PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 24100

Electricity & Optics

Lecture 28 – Review of chapters 29-33
(Lectures 19-27)

Fall 2012 Semester

Matthew Jones

ANNOUNCEMENT

***Exam 1**: Friday December 14, 2012, 8 AM – 10 AM

*__Location:__ Elliot Hall of Music

*Covers all readings, lectures, homework from Chapters 29 through 33.

*The exam will be multiple choice.

Be sure to bring your student ID card and a hand-written one-page (two sided) crib sheet plus the crib sheets that you prepared for exams 1 and 2.

NOTE THAT FEW EQUATIONS WILL BE GIVEN – YOU ARE REMINDED THAT IT IS YOUR RESPONSIBILITY TO CREATE WHATEVER TWO-SIDED CRIB SHEET YOU WANT TO BRING TO THIS EXAM.

The equation sheet that will be given with the exam is posted on the course homepage. Click on the link on the left labeled "EquationSheet"

Review of Chapters 29-33

This lecture reviews some, but not all of the material that will be on the final exam covering Chapters 29-33.

Alternating Current Circuits

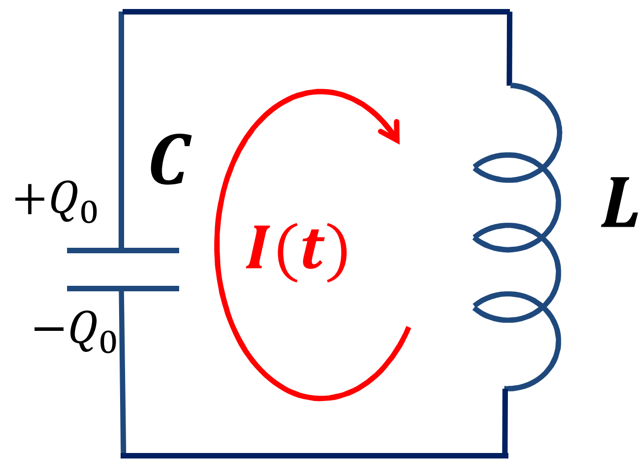
Stored energy:

$$U_e = \frac{1}{2C}Q^2 = \frac{1}{2}CV^2$$

$$U_m = \frac{1}{2}LI^2$$

Oscillation frequency:

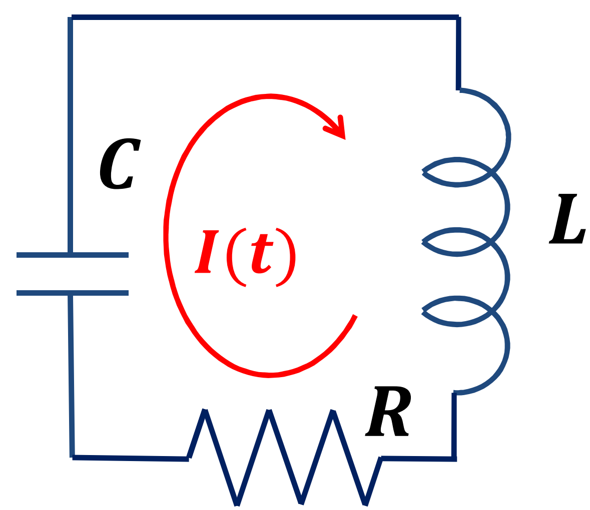
$$\omega_0 = \frac{1}{\sqrt{LC}}$$



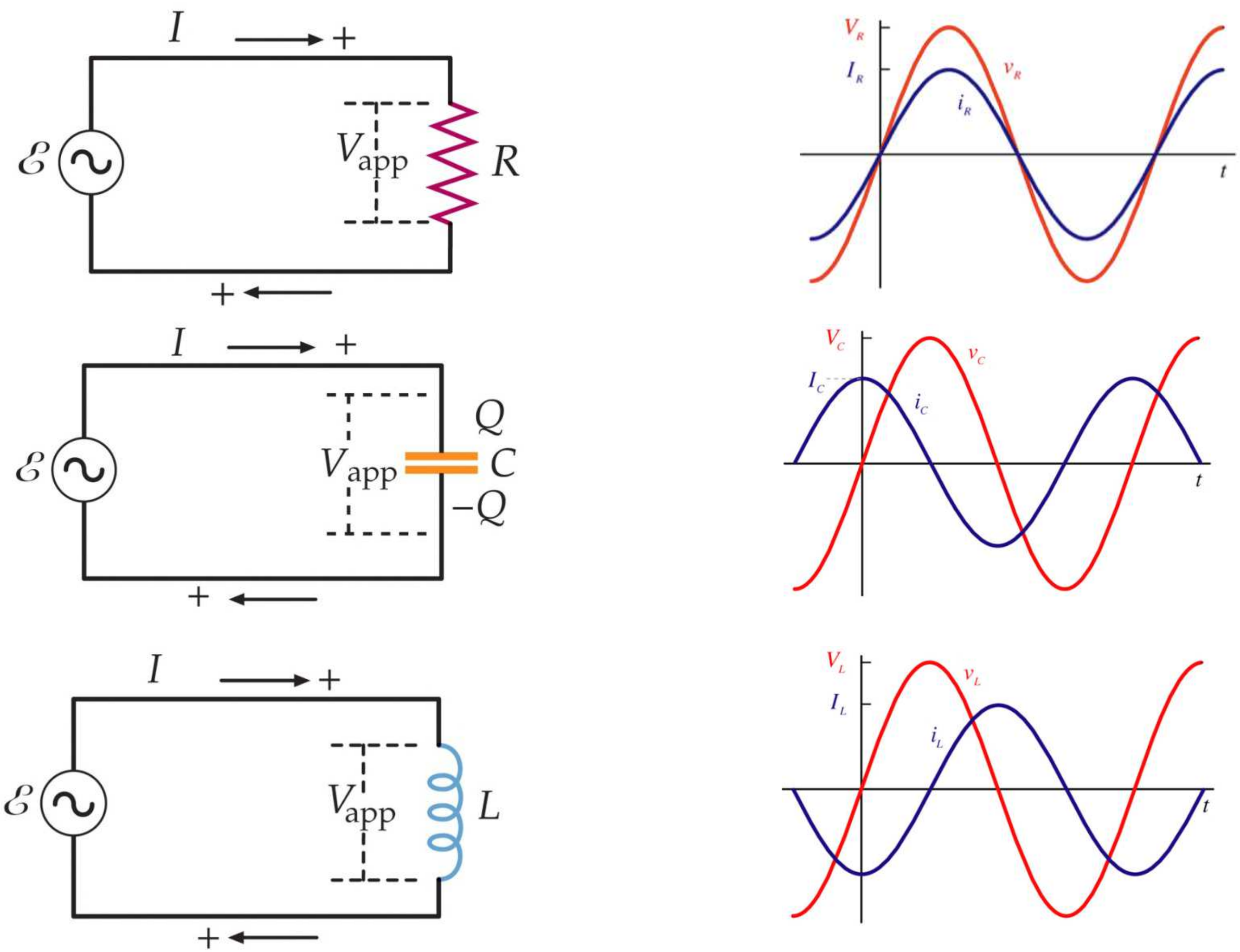
RLC Circuit:

$$I(t) = I_0 e^{-Rt/2L}\cos(\omega t + \varphi)$$

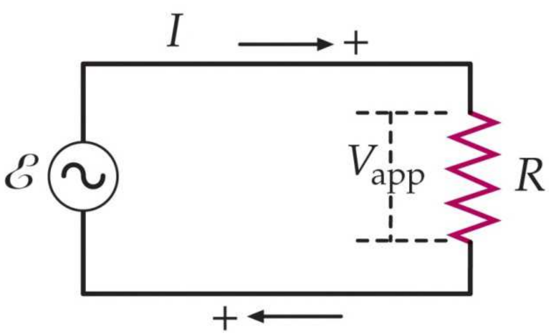
$$\omega = \sqrt{(\omega_0)^2 - \left(\frac{R}{2L}\right)^2}$$



Time Varying EMF



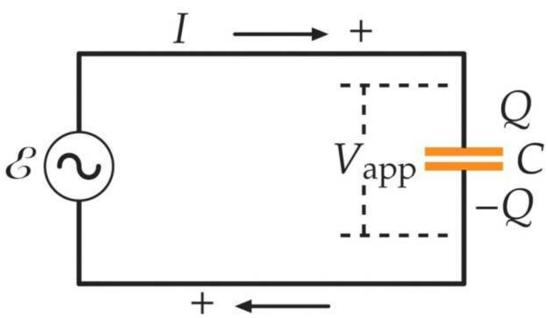
Relation between $I(t)$ and $\mathcal{E}(t)$



$$\mathcal{E}(t) = \mathcal{E}_0 \sin \omega t$$

$$I_R(t) = \frac{\mathcal{E}_0}{R} \sin \omega t$$

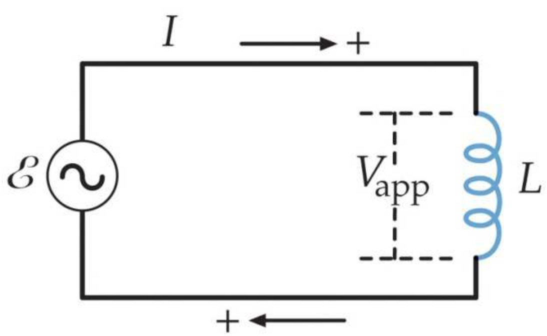
Voltage and current are in phase.



$$I_C(t) = \frac{\mathcal{E}_0}{X_C} \sin(\omega t + 90°)$$

$$X_C = 1/\omega C$$

"Voltage lags the current by 90 degrees"

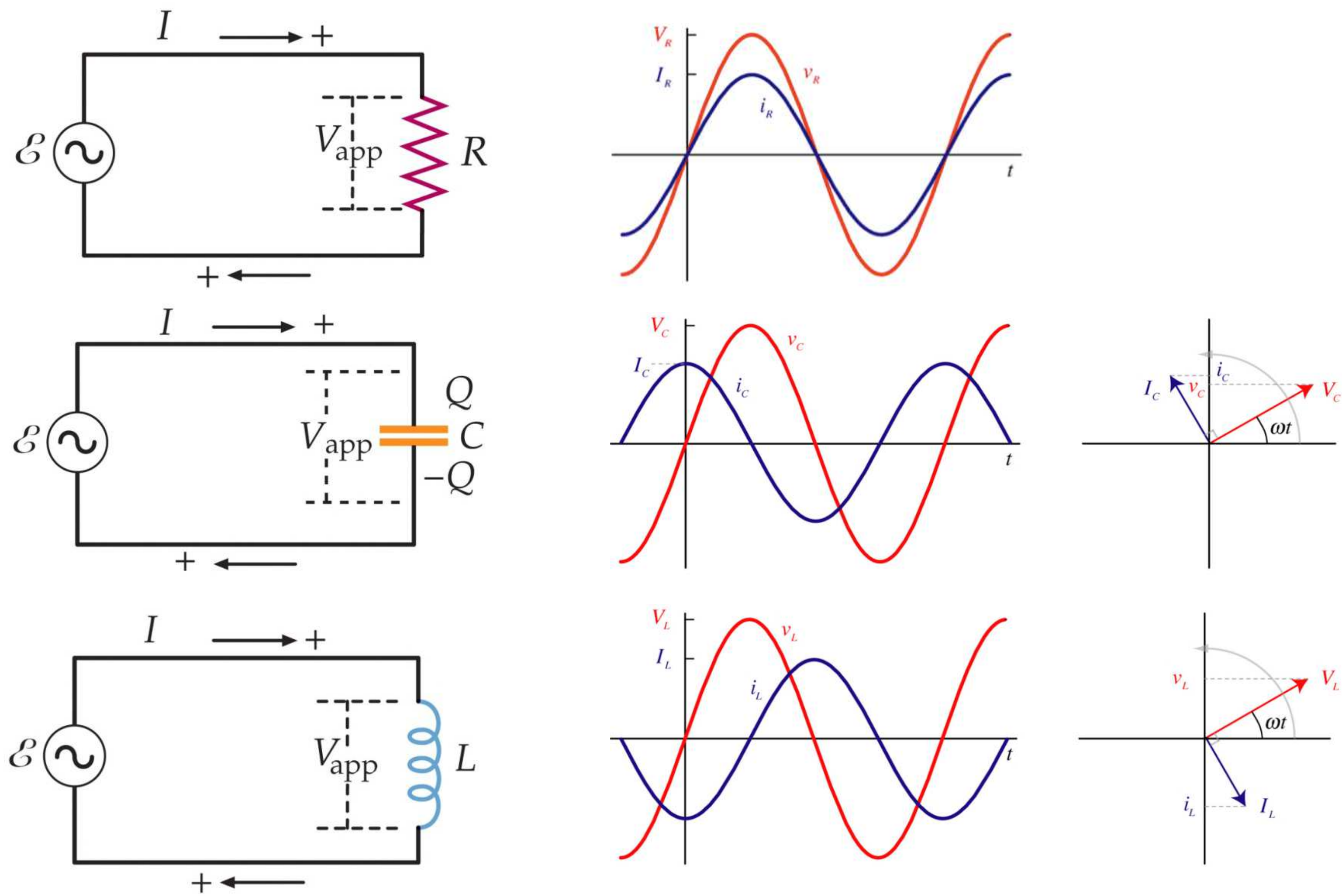


$$I_L(t) = \frac{\mathcal{E}_0}{X_L} \sin(\omega t - 90°)$$

$$X_L = \omega L$$

"Voltage leads the current by 90 degrees"

Phasor Diagrams

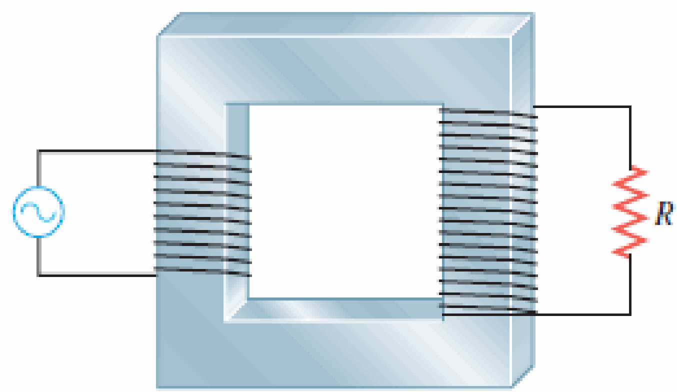


Transformers

Primary winding:
N_P turns.

Applied EMF:

$$V_P = -N_P \frac{d\Phi}{dt}$$



Secondary winding:
N_S turns.

Induced EMF:

$$V_S = -N_S \frac{d\Phi}{dt}$$

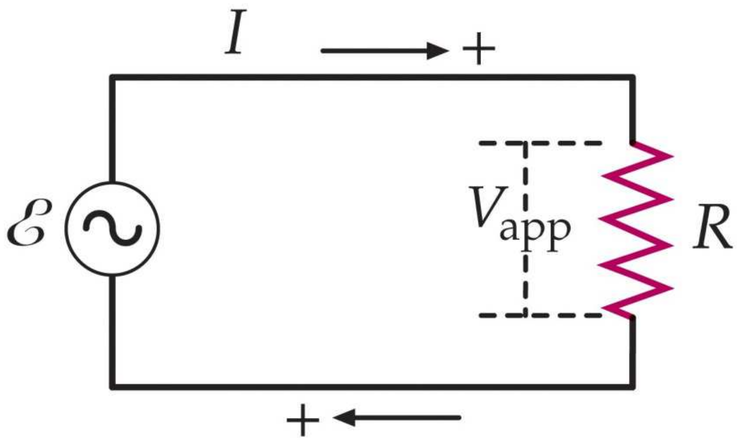
Equal flux Φ through both windings.

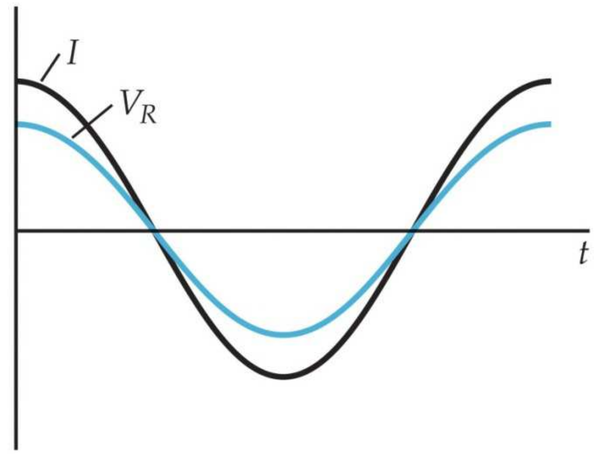
$$\frac{V_P}{N_P} = \frac{V_S}{N_S}$$

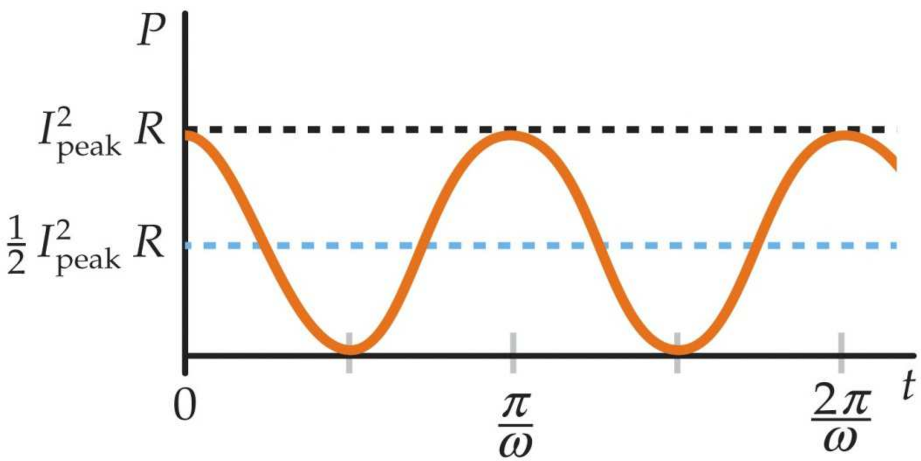
Secondary EMF: $V_S = V_P \frac{N_S}{N_P}$

Step-up transformer: $N_S/N_P > 1$ and $V_S > V_P$
Step-down transformer: $N_S/N_P < 1$ and $V_S < V_P$

RMS Voltage and Current







$$I_{peak} = \frac{V_{R\,peak}}{R}$$

$$P_{av} = (I_{RMS})^2 R$$

$$I_{RMS} = \sqrt{\frac{1}{2}\left(I_{peak}\right)^2} = \frac{I_{peak}}{\sqrt{2}}$$

(for sinusoidal waveforms)

Maxwell's Equations

$$\oint_S \hat{n} \cdot \vec{E}\, dA = \frac{Q_{inside}}{\epsilon_0}$$

Gauss's Law

$$\oint_S \hat{n} \cdot \vec{B}\, dA = 0$$

"No magnetic monopoles"

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt}$$

Faraday's Law

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_e}{dt}$$

Ampere's Law with
displacement current

Maxwell's Equations in Free Space

$$\left.\begin{aligned}\oint_C \vec{E}\cdot d\vec{\ell} &= -\frac{d\phi_m}{dt}\\ \oint_C \vec{B}\cdot d\vec{\ell} &= \mu_0\varepsilon_0\frac{d\phi_e}{dt}\end{aligned}\right\} \quad \frac{\partial^2 E_x}{\partial z^2} = \mu_0\epsilon_0\frac{\partial^2 E_x}{\partial t^2}$$

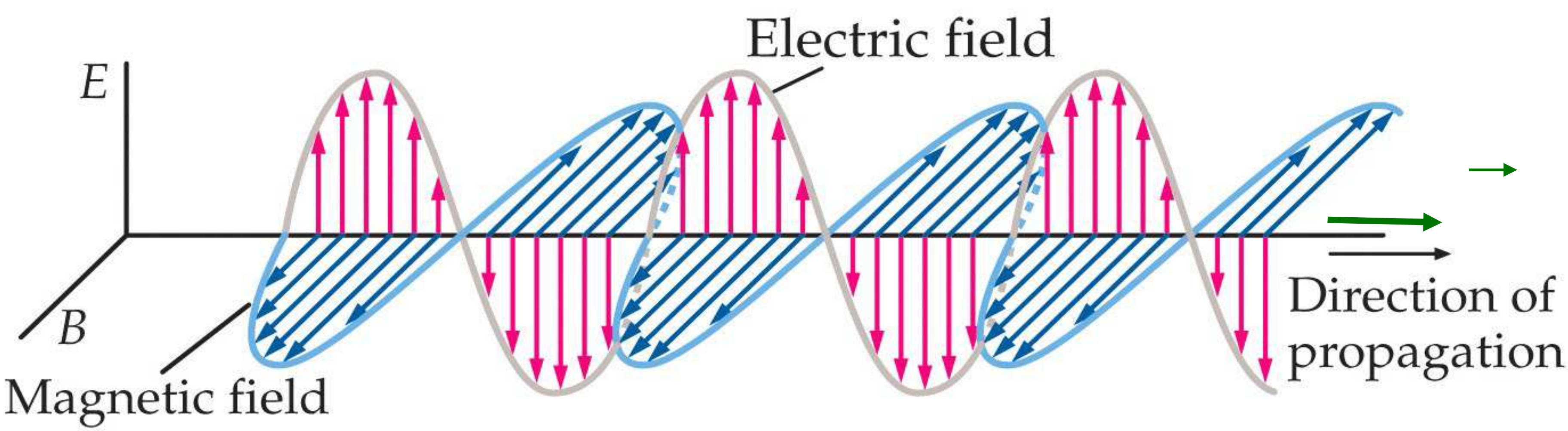
$$E_x(z,t) = E_0 \sin(kz - \omega t)$$

$$\omega = 2\pi f = kc = 2\pi c/\lambda$$

$$c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = \lambda f$$

Characteristics

- $\vec{E}$ and $\vec{B}$ are perpendicular
- If $E = E_0 \sin(kz - \omega t)$ then $B = B_0 \sin(kz - \omega t)$
- The Poynting vector, $\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$ is in the direction of propagation
- $\vec{E}$ and $\vec{B}$ are perpendicular to $\vec{S}$



Energy of Electromagnetic Waves

- Energy stored in electromagnetic waves:

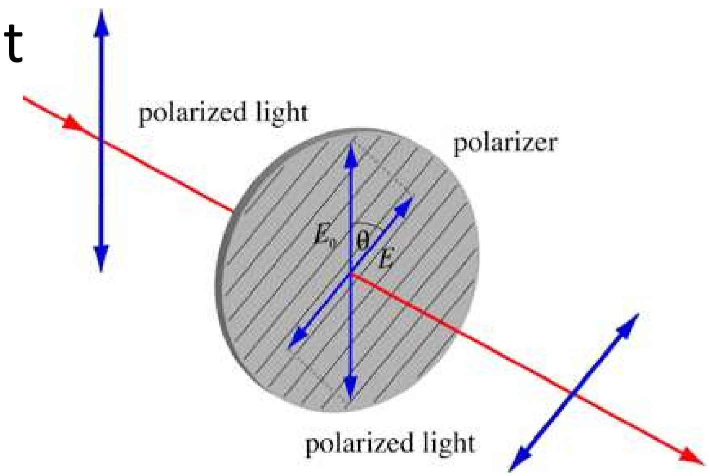
$$u_e = \frac{1}{2}\epsilon_0 E^2 \qquad u_m = \frac{1}{2\mu_0}B^2$$

$$u_m = \frac{1}{2\mu_0}B^2 = \frac{1}{2}\epsilon_0 E^2 = u_e \qquad B = E/c = E\sqrt{\mu_0\epsilon_0}$$

- Light intensity: $I = \frac{\langle E^2 \rangle}{\mu_0 c}$
  - Power per unit area
- Radiation pressure: $P_r = \frac{I}{c}$
  - Force per unit area

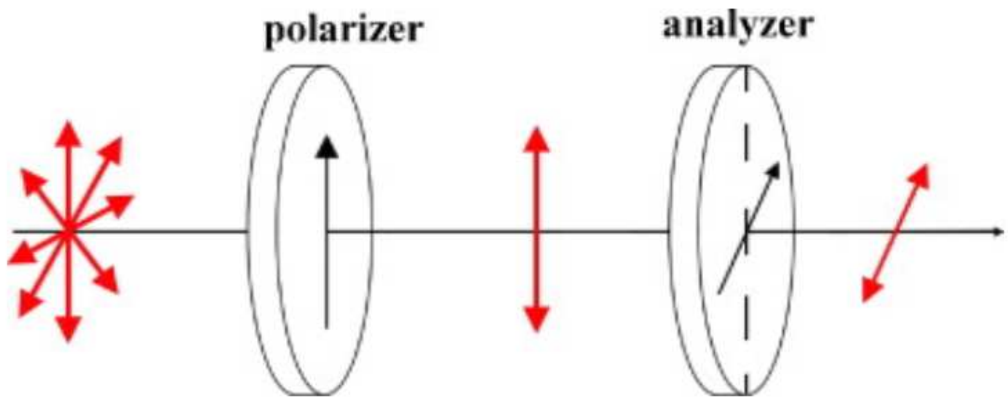
Polarized Light

Polarizers transmit only the component of $\vec{E}$ parallel to the polarizing axis.



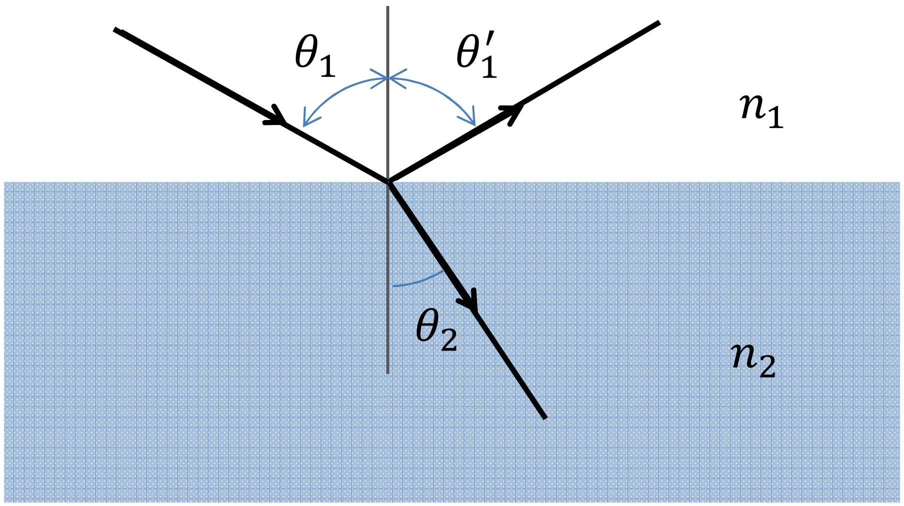
If the incident light is un-polarized, the intensity is reduced by 1/2.

Two polarizers:



Malus's Law: $I = I_0 \cos^2\theta$

Geometric Optics



Reflection:

$$\theta_1' = \theta_1$$

Refraction:

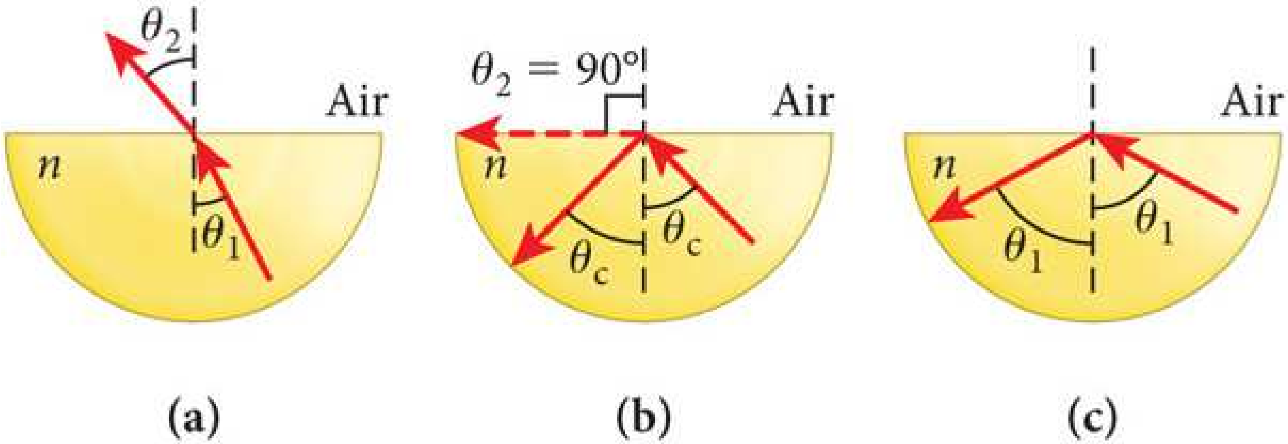
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

(Snell's law)

In a material with index of refraction, $n > 1$:

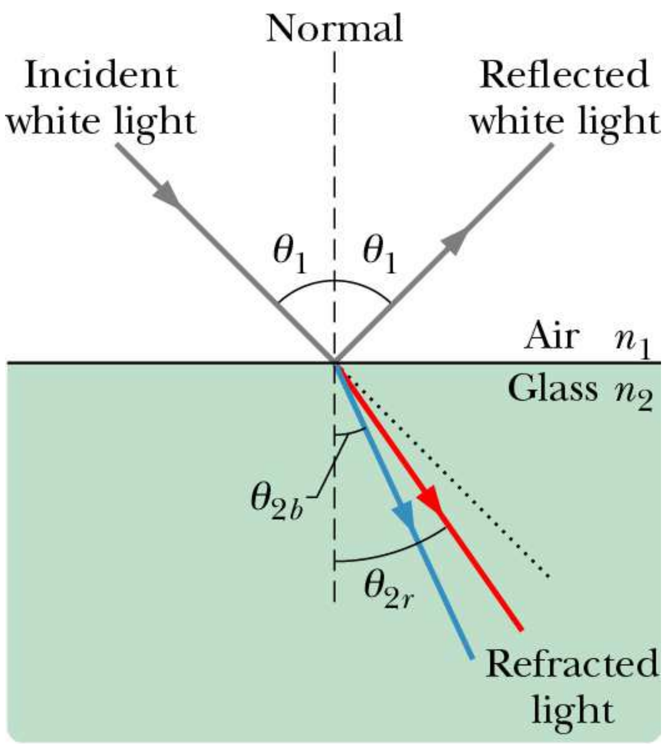
- Speed of light: $v = \frac{c}{n}$
- Wavelength: $\lambda' = \lambda/n$

Total Internal Reflection

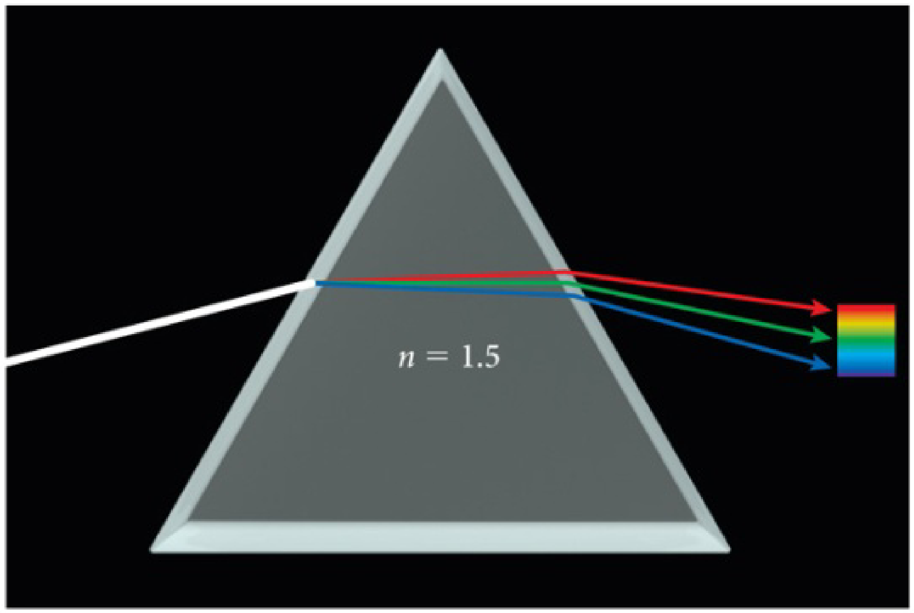


Critical angle θ_C defined by $n \sin \theta_C = 1$.
At angles greater than θ_C, all light is reflected from the surface.

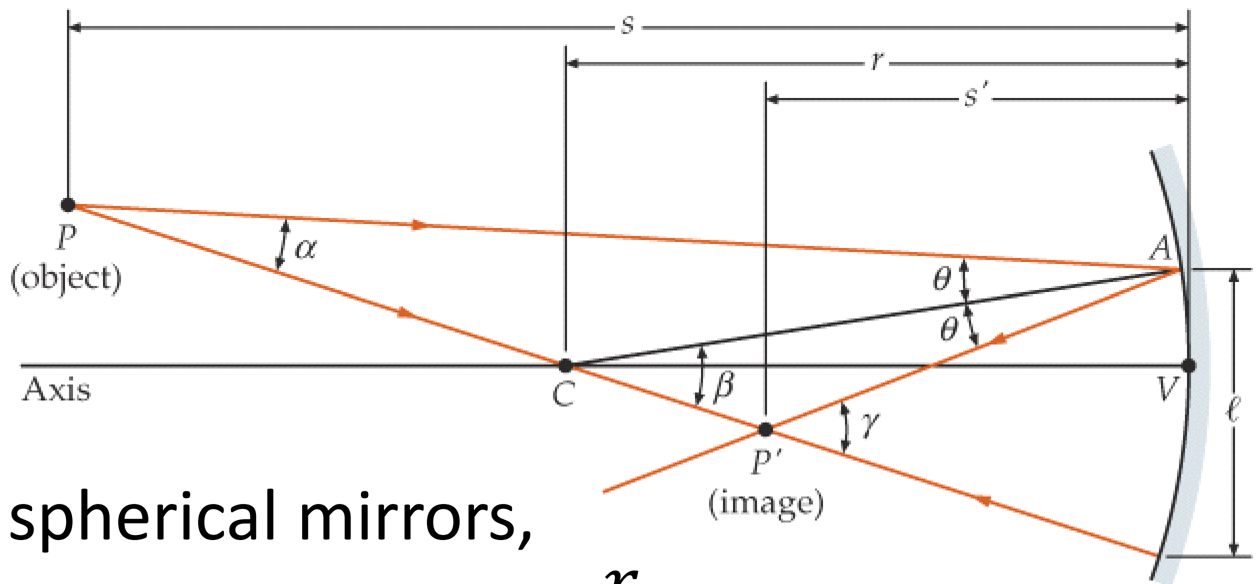
Chromatic Dispersion



- The index of refraction depends on the wavelength of light
  - It is usually larger at shorter wavelengths.



Optical Images from Mirrors



For spherical mirrors,

$$f = \frac{r}{2}$$

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

$$m = -\frac{s'}{s}$$

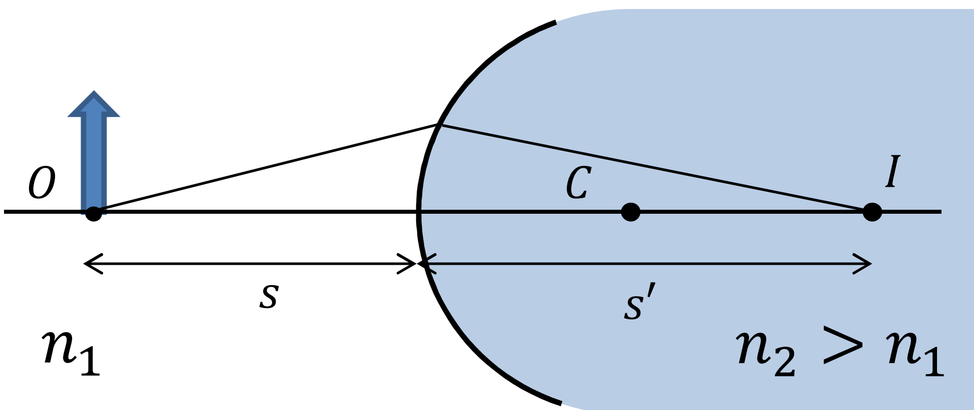
Concave mirrors:
$r > 0$ and $f > 0$

Convex mirrors:
$r < 0$ and $f < 0$

Refraction from one Surface

- Snell's Law:

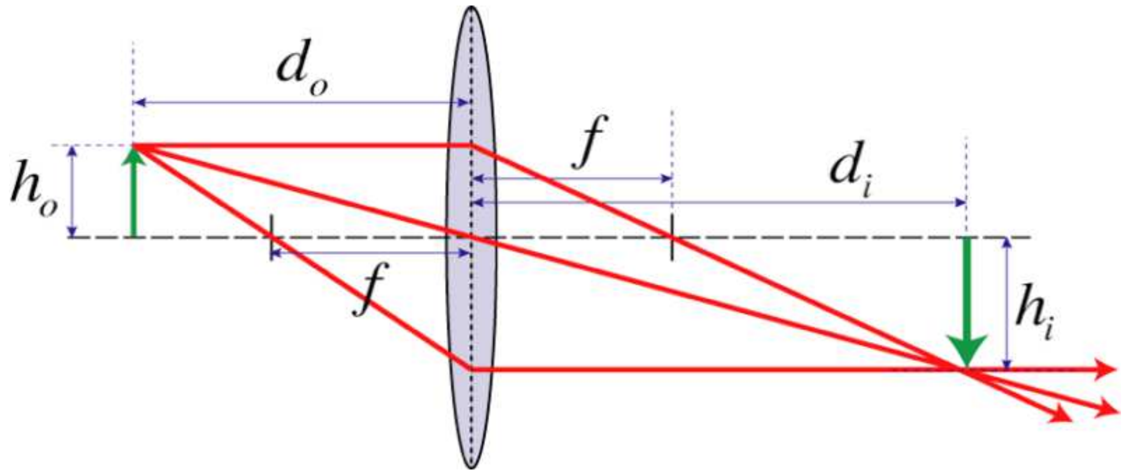
$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \rightarrow n_1\theta_1 = n_2\theta_2$$



$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$

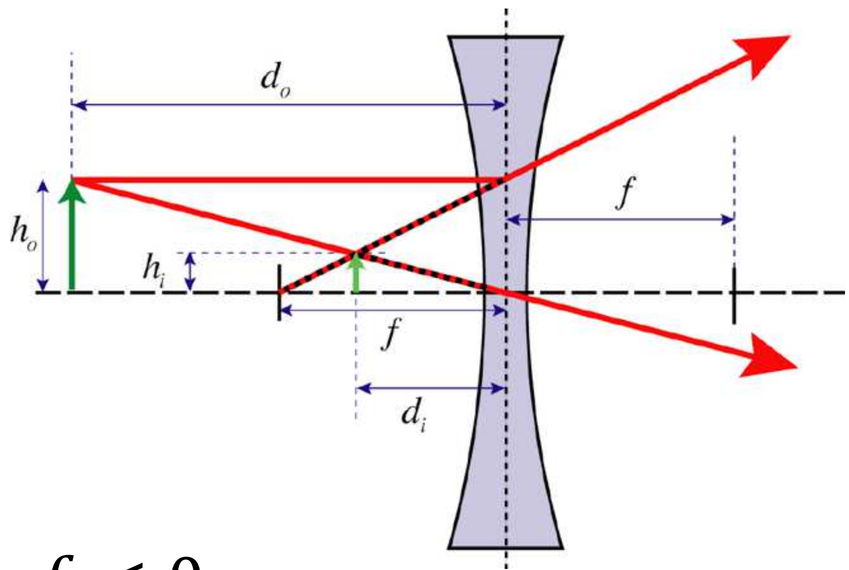
If the surface is concave, then $r < 0$

Optical Images from Lenses



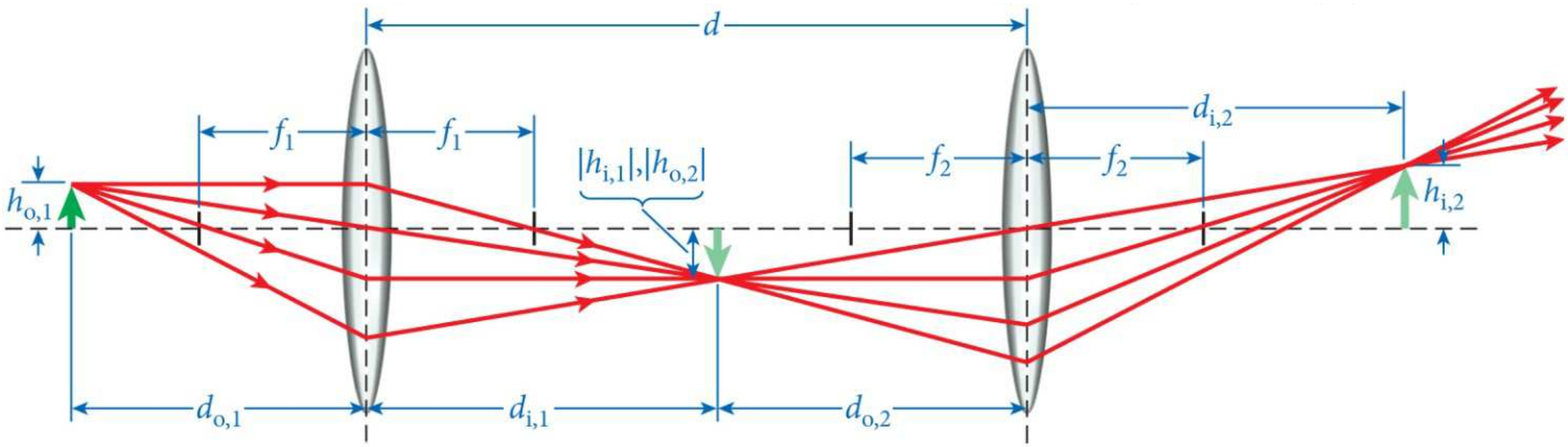
- Lens-maker's formula: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
- Thin lens equation: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$
- Magnification: $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

Optical Images from Lenses



- Concave lenses have $f < 0$
- Lens-maker's formula: $\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$
- Thin lens equation: $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$
- Magnification: $m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

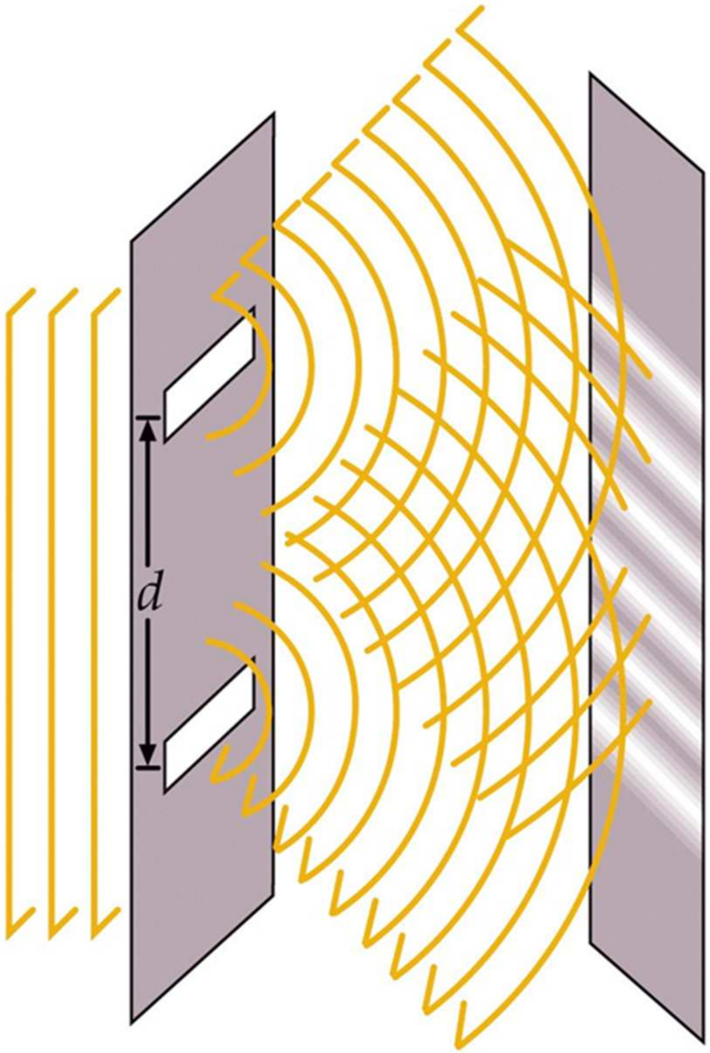
Systems of Lenses



$$\frac{1}{d_{o,1}}+\frac{1}{d_{i,1}}=\frac{1}{f_1} \qquad \frac{1}{d_{o,2}}+\frac{1}{d_{i,2}}=\frac{1}{f_2}$$

$$m=m_1m_2=\left(\frac{h_{i,1}}{h_{o,1}}\right)\left(\frac{h_{i,2}}{h_{o,2}}\right)=\left(\frac{h_{i,2}}{h_{o,1}}\right)$$

Interference and Diffraction



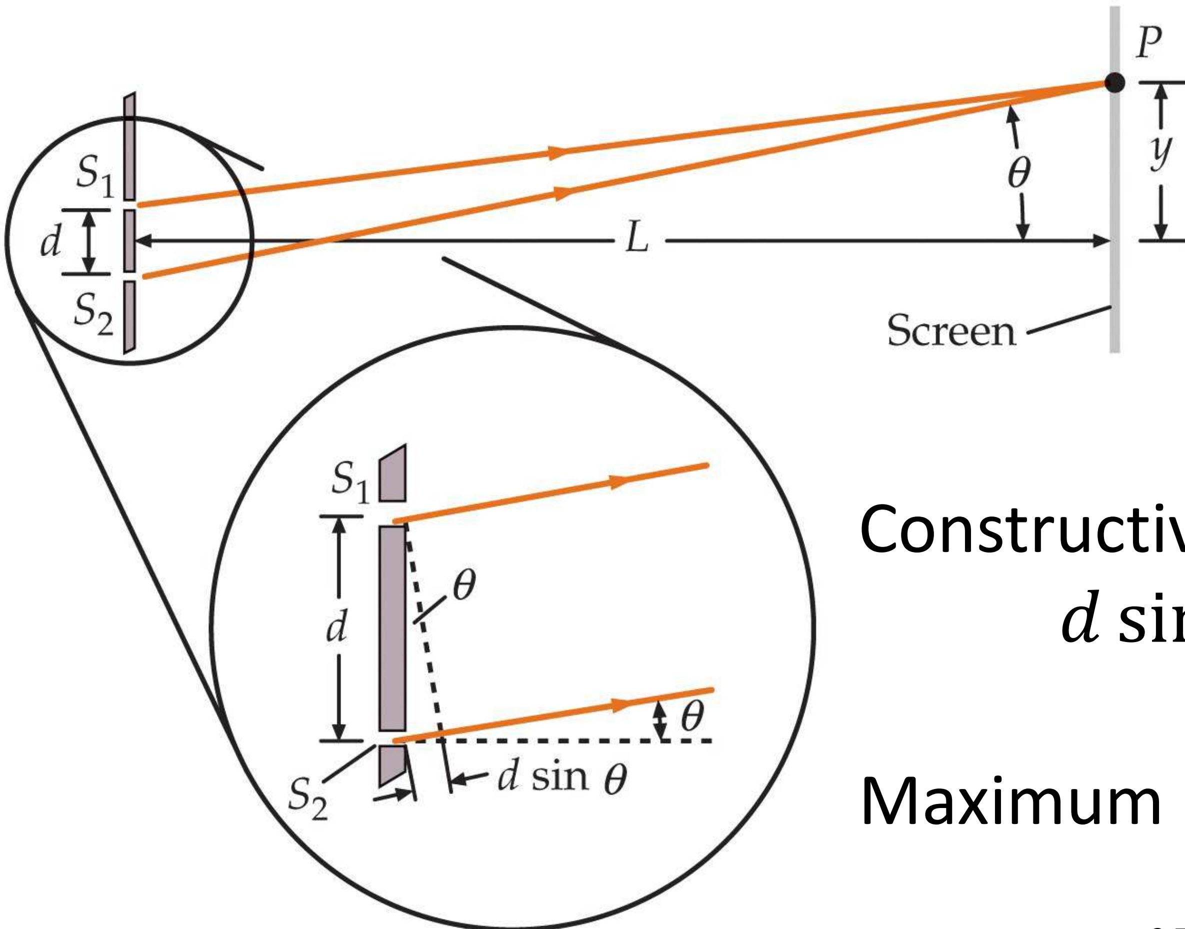
Huygens' principle:

- Each point on a propagating wave-front acts like a source of spherical waves
- These interfere destructively except in the forward region or when obscured by an obstacle.

Interference

- Destructive interference:
  - Path length differs by $\Delta x = m\frac{\lambda}{2}$ where $m = 0,1,2,$
- Constructive interference:
  - Path length differs by $\Delta x = m\lambda$
- Phase differences caused by
  - Different path lengths
  - Different indices of refraction
  - Reflection from a surface with larger n

Interference



$$\tan \theta = y/L$$

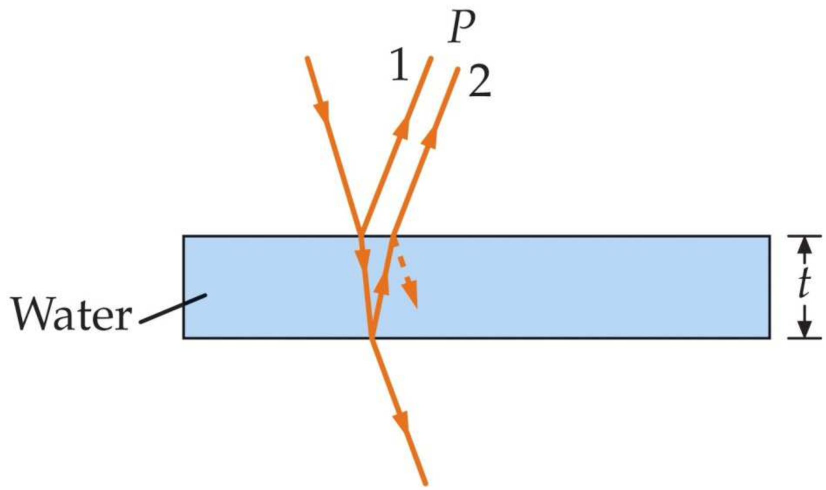
Constructive interference:

$$d \sin \theta = m\lambda$$

Maximum intensity occurs at

$$y = m \frac{\lambda L}{d}$$

Interference from Thin Films

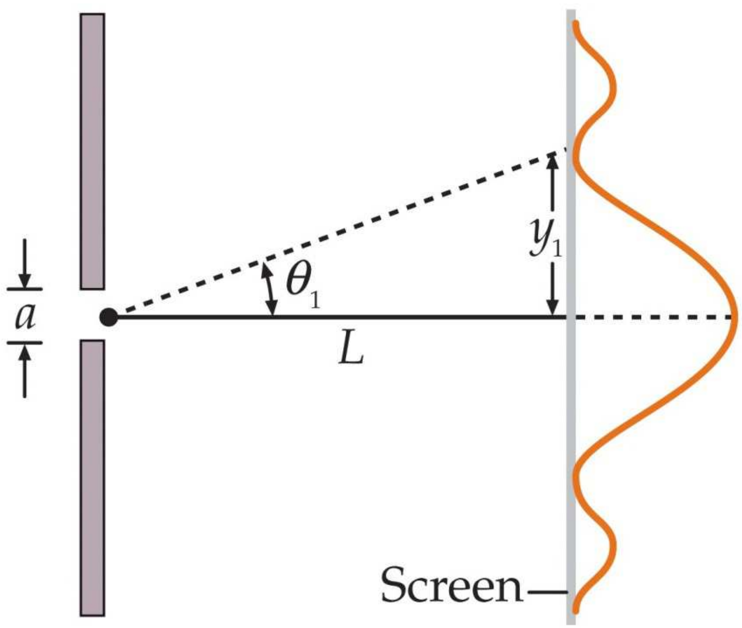


- Light reflected from the top surface has a phase shift of $\lambda/2$.
- Wavelength in the film: $\lambda' = \lambda/n$
- Number of wavelengths in distance $2t$ is $2t/\left(\frac{\lambda}{n}\right)$
- Bright fringes when $2t = (m + 1/2)\frac{\lambda}{n}$
- Dark fringes when $2t = m\frac{\lambda}{n}$

Diffraction

Position of minima for light transmitted through an single slit of width a:

$$y_{min} = mL\frac{\lambda}{a} \qquad m = 1,2,3,\ldots$$



For a circular aperture of diameter D:

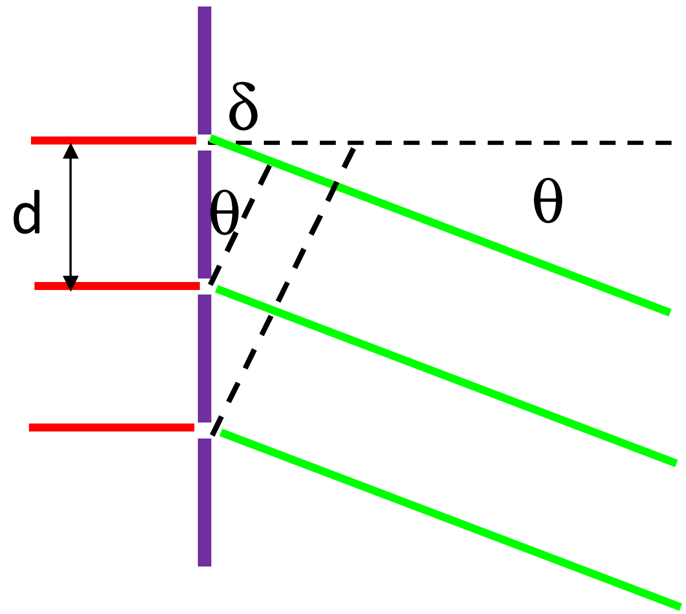
$$\sin\theta_{min} = 1.22\frac{\lambda}{D}$$

Rayleigh's criteria:

- Images are resolvable when

$$\Delta\theta > \theta_R = \sin^{-1}\left(\frac{1.22\lambda}{D}\right)$$

Diffraction Gratings



N lines per unit length.

- Constructive interference when

$$\delta = d \sin\theta = m\lambda$$

- Width of individual lines is

$$\Delta\theta = \frac{\lambda}{Nd}$$

- Resolving power:

$$R = \frac{\lambda}{|\Delta\lambda|} = \frac{m\lambda}{d\ \Delta\theta} = mN$$

Main application:

Determining λ by measuring θ when N is known.

The Very Last Clicker Question

- My favorite part of the course was:

(A) Charges, potential, electric fields

(B) Magnetic fields, induction, RLC, etc.

(C) Optics: lens, mirrors, interference, diffraction, etc.

(D) I don't know and after the final I never want to think about this material again.

(E) All of the material and I hope to use some or all this material in my future career.