Purdue University Department of Physics

Physics 24100

Electricity & Optics

Lecture 7 – Chapter 23 sec. 4-5

Fall 2012 Semester

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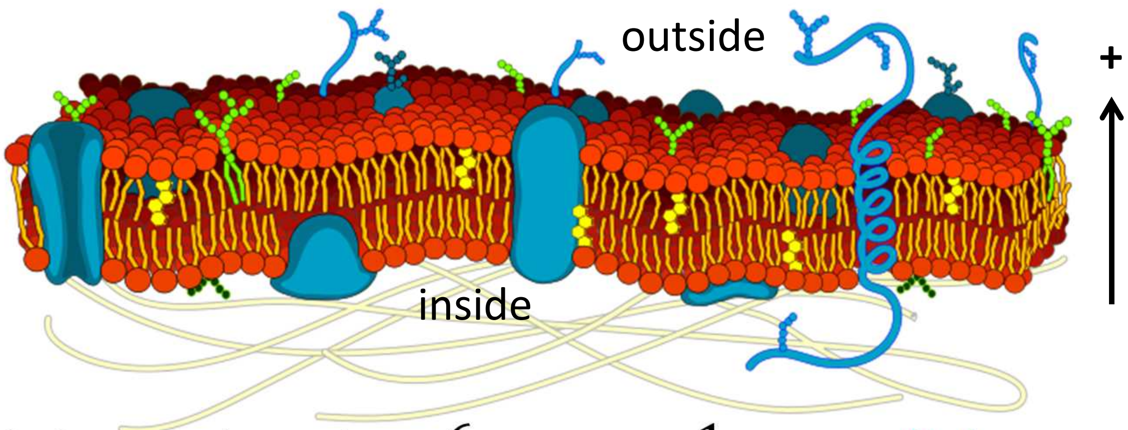
Thursday's Clicker Question

- Assume that the electric field in a cell membrane is constant and the electric potential on the *inside* of the membrane is **50 mV** *lower* than the *outside*.
- What is the electric field in a cell membrane that is **5 nm** thick?



This is the "+" direction. A positive E-field would point out.

$$V_{outside} = 0\ \text{V}$$
$$V_{inside} = -50\ \text{mV}$$
$$\Delta x = 5\ \text{nm}$$

(a) $2.5 \times 10^{-6}\ \text{V} \cdot \text{m}^{-1}$

(b) $-10^{7}\ \text{V} \cdot \text{m}^{-1}$

(c) $+10^{7}\ \text{V} \cdot \text{m}^{-1}$

(d) $2.5 \times 10^{6}\ \text{V} \cdot \text{m}^{-1}$

Clicker Question

- Direction of the $\vec{E}$ field:
  - The $\vec{E}$ field points "downhill" because $\vec{E} = -\vec{\nabla}V$
  - If the potential is lower inside the cell then $\vec{E}$ is pointing in.
  - This is opposite the sign convention (where + pointed out)
  - Therefore, the E will be negative.

Clicker Question

- Magnitude of the $\vec{E}$ field:
  - $\vec{E}$ and V are related by $\vec{E} = -\vec{\nabla}V$
  - For a constant $\vec{E}$ field in, say, the x-direction,

$$E = -\frac{\Delta V}{\Delta x}$$

  - For the cell, $\Delta V = 50\, mV$ and $\Delta x = 5\, nm$
    - When measured using the sign convention provided.
  - Therefore,

$$E = -\frac{5 \times 10^{-2}\, V}{5 \times 10^{-9}\, m} = -10^{7}\, V/m$$

Electric Potential

- Relation between $\vec{E}$ and V:

$$\vec{E} = -\vec{\nabla}V = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$$

- The components are:

$$E_x = -\frac{\partial V}{\partial x},\ E_y = -\frac{\partial V}{\partial y},\ E_z = -\frac{\partial V}{\partial z}$$

- We can add a constant to V and not change $\vec{E}$.
- For a point charge, we defined

$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$ so that $V(r) \to 0$ when $r \to \infty$.

Electric Potential

- This is a convenient definition when the potential does "vanish at infinity" but for some field configurations do not.
- Consider a constant electric field in the x-direction:

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{\ell} = -E_x(x_b - x_a)$$

When we let $x_a \to \infty$ we get $\Delta V \to \infty$ which doesn't make sense…

- A better choice would be to define $V = 0$ when $x = 0$.
- That's what we did before:

$$V(x) = -E\,x$$

- If the field doesn't "vanish at infinity", don't use "infinity" as a reference point to define $V(x)$.

Electric Potential due to several Point Charges

- Electric potential for a single point charge:

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$ (when $r_{ref} \to \infty$)

- Electric potential at point $\vec{x}$ due to several point charges:

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{Q_i}{r_i}$$

where $r_i = |\vec{x} - \vec{x}_i|$ is the distance to each charge.

- What about a continuous distribution of charge?

Continuous Charge Distributions

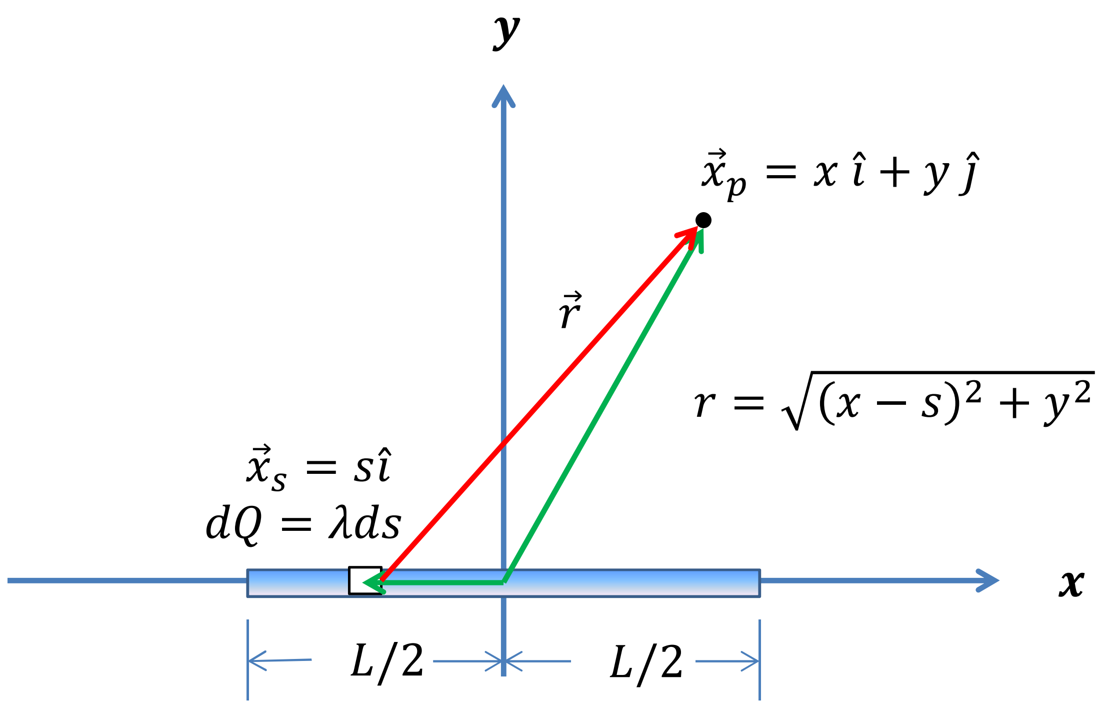
$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{Q_i}{r_i} \quad \Rightarrow \quad V(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int \frac{dQ}{r}$$

Continuous Charge Distributions

The same general guidelines as for calculating the electric field:

1. Pick an appropriate coordinate system…
   – Cartesian? Cylindrical? Spherical? Something else?
2. Draw a big diagram and label:
   – Source and field points
   – Useful integration variables
   – Element of charge, dQ
3. Evaluate the integral
4. Check limiting behavior

Continuous Line of Charge



1. Coordinate system
2. Field point
3. Source point and integration variable
4. Express dQ and r in terms of integration variable

Continuous Line of Charge

- Write down the integral:

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int \frac{dQ}{r}$$

- In this case, the integration variable is s and,

$$dQ = \lambda ds$$

$$r = \sqrt{(x-s)^2 + y^2}$$

- The integral is:

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int_{-L/2}^{L/2} \frac{\lambda\, ds}{\sqrt{(x-s)^2 + y^2}}$$

Continuous Line of Charge

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{\lambda\, ds}{\sqrt{(x-s)^2 + y^2}}$$

- This is of the form

We will use "$\log x$ " for base e.
We will use "$\log_{10} x$" for base 10.

$$\int \frac{du}{\sqrt{u^2 + a^2}} = \log\left(u + \sqrt{u^2 + a^2}\right)$$

but we have to make a change of variables…

- Let $u = x - s$, then $du = -ds$
  - When $s = -L/2, u = x + L/2$
  - When $s = +L/2, u = x - L/2$

Continuous Line of Charge

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0} \int_{-L/2}^{L/2} \frac{\lambda\, ds}{\sqrt{(x-s)^2 + y^2}}$$

$$V(\vec{x}) = \frac{-\lambda}{4\pi\epsilon_0} \int_{x+L/2}^{x-L/2} \frac{du}{\sqrt{u^2 + y^2}}$$

- Now we can use:

$$\int \frac{du}{\sqrt{u^2 + a^2}} = \log\left(u + \sqrt{u^2 + a^2}\right)$$

- Remember that $\log(b) - \log(a) = \log(b/a)$

Continuous Line of Charge

$$V(\vec{x}) = \frac{-\lambda}{4\pi\epsilon_0}\int_{x+L/2}^{x-L/2} \frac{du}{\sqrt{u^2+y^2}}$$

$$= \frac{\lambda}{4\pi\epsilon_0}\int_{x-L/2}^{x+L/2} \frac{du}{\sqrt{u^2+y^2}}$$

Exchange limits of integration…

$$V(\vec{x}) = \frac{\lambda}{4\pi\epsilon_0}\log\left(\frac{x+L/2+\sqrt{(x+L/2)^2+y^2}}{x-L/2+\sqrt{(x-L/2)^2+y^2}}\right)$$

- Limiting behavior? We expect it to look like

$$V(r) \sim \frac{\lambda L}{r}$$

Continuous Line of Charge

- Consider points where $y = 0$:

$$V(\vec{x}) = \frac{\lambda}{4\pi\epsilon_0} \log\left(\frac{x + L/2 + \sqrt{(x+L/2)^2}}{x - L/2 + \sqrt{(x-L/2)^2}}\right)$$

- When $x \gg L/2$, $\sqrt{(x \pm L/2)^2} = x \pm L/2$

$$V(\vec{x}) = \frac{\lambda}{4\pi\epsilon_0} \log\left(\frac{x + L/2}{x - L/2}\right) = \frac{\lambda}{4\pi\epsilon_0} \log\left(\frac{1 + L/2x}{1 - L/2x}\right)$$

- Taylor series: $\log\left(\frac{1+u}{1-u}\right) = 2\left(u + \frac{u^3}{3} + \frac{u^5}{5} + \cdots\right)$

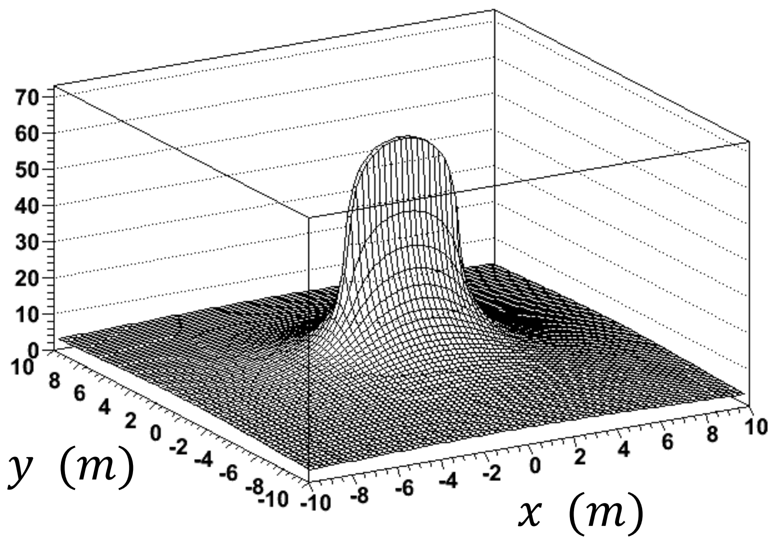
Google "power series logarithm"

$$V(\vec{x}) \to \frac{\lambda}{4\pi\epsilon_0} \frac{L}{x}$$

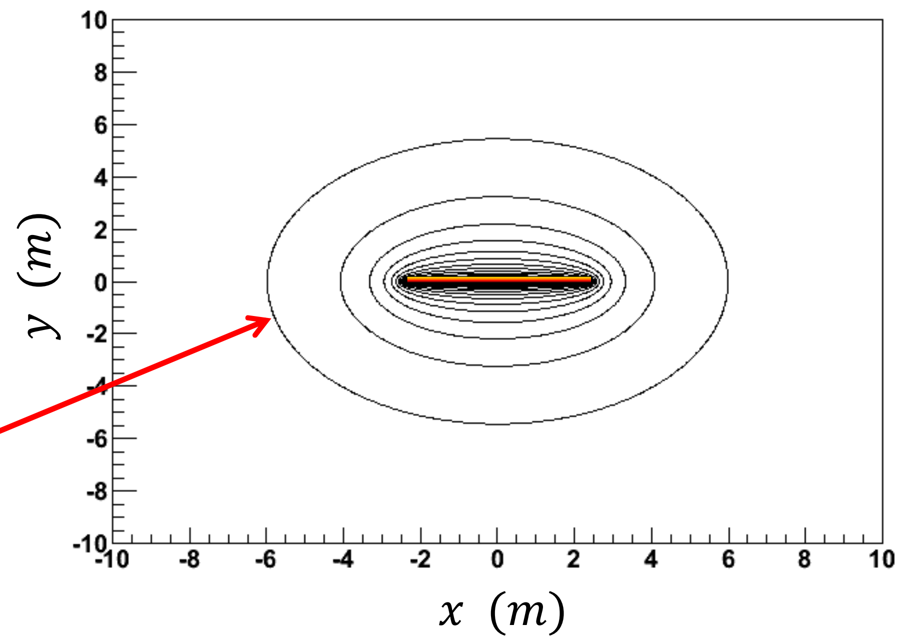
Continuous Line of Charge

- The electric potential is a surface:

$$\lambda = 1\,nC/m$$
$$L = 5\,m$$

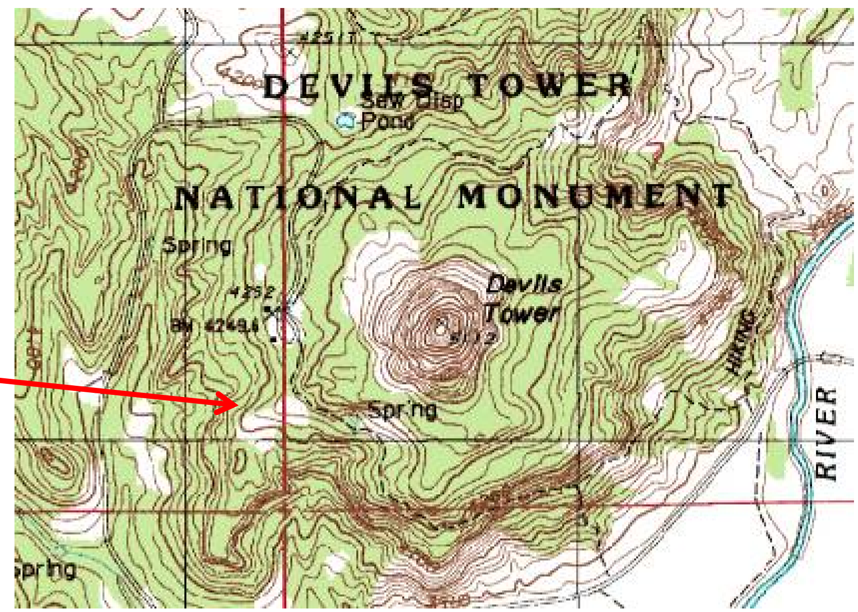


Equipotential contours

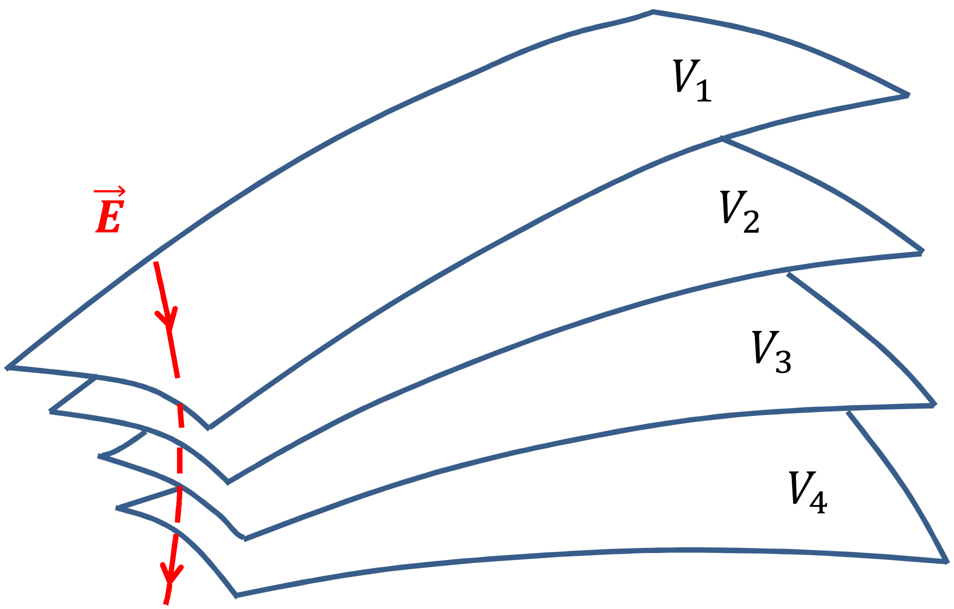


Electric Potential Surfaces

Lines of equal elevation



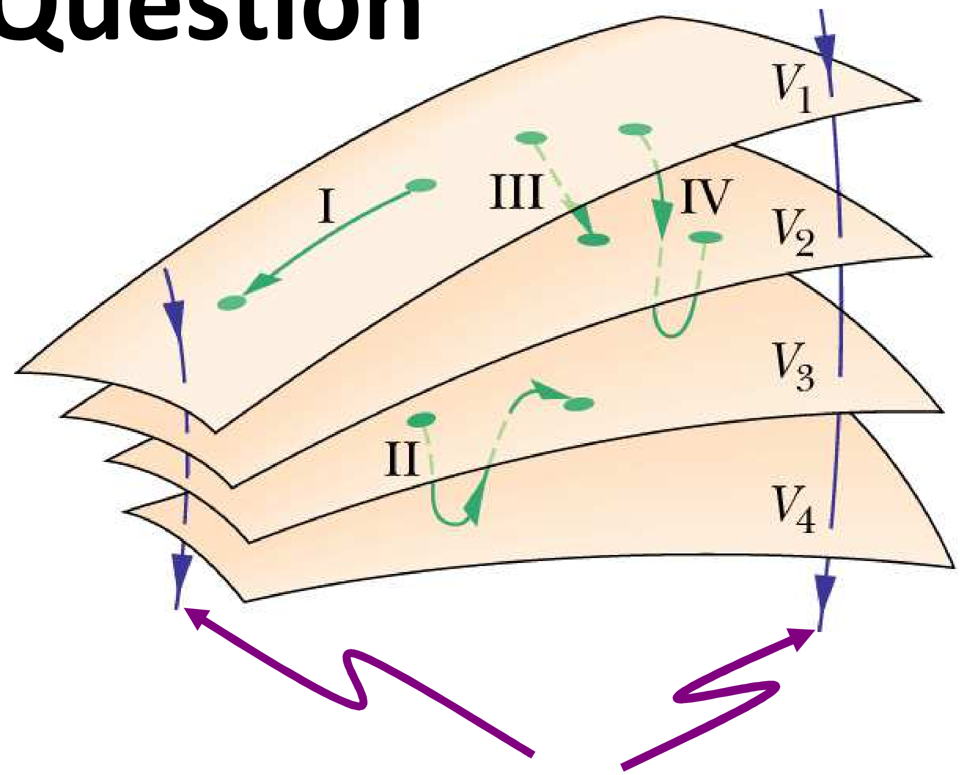
Equipotential Surfaces



- This is harder to visualize in three dimensions.
- Electric field lines are always perpendicular to equipotential surfaces.

Clicker Question

Which of the following statements is true about the work done by the electric field in moving a positive charge along the paths?



electric field lines

$$V_1 = 100\ V$$
$$V_2 = 80\ V$$
$$V_3 = 60\ V$$
$$V_4 = 40\ V$$

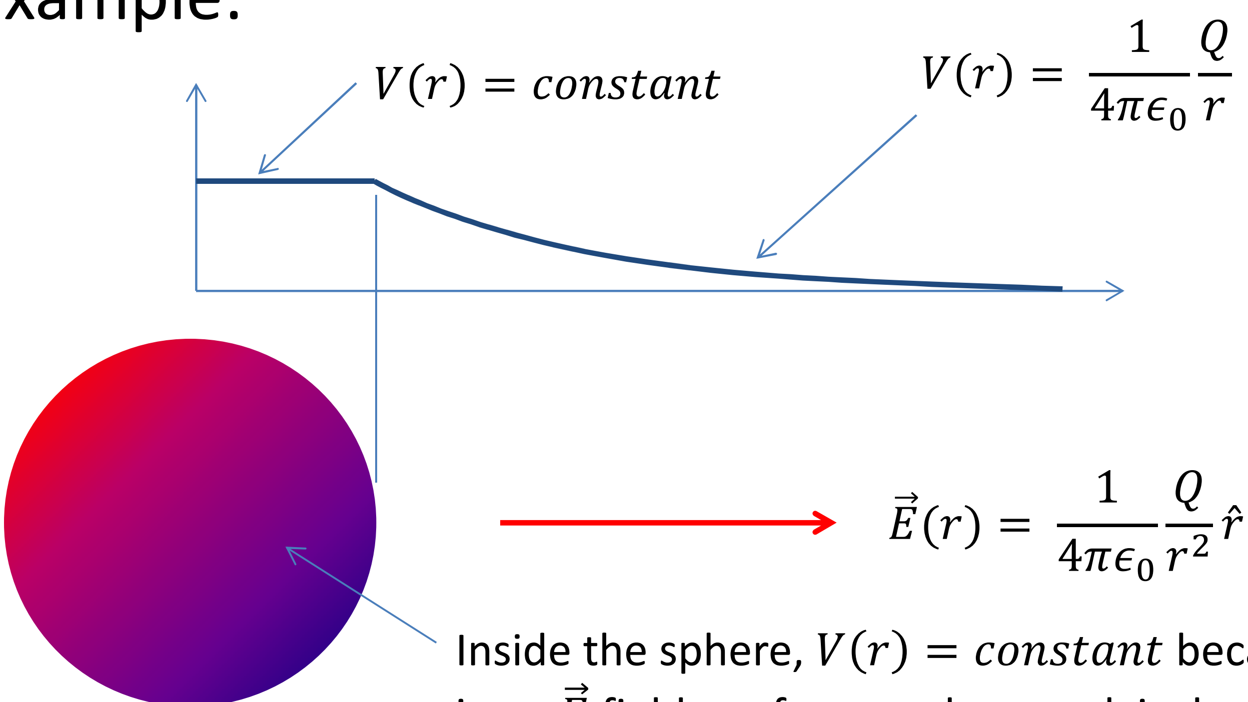
(A) III = IV > 0, I = II = 0

(B) I = II = 0, III > IV

(C) III = IV < 0, I = II = 0

Electric Potential Inside a Conductor

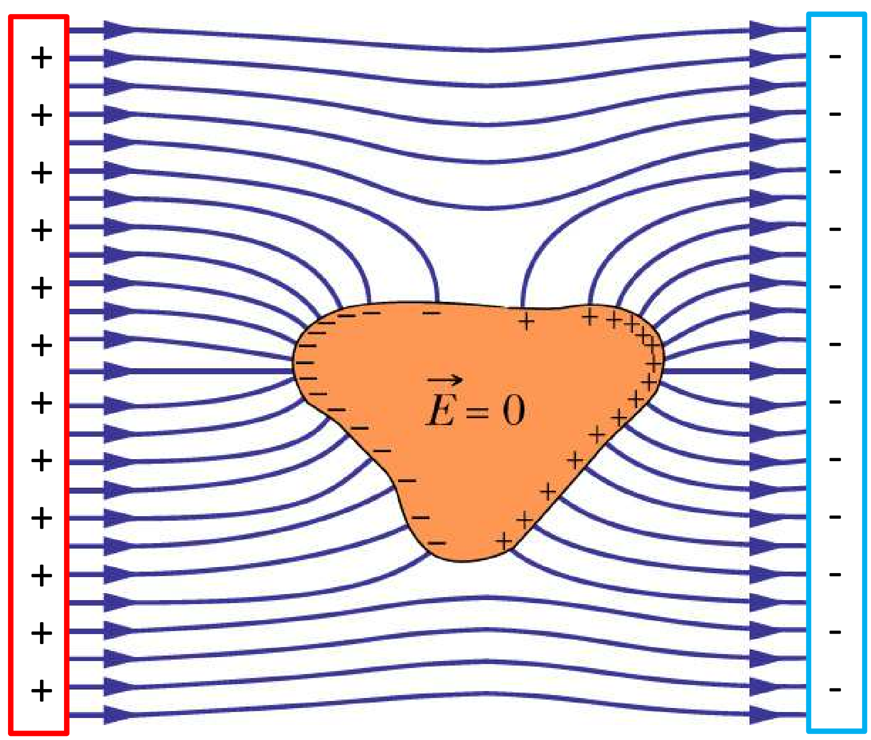
- There is no electric field in a conductor (if it is in electrostatic equilibrium)
- *This does not mean that the electric potential is zero!*
- Example:



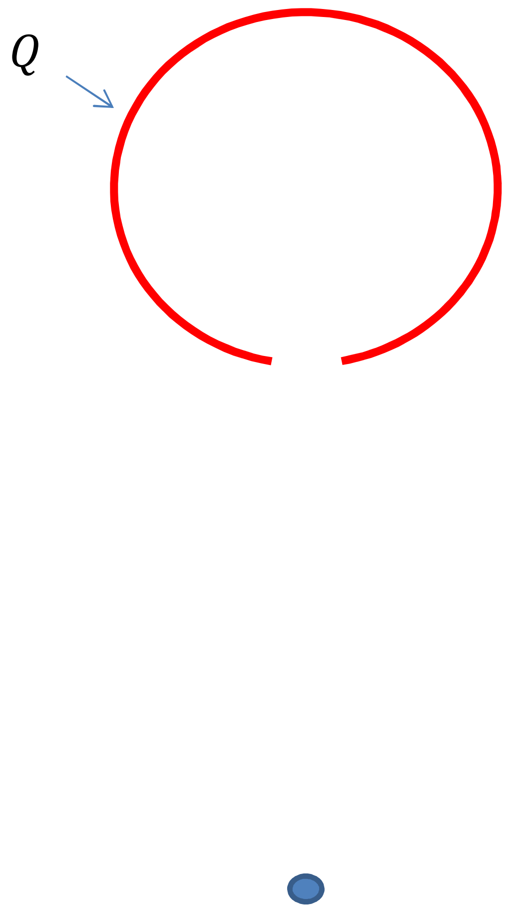
Inside the sphere, $V(r) = constant$ because there is no $\vec{E}$ field, no force and no work is done.

Electric Potential Outside a Conductor

- If $V = constant$ inside a conductor then the outside must be an equipotential surface.
- The $\vec{E}$ field must be perpendicular to the surface.

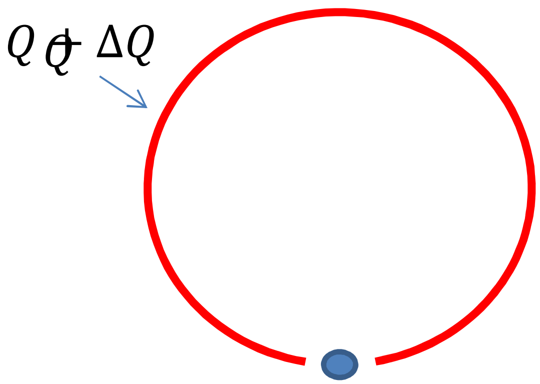


The Van de Graaff Generator



- Initial charge on the dome is Q
  - All the charge is on the outer surface.
  - Electric potential is $V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$
- A charge ΔQ is brought towards the dome.
  - Work done is $\Delta U = \frac{1}{4\pi\epsilon_0}\frac{Q\Delta Q}{R}$

The Van de Graaff Generator

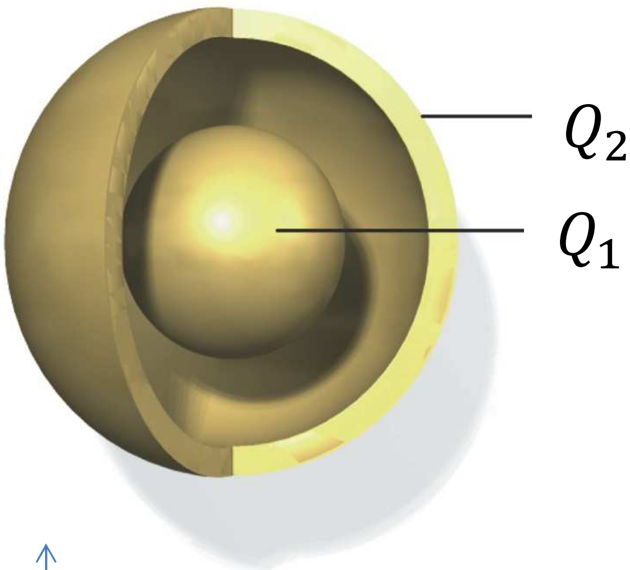


- The charge ΔQ is brought inside the dome.
  - Work done is $U = 0$
  - A surface charge of $-\Delta Q$ is induced on the inner surface
  - Outer surface must have charge $Q + \Delta Q$
- If the charge is in electrical contact with the inner surface (via a spark) then there is no net charge on the surface.
  - The outer surface retains the charge $Q + \Delta Q$.

Example

- Adding charge ΔQ requires work $\Delta U = \frac{1}{4\pi\epsilon_0}\frac{Q\Delta Q}{R}$
- What is the total work required to charge a Van de Graaff generator to an electric potential V if it was initially uncharged?
- What is the electric field at the surface once it is fully charged?

Final Clicker Question



If $Q_2 = 1\ \mu C$ and $Q_1 = -2\ \mu C$ which graph most accurately shows $V(r)$?