# Physics 422 - Spring 2013 - Assignment #1, Due January 18$^{\text{th}}$

**1.** Consider the differential equation

$$\frac{d^2x}{dt^2} + \omega^2 x \quad = \quad 0 \tag{1}$$

If a general solution is written in the form of the infinite series,

$$x(t) = \sum_{n=0}^{\infty} a_n t^n$$

what constraints must the coefficients $a_n$ satisfy if $x(t)$ is a solution of Equation 2? Hint: re-write the series in the form

$$x(t) = \sum_{n=0,2,4,\ldots} a_n t^n + \sum_{n=1,3,5,\ldots} b_n t^n,$$

subsititue it into Equation 2, and equate terms with equal powers of $n$.

**2.** Verify that the differential equation

$$\frac{d^2x}{dt^2} + \omega^2 x \quad = \quad 0 \tag{2}$$

has as its solution

$$x(t) = A\cos(\omega t) + B\sin(\omega t)$$

where $A$ and $B$ are arbitrary constants. Show that this solution can be written in the form

$$x(t) = C\cos(\omega t + \alpha) = C\mathrm{Re}\left[e^{i(\omega t+\alpha)}\right]$$

and express $C$ and $\alpha$ as functions of $A$ and $B$.

**3.** Using Euler's formula

$$e^{i\theta} = \cos\theta + i\sin\theta \tag{3}$$

show that we can write

(a) $\cos\theta = (e^{i\theta} + e^{-i\theta})/2$

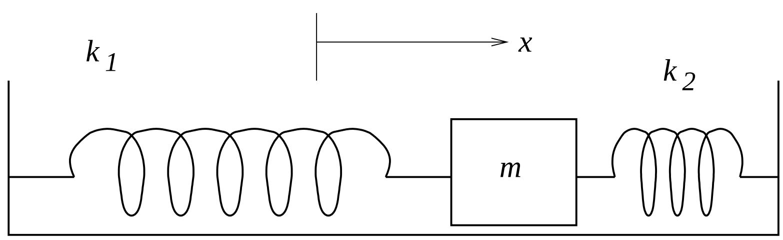
(b) $\sin\theta = (e^{i\theta} - e^{-i\theta})/2i$

(c) $\sin^2\theta + \cos^2\theta = 1$

(d) $\cos^2\theta - \sin^2\theta = \cos 2\theta$

(e) $2\sin\theta\cos\theta = \sin 2\theta$

**4.** Consider an object of mass $m$ that is connected to two springs, with spring constants $k_1$ and $k_2$ as shown:




where $x$ measures the displacement of the object from its equilibrium position. Determine the angular frequency, $\omega$, with which the object will oscillate.