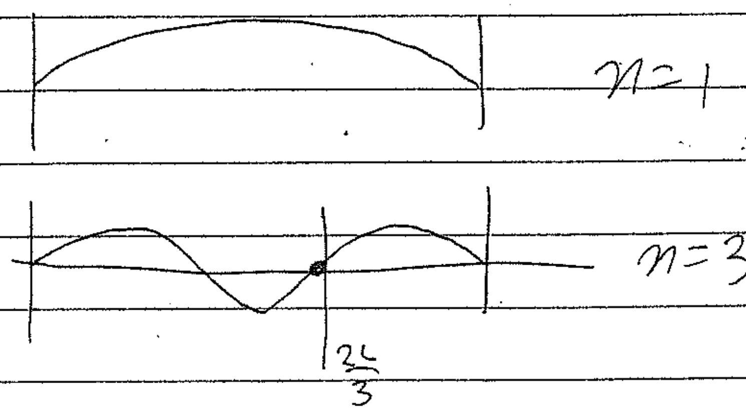
(a) Lowest frequency is $\omega_1 = \frac{\pi}{L}\sqrt{\frac{T}{\mu}}$

or $f_1 = \omega_1/2\pi = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$.

(b) ~~The only~~ All the normal modes have $y(L/2, t) \neq 0$. Therefore, touching the string at $L/2$ would not produce any sound.

Normal modes

(c) ~~Solutions~~ are like this



So touching the string at $2L/3$ selects only the $n=3$ normal mode.

In this case

$$\omega_3 = \frac{3\pi}{L}\sqrt{\frac{T}{\mu}}$$

or $f_3 = \frac{3}{2L}\sqrt{\frac{T}{\mu}}$.

6. The signal from the voltage source propagates to the end of the cable where it reflects without inversion.

The voltage seen at the end of the cable is now 2V because it is the sum of the incident and reflected signals.

The reflected signal propagates back to the source where it is reflected with inversion. This inverted signal propagates back to the open end of the cable where it cancels the reflected signal at that end leaving an observed voltage of V.

(10)

The signal at the open end of the cable will look like this: