Physics 24100
Electricity & Optics
Lecture 6 – Chapter 23 sec. 1-3

Fall 2012 Semester

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Question

- An isolated conductor of arbitrary shape has a net charge of +10 Coulombs.
- Inside the conductor is a cavity that contains a point charge of -3 Coulombs.
- What is the net charge on the inner surface of the cavity?

(a) -13 Coulombs

(b) -3 Coulombs

(c) 0 Coulombs
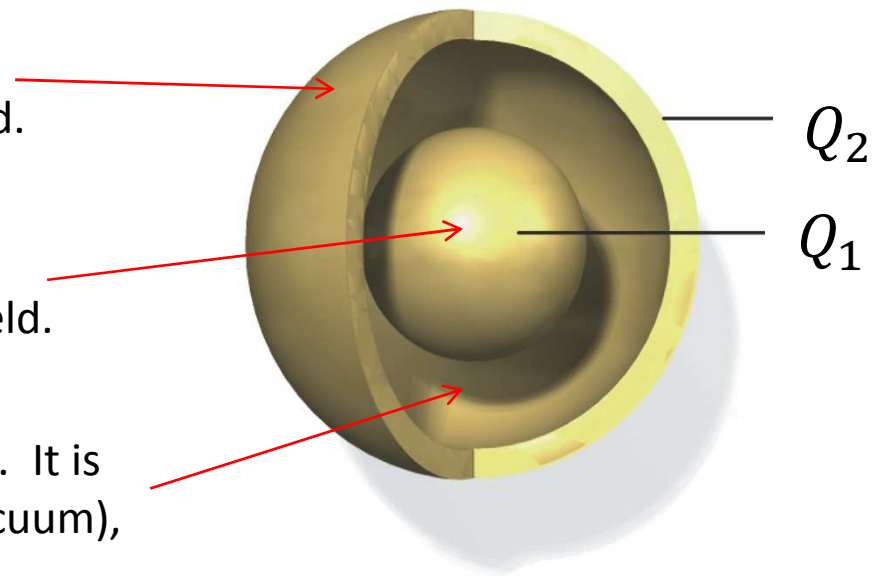
(d) +3 Coulombs

(e) +13 Coulombs

Conductors

- There is no electric field inside a conductor in electrostatic equilibrium.

- Some clarification:

  "Inside a conductor" only refers to the conducting material.

  It does not refer to any cavities inside a conductor.

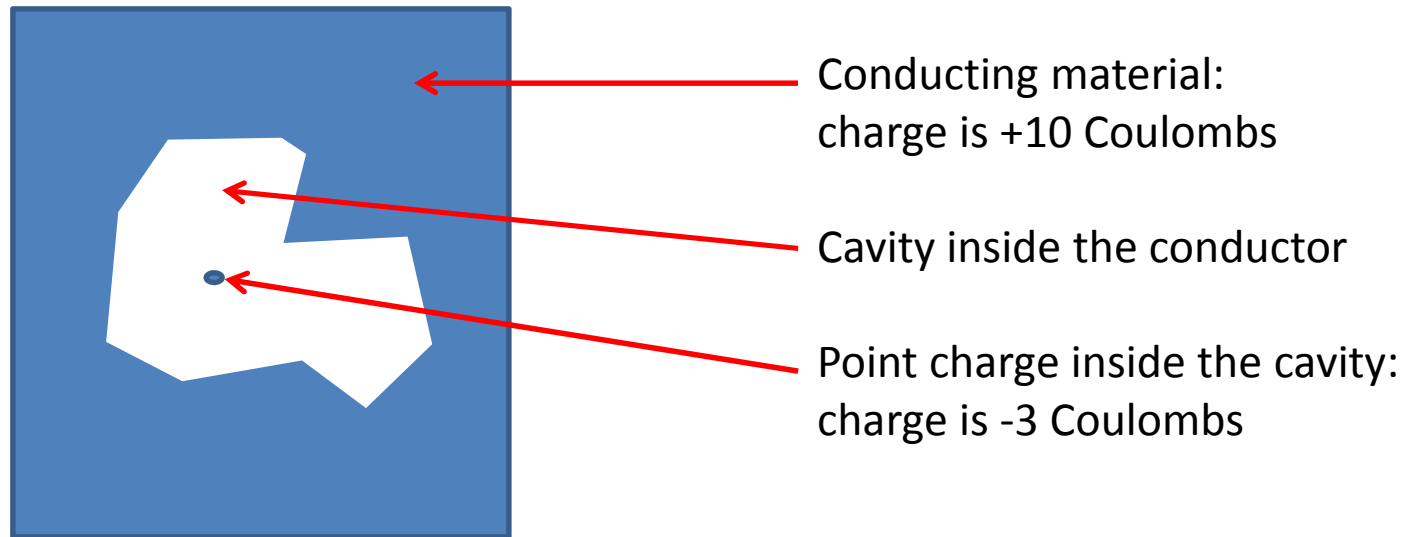This "spherical conducting shell" is conducting material: no electric field.

This "solid conducting sphere" is conducting material: no electric field.

Q_2

Q_1

This is a gap between the conductors. It is not conducting material (ie, air or vacuum), so it can have an electric field!
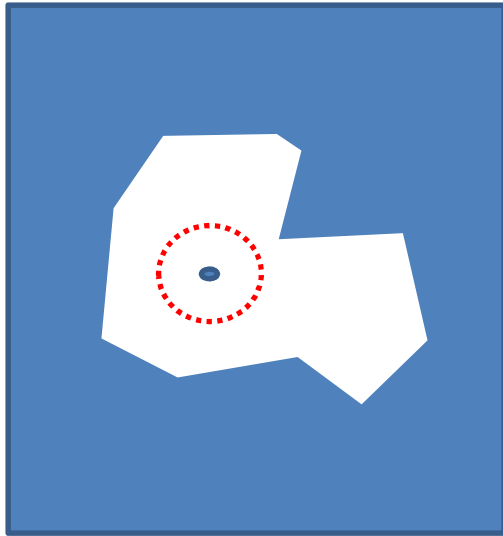
Tuesday's Question

- Surface charge on the inside of the cavity:

Conducting material: charge is +10 Coulombs

Cavity inside the conductor

Point charge inside the cavity: charge is -3 Coulombs

- Let's use Gauss's law to figure out what the net charge is on each surface.

Tuesday's Question
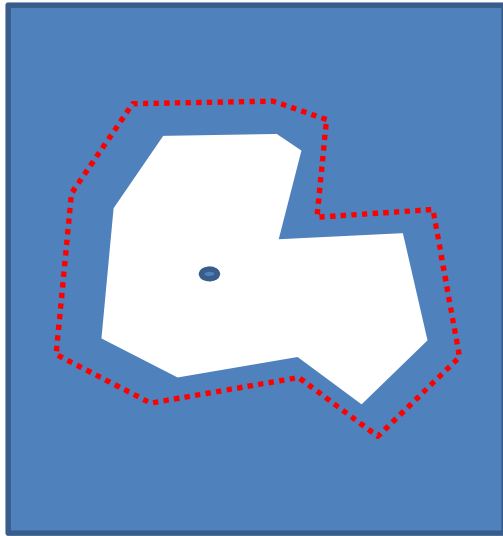
- Apply Gauss's Law to just the point charge.

- Gauss's Law: $\oint_S \hat{n} \cdot \vec{E} \, dA = \dfrac{Q_{inside}}{\epsilon_0}$
- Spherical Gaussian surface around the charge: $Q_{inside} = -3$ C.
- Electric field:

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{(-3\ C)}{r^2} \hat{r}$$
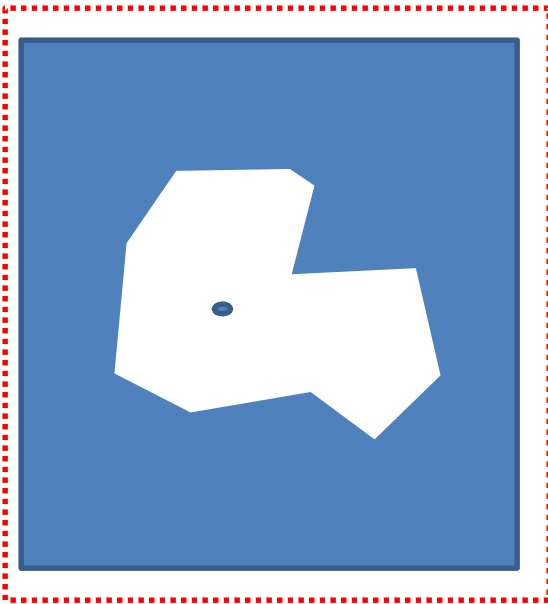
Tuesday's Question

- Apply Gauss's Law to the whole cavity.

- The Gaussian surface is always inside the conductor so $\vec{E} = 0$ everywhere on the surface.

- Gauss's Law: $\oint_S \hat{n} \cdot \vec{E}\, dA = \dfrac{Q_{inside}}{\epsilon_0}$

- Since $\oint_S \hat{n} \cdot \vec{E}\, dA = 0$ we must have $Q_{inside} = 0$.

- But the point charge is -3 C...

- The charge on the inner surface of the cavity must be +3 Coulombs.
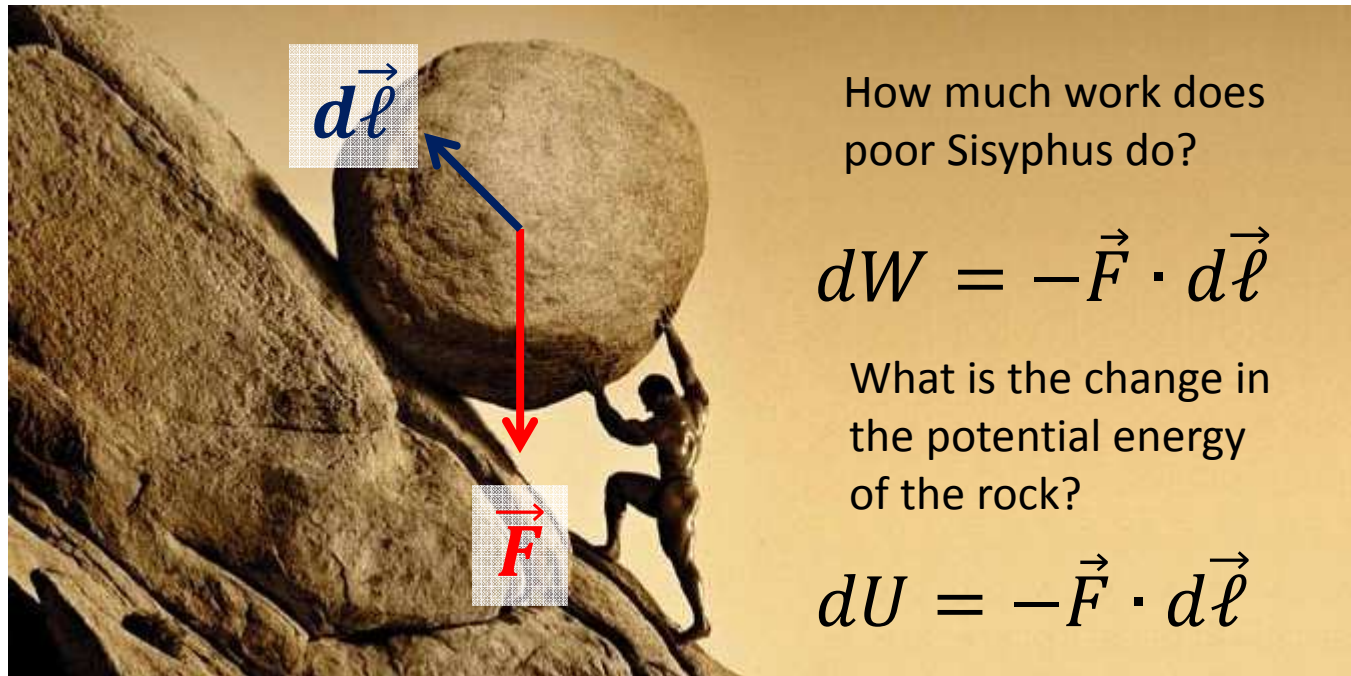
Tuesday's Question

- Apply Gauss's Law to the entire system.

- Gauss's Law: $\oint_S \hat{n} \cdot \vec{E}\, dA = \frac{Q_{inside}}{\epsilon_0}$

- No electric field inside the conductor, so the charge can only be on the inner and outer surfaces.

- We were told that the total charge was +10 Coulombs.

- We worked out that +3 Coulombs was on the inner surface.

- We must have +7 Coulombs on the outer surface.

Work done by a Force

- Definitions from mechanics (Physics 152/172):

How much work does poor Sisyphus do?

$$dW = -\vec{F} \cdot d\vec{\ell}$$

What is the change in the potential energy of the rock?

$$dU = -\vec{F} \cdot d\vec{\ell}$$

- *Conservative force*: Net work independent of the path.
  - Total work done: $W_{net} = U_f - U_i$ (change in potential energy)

Work Done by Electrostatic Force

- An electric field exerts a force on a charge.

$$d\vec{\ell} \longleftarrow \overset{q > 0}{\bullet} \longrightarrow \vec{F} \qquad \vec{E}$$

- If we push it to the left, we do work on the charge:
$$dW = -\vec{F} \cdot d\vec{\ell} > 0$$

  – The charge *gains* potential energy.
- If we release the charge, it accelerates to the right
  – The charge *loses* potential energy.
  – It gains kinetic energy (total energy is conserved).

Work Done by an Electric Field

- The force is related to the electric field:

$$\vec{F} = q\,\vec{E}$$

- The change in potential energy per unit charge is called the electrical potential difference:

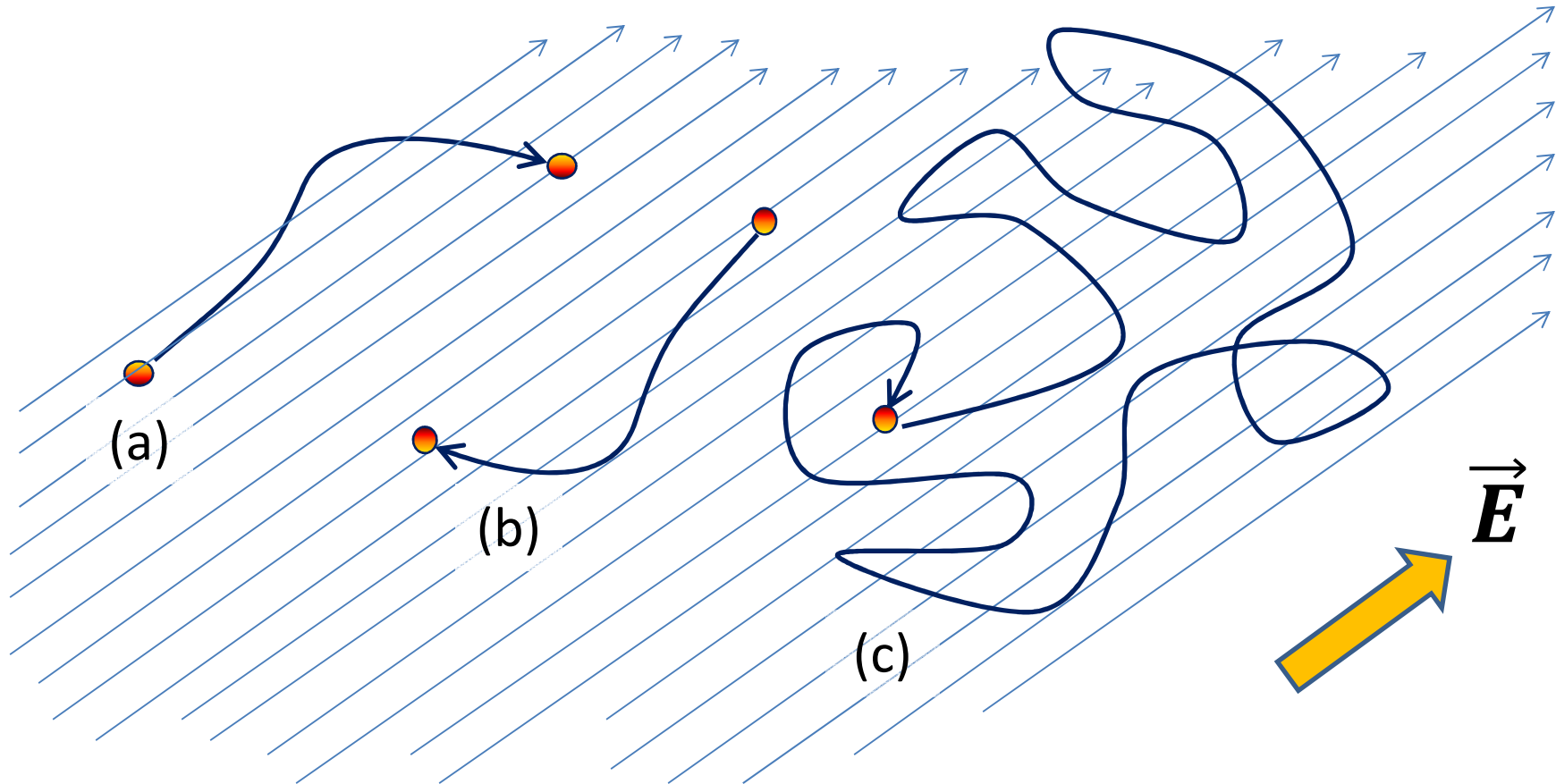$$dV = \frac{dU}{q} = -\vec{E} \cdot d\vec{\ell}$$

- As a unit charge is moved from point a to point b the potential difference is:

$$\Delta V = \int_a^b dV = -\int_a^b \vec{E} \cdot d\vec{\ell} = V_b - V_a$$
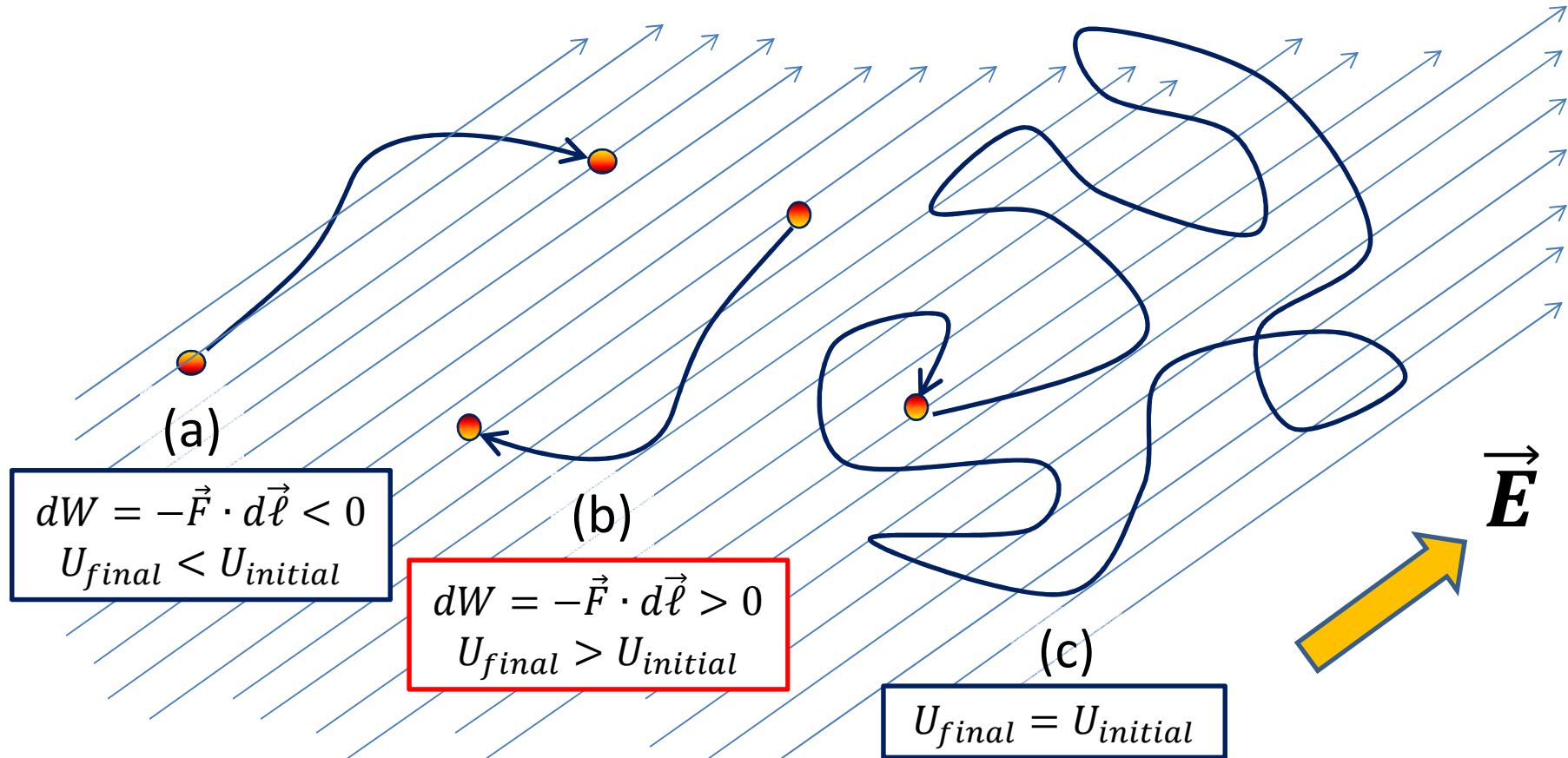
We can do this because the force is conservative!

Clicker Question

- Which path will give the biggest *increase* in the potential energy of a charge $q > 0$ in the constant electric field $\vec{E}$?

Clicker Question

- Which path will give the biggest *increase* in the potential energy of a charge q in the constant electric field $\vec{E}$?

(a)

$$dW = -\vec{F} \cdot d\vec{\ell} < 0$$
$$U_{final} < U_{initial}$$

(b)

$$dW = -\vec{F} \cdot d\vec{\ell} > 0$$
$$U_{final} > U_{initial}$$

(c)

$$U_{final} = U_{initial}$$

$$\vec{E}$$

Potential Energy vs Electric Potential

- Electric potential difference: **SI units: Joules per Coulomb**

$$\Delta V = \int_a^b dV = -\int_a^b \vec{E} \cdot d\vec{\ell} = V_b - V_a$$

  – This only depends on the electric potential at points a and b…

  – Electric potential is a *property of the field*.

- Difference in potential energy:

$$\Delta U = q\, \Delta V \qquad \text{SI units: Joules}$$

  – This depends on the charge, q…

  – It is not a property of the field.

SI Units

- Units of electric potential: **Joules per Coulomb**

- We will use this so frequently that we define:
   1 Volt = 1 Joule/Coulomb
- Electric field:
   1 Newton/Coulomb = 1 Volt/meter

Properties of the Electric Potential

- The electric field points in the direction of *decreasing* electric potential.
    - As Sisyphus' rock rolls down hill (direction of $\vec{F}$) it loses potential energy.
- So far, our definition only referred to changes in potential energy and differences in electric potential.
    - You can add an arbitrary constant to the electric potential without changing the potential difference.
    - But it must be the same value at all points in space.
- We usually define the *electric potential* as the potential difference relative to a convenient "reference point".
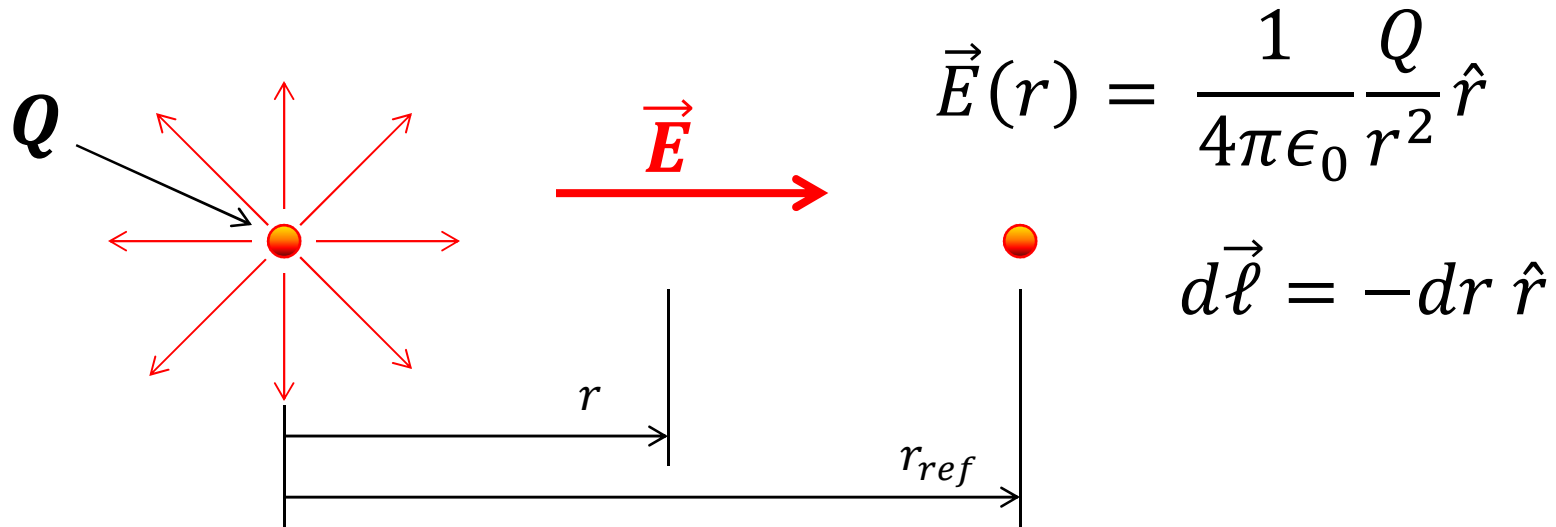
Calculating Electric Potential

- If the electric potential is a property of the field, how do we calculate it?

- The electric potential of a field at a point $\vec{x}$ is the work per unit charge required to move from the reference point, $\vec{x}_{ref}$, to the point, $\vec{x}$:

$$\frac{W}{q} = V(\vec{x}) - V_{ref} = -\int_{\vec{x}_{ref}}^{\vec{x}} \vec{E} \cdot d\vec{\ell}$$

- Maybe it would be nice to pick $\vec{x}_{ref}$ so that $V_{ref} = 0$.

Electric Potential due to a Point Charge

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

$$\vec{E}$$

$$d\vec{\ell} = -dr\,\hat{r}$$

Q

r

r_{ref}

$$V(r) - V(r_{ref}) = -\int_{r_{ref}}^{r}\vec{E}(r)\cdot d\vec{\ell}$$

$$= -\frac{Q}{4\pi\epsilon_0}\int_{r_{ref}}^{r}\frac{dr}{r^2} = \frac{Q}{4\pi\epsilon_0}\left(\frac{1}{r} - \frac{1}{r_{ref}}\right)$$

- We can make $V(r_{ref}) = 0$ if we let $r_{ref} \to \infty$.

Electric Potential due to several Point Charges

- Electric potential for a single point charge:

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \qquad \text{(when } r_{ref} \to \infty\text{)}$$

- Electric potential at point $\vec{x}$ due to several point charges:

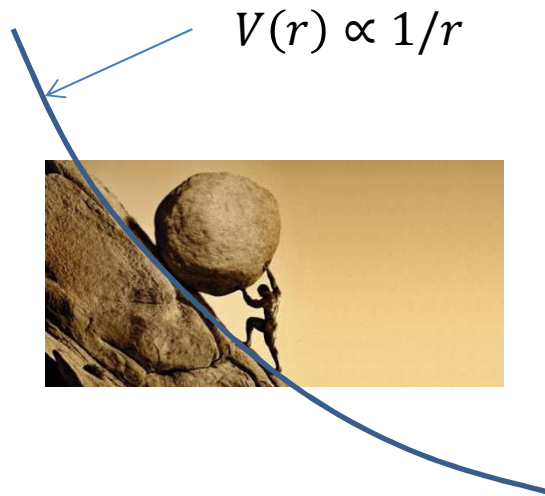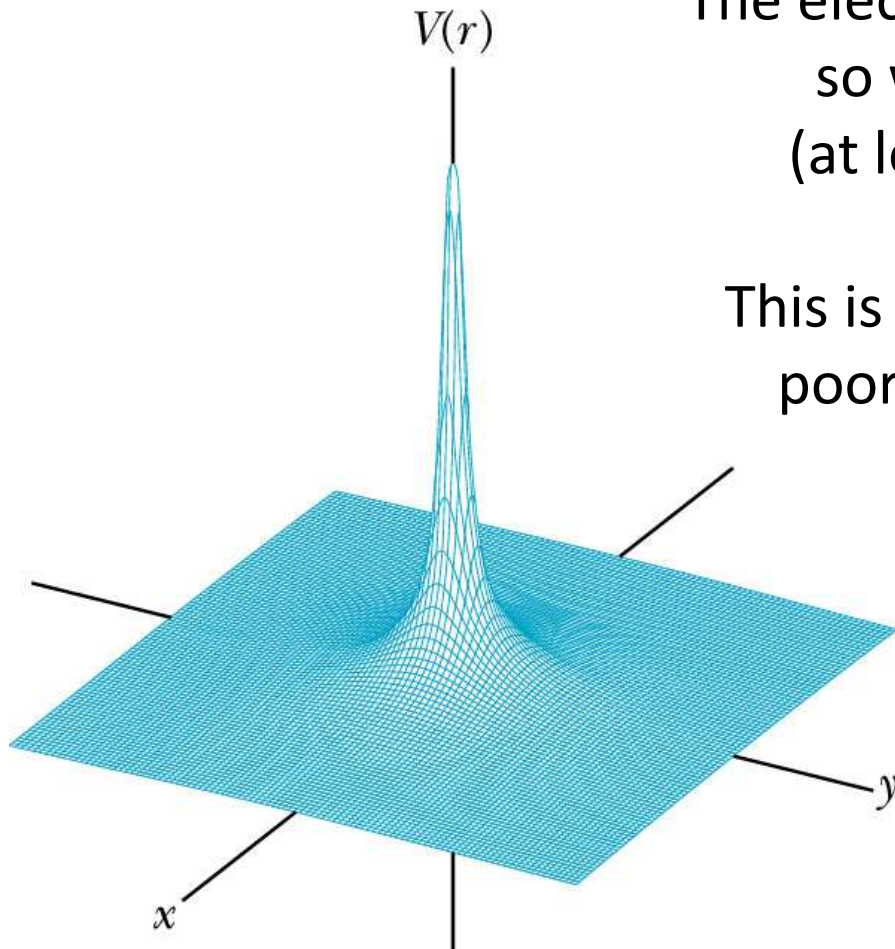$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{Q_i}{r_i}$$

where $r_i = |\vec{x} - \vec{x}_i|$ is the distance to each charge.

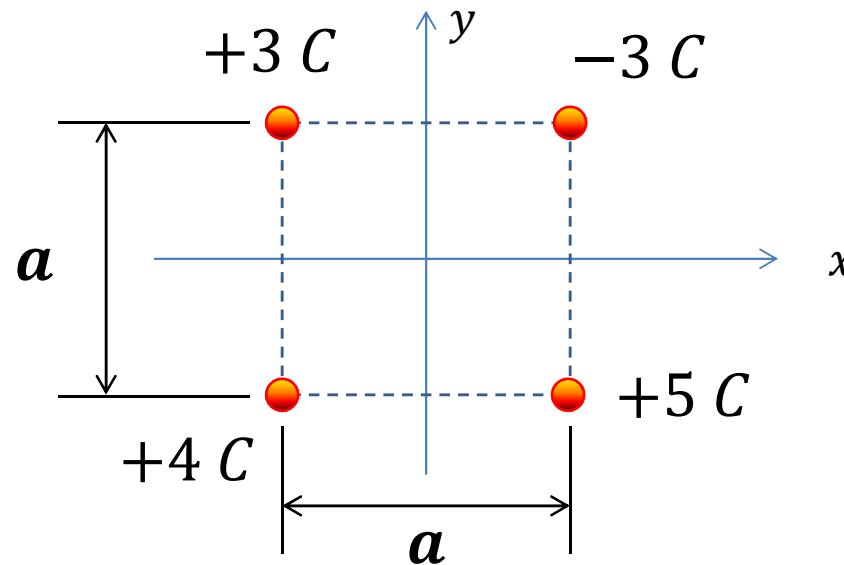- $V(\vec{x})$ is a *scalar* function (no direction).

Electric Potential

$V(r)$

The electric potential is a scalar function, so we can graph it as a surface.
(at least as a function of x and y).

This is the shape of the mountain that poor Sisyphus has to push his unit charge up.

$V(r) \propto 1/r$

x

y

Clicker Question

- What is the electric potential at the center of a square with charges at each corner?

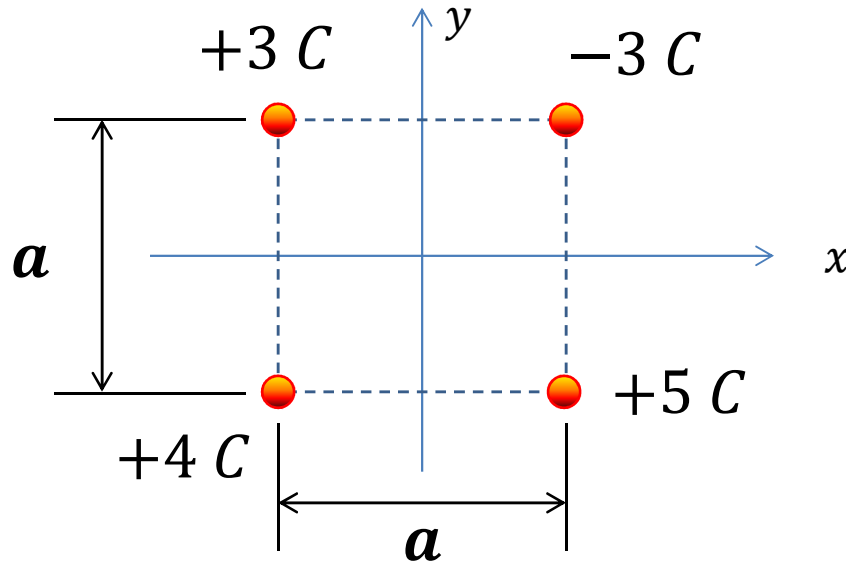(a) $V(\vec{0}) = \dfrac{1}{4\pi\epsilon_0} \dfrac{(9\,C)}{a/\sqrt{2}}$

(b) $V(\vec{0}) = \dfrac{1}{4\pi\epsilon_0} \dfrac{(9\,C)}{a^2/2}$

(c) $V(\vec{0}) = -\dfrac{1}{4\pi\epsilon_0} \dfrac{(9\,C)}{a/\sqrt{2}}$

(d) $V(\vec{0}) = \dfrac{1}{4\pi\epsilon_0} \dfrac{(15\,C)}{a/\sqrt{2}}$

Clicker Question

$+3\,C$ $-3\,C$

$+4\,C$ $+5\,C$

$$V(\vec{x}) = \frac{1}{4\pi\epsilon_0} \sum_i \frac{Q_i}{r_i}$$

r_i is the same for all points:

$$r_i = \frac{a\sqrt{2}}{2} = a/\sqrt{2}$$

Sum of the charges is +9 C.

(a) $V(\vec{0}) = \dfrac{1}{4\pi\epsilon_0} \dfrac{(9\,C)}{a/\sqrt{2}}$

(b) $V(\vec{0}) = \dfrac{1}{4\pi\epsilon_0} \dfrac{(9\,C)}{a^2/2}$

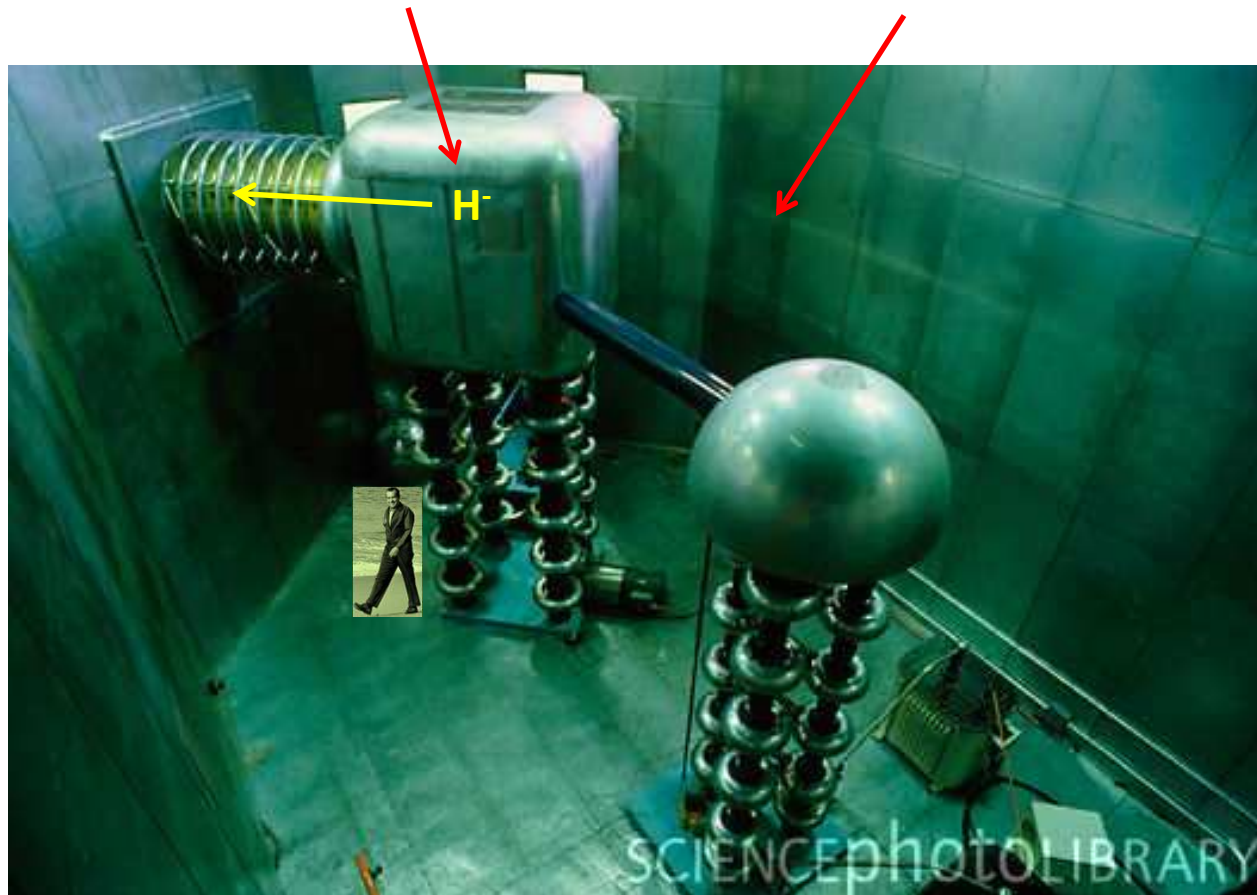(c) $V(\vec{0}) = -\dfrac{1}{4\pi\epsilon_0} \dfrac{(9\,C)}{a/\sqrt{2}}$

(d) $V(\vec{0}) = \dfrac{1}{4\pi\epsilon_0} \dfrac{(15\,C)}{a/\sqrt{2}}$

Example

This enclosure has an electric potential of **-750 kV**.

The walls of this room have an electric potential of zero volts.

Negatively charged hydrogen ions are produced in the enclosure.

How much kinetic energy do they have when the leave the room?

Example

- Potential energy inside the enclosure:

$$U_{inside} = q \, V_{inside}$$

$$q = -1.602 \times 10^{-19} \text{ Coulombs}$$

$$V_{inside} = -750 \times 10^3 \text{ Volts}$$

$$= -750 \times 10^3 \text{ J/C}$$

$$U_{inside} = (-1.602 \times 10^{-19} \text{ C})(-750 \times 10^3 \text{ J} \cdot \text{C}^{-1})$$

- Potential energy outside the enclosure:

$$U_{outside} = 0$$

- Energy is conserved: loss of potential energy = gain in kinetic energy.

$$\boldsymbol{T = 1.20 \times 10^{-13} \text{ Joules}}$$

Other Units for Energy

- When we discuss the energy of elementary charged particles with $e = 1.602 \times 10^{-19}\ C$ measuring energy in Joules is inconvenient.

- Instead, we use "electron-Volts":
$$1\ eV = 1.602 \times 10^{-19}\ \text{Joules}$$

- This is the energy that an electron gains or loses when its moves through an electric potential difference of 1 volt.

- In the previous example, the hydrogen ions would leave the room with a kinetic energy of **750 keV**.

Electric Field (again)

- We can calculate the electric field from the electric potential:

$$\vec{E} = -\vec{\nabla} V = -\left(\frac{\partial V}{\partial x}\hat{\imath} + \frac{\partial V}{\partial y}\hat{\jmath} + \frac{\partial V}{\partial z}\hat{k}\right)$$

- This is called the "gradient" of the electric potential.

- Useful relations:

$$\frac{\partial V(r)}{\partial x} = \frac{\partial V}{\partial r}\frac{\partial r}{\partial x}$$

The "chain rule"...
Also works for y and z.

$$r = \sqrt{x^2 + y^2 + z^2} \implies \frac{\partial r}{\partial x} = \frac{x}{r} \quad \frac{\partial r}{\partial y} = \frac{y}{r} \quad \frac{\partial r}{\partial z} = \frac{z}{r}$$

$$\vec{\nabla} r = \hat{r}$$

Electric Field from a Point Charge

- Electric potential:

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

- Electric field:

$$\vec{E} = -\vec{\nabla}V = -\frac{\partial V}{\partial r}\vec{\nabla}r = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$$

Constant Electric Field

- What electric potential produces a constant electric field, $\vec{E} = E\hat{\imath}$?
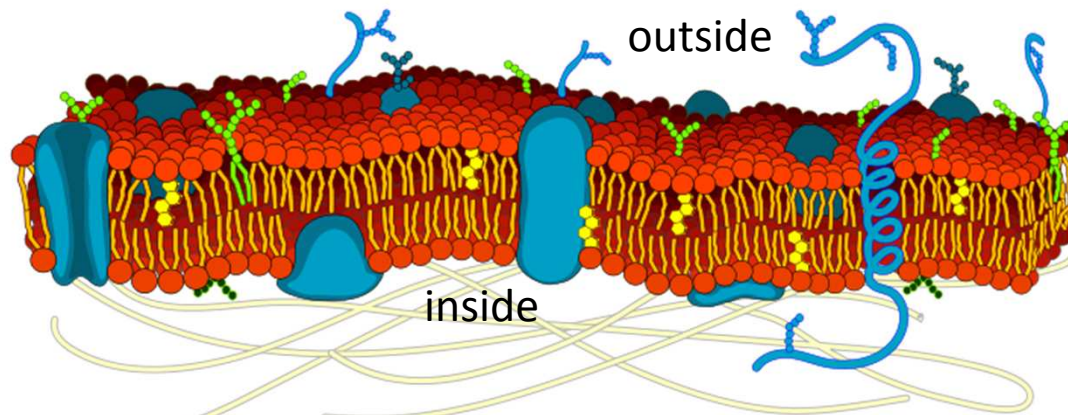- Electric field:

$$\vec{E} = -\vec{\nabla}V$$

  - What function will have $\frac{\partial V}{\partial x} = -E, \frac{\partial V}{\partial y} = 0, \frac{\partial V}{\partial z} = 0$?
  - How about $V(\vec{x}) = -Ex$…

- A *linear* electric potential produces a *constant* electric field.

Final Clicker Question

- Assume that the electric field in a cell membrane is constant and the electric potential on the *inside* of the membrane is **50 mV** *lower* than the *outside*.
- What is the electric field in a cell membrane that is **5 nm** thick?

outside

inside

+

This is the "+" direction. A positive E-field would point out.

$$V_{outside} = 0 \text{ V}$$
$$V_{inside} = -50 \text{ mV}$$
$$\Delta x = 5 \text{ nm}$$

(a) $2.5 \times 10^{-6} \text{ V} \cdot \text{m}^{-1}$

(b) $-10^7 \text{ V} \cdot \text{m}^{-1}$

(c) $+10^7 \text{ V} \cdot \text{m}^{-1}$

(d) $2.5 \times 10^6 \text{ V} \cdot \text{m}^{-1}$