Physics 422 - Spring 2013 - Assignment #1, Due January 24$^{\text{th}}$

1. Show that the complex valued function

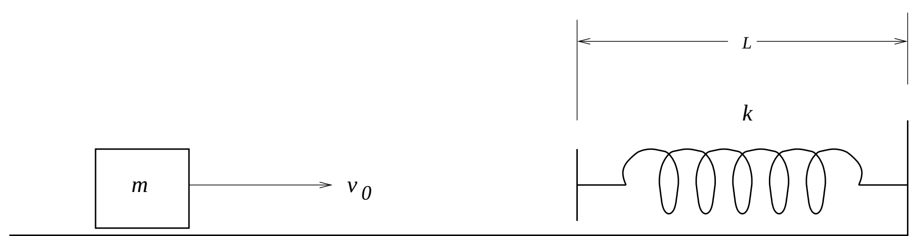
$$z(t) = ae^{i\alpha}e^{i\omega t} + be^{i\beta}e^{i\omega t}$$

can be written in the form

$$z(t) = re^{i(\omega t+\varphi)}$$

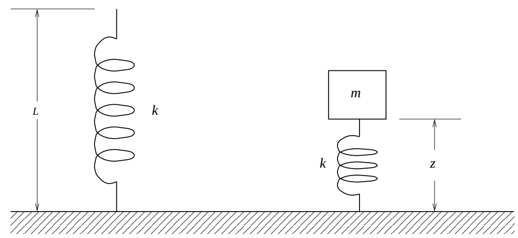
and find expressions for r and φ in terms of the real numbers a, b, α and β.

2. A mass is sliding along a frictionless surface with velcity v_0 when at time $t = 0$ it hits, and sticks to, a spring with spring constant k and uncompressed length L as shown:



(a) Draw a diagram that clearly illustrates your definition of the quantity $x(t)$, which represents the position of the mass at time $t > 0$.
(b) Solve the equation of motion for the mass at times $t > 0$.

3. Suppose a mass m is placed on top of a spring, with spring constant k, which has an uncompressed length L as shown:



(a) The weight of the mass compresses the spring. What will be the compressed length of the spring, z_0, when the mass is in equilibrium?
(b) Draw a free-body diagram showing the forces that act on the mass when the spring is compressed to a length z, where z is not nececessarily equal to z_0?
(c) What is the net force on the mass when the spring is compressed to an arbitrary length z?
(d) Using Newton's second law, derive the equation of motion for the mass in terms of z. Is the equation of motion of the form $\ddot{z} + \omega^2 z = 0$?
(e) Show that the equation of motion can be written in the form $\ddot{z}' + \omega^2 z' = 0$ by a suitable change of variables.
(f) If the mass is released from rest on the uncompressed spring at time $t = 0$, find $z(t)$ for times $t > 0$.