PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 24100

Electricity & Optics

Lecture 21 – Chapter 30 sec. 1-4

Fall 2012 Semester

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Question

- An LC circuit has $C = 100\ pF$ and $L = 100\ \mu H$.
- If it oscillates with an amplitude of 100 mV, what is the amplitude of the current?

(a) $100\ mA$

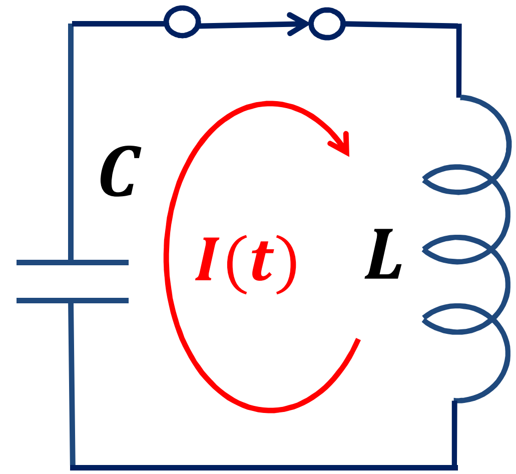
(b) $100\ \mu A$

(c) $10\ mA$

(d) $10\ \mu A$

Question

- The energy bounces back and forth between the capacitor and the inductor.



$$U_e = \frac{1}{2} C V^2$$

$$U_m = \frac{1}{2} L I^2$$

$$I = \sqrt{\frac{CV^2}{L}} = \sqrt{\frac{(10^{-10}\ F)(10^{-2}\ V^2)}{(10^{-4}\ H)}} = \sqrt{(10^{-8}\ A^2)}$$

$$= 10^{-4}\ A$$

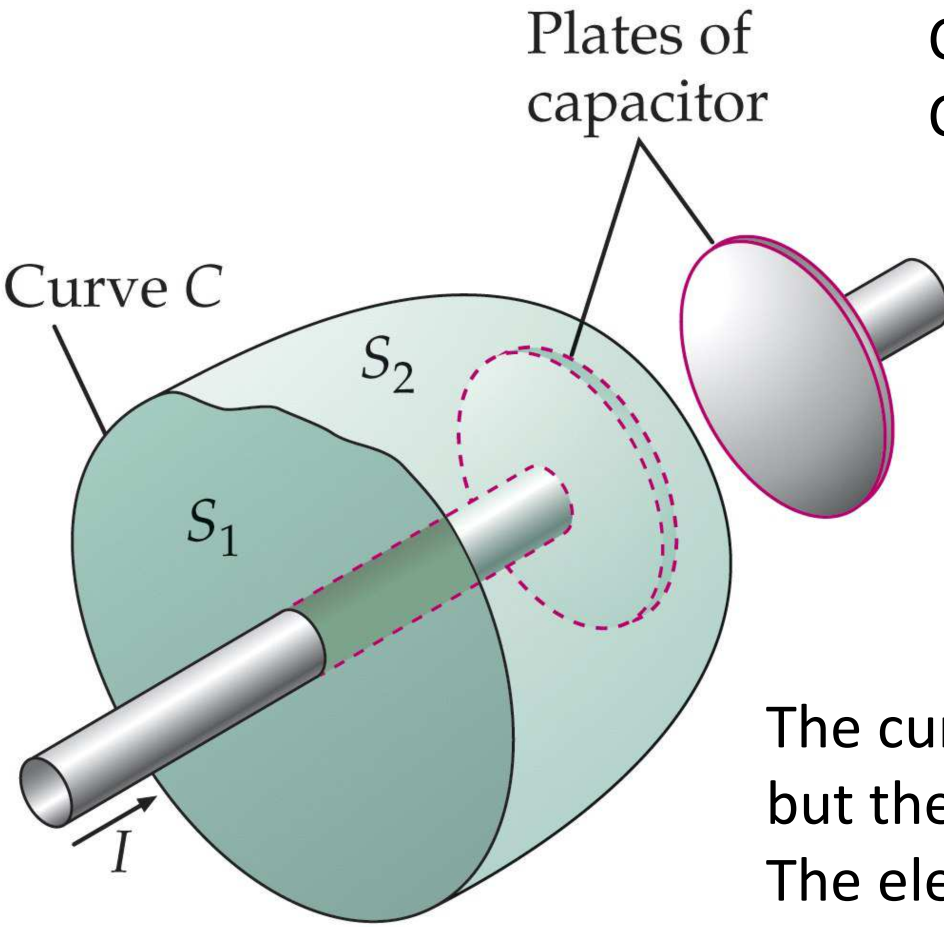
$$= 0.1\ mA$$

$$100\ \mu A$$

Maxwell's Equations (so far)

- $\oint_S \hat{n} \cdot \vec{E}\, dA = \frac{Q_{inside}}{\epsilon_0}$ (Gauss's law)
- $\oint_S \hat{n} \cdot \vec{B}\, dA = 0$ (Gauss's law for magnetism)
- $\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\int_S \hat{n} \cdot \vec{B} dA$ (Faraday's law)
- $\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I$ (Ampere's law)

The Problem with Ampere's Law



Current through S_1 is I
Current through S_2 is 0!

The current through S_2 is zero, but the electric flux is not zero. The electric flux changes as charge flows onto the capacitor.

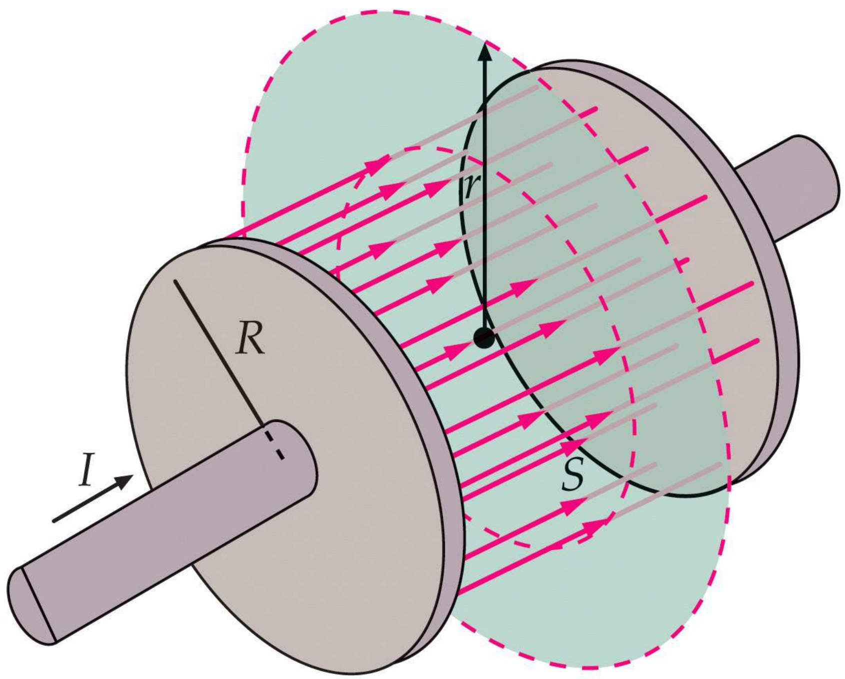
Maxwell's Displacement Current

- We can think of the changing electric flux through S_2 as if it were a current:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt} = \epsilon_0 \frac{d}{dt} \int_{S_2} \vec{E} \cdot \hat{n} \, dA$$

$$\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}} = \boldsymbol{\mu_0}(\boldsymbol{I} + \boldsymbol{I_d}) = \boldsymbol{\mu_0}\mathbf{I} + \boldsymbol{\mu_0}\boldsymbol{\epsilon_0} \frac{\boldsymbol{d}}{\boldsymbol{dt}} \int_{S_2} \vec{\boldsymbol{E}} \cdot \hat{\boldsymbol{n}} \, \boldsymbol{dA}$$

Calculating Displacement Current



Parallel plate capacitor

Gauss's law: (Lecture 5)

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

Electric flux:

$$\phi_e = EA = \frac{Q}{\epsilon_0}$$

Change in electric flux:

$$\frac{d\phi_e}{dt} = \frac{1}{\epsilon_0}\frac{dQ}{dt} = \frac{I}{\epsilon_0}$$

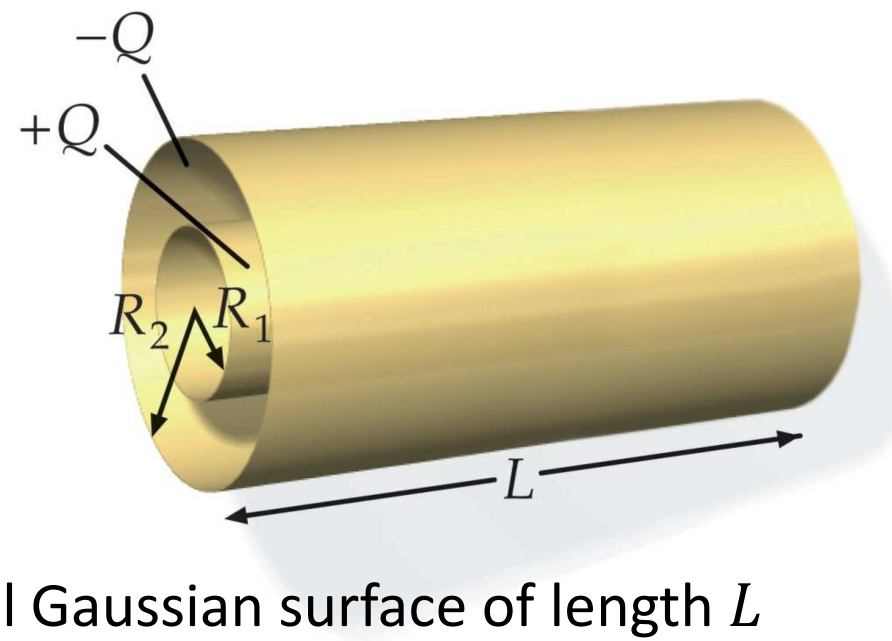
Displacement current:

$$I_d = \epsilon_0 \frac{d\phi_e}{dt} = I$$

Coaxial Capacitor

In Lecture 8 we calculated the electric field between the inner and outer conductors using Gauss's law:

$$\oint_S \hat{n} \cdot \vec{E}\, dA = \frac{Q_{inside}}{\epsilon_0}$$



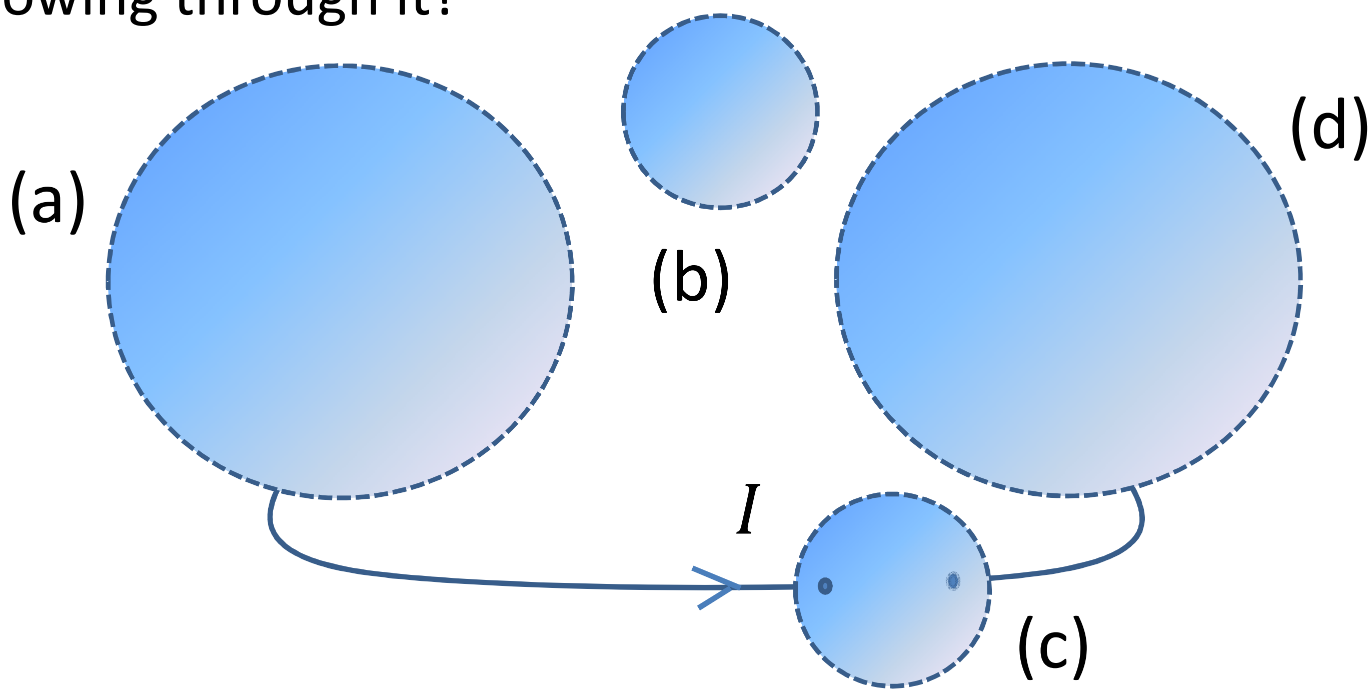
We choose S to be a cylindrical Gaussian surface of length L and radius r with $R_1 < r < R_2$.
But the left-hand-side is just ϕ_e so the displacement current is

$$I_d = \epsilon_0 \frac{d\phi_e}{dt} = \frac{dQ_{inside}}{dt}$$

The real current flows into the ends of the Gaussian surface, the displacement current flows out the sides.

Clicker Question

- Suppose charge flows between two spherical conductors. Which surface will have the largest displacement current flowing through it?



Clicker Question

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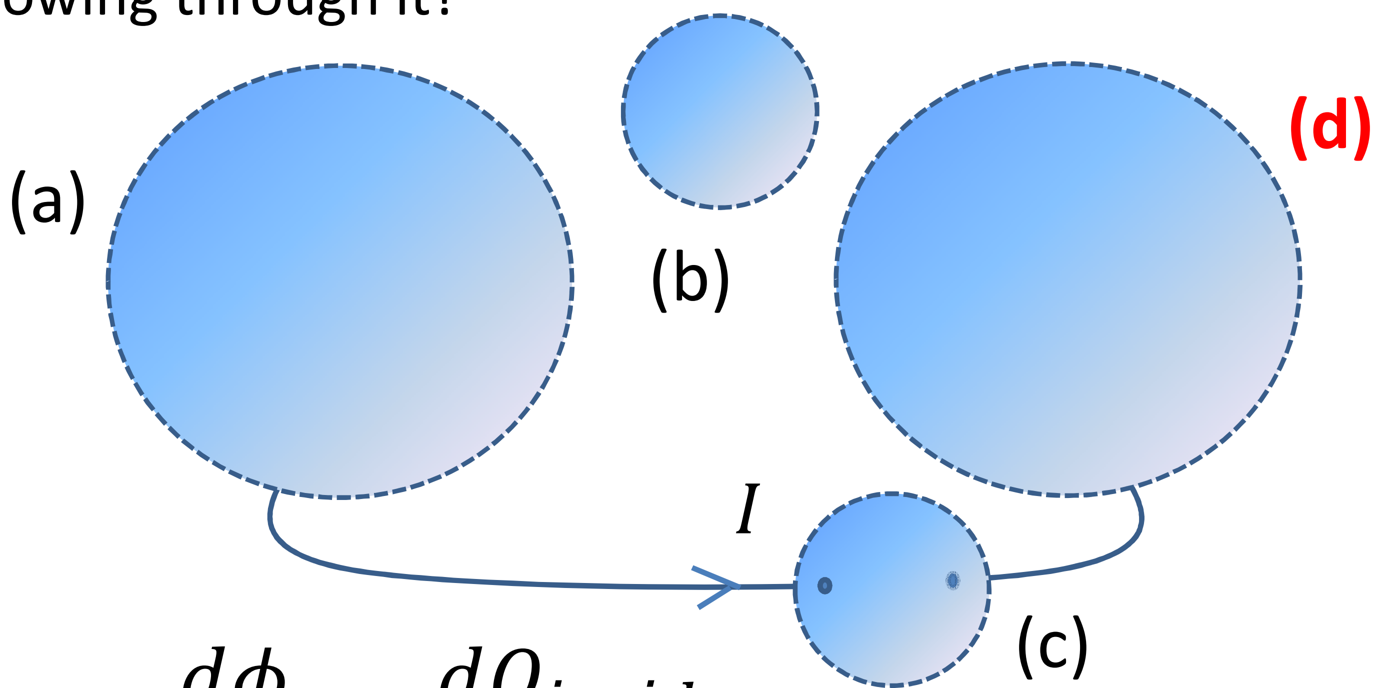
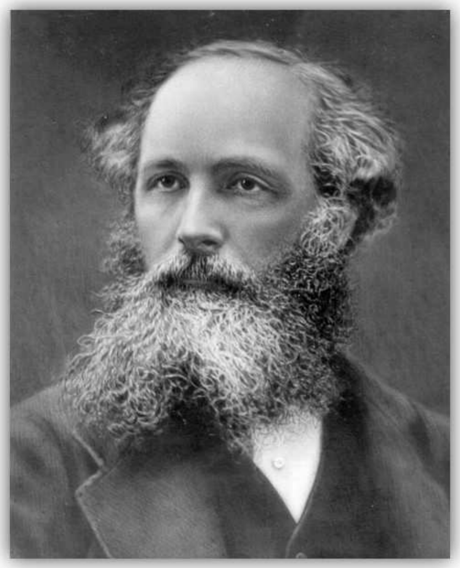


$$I_d = \epsilon_0 \frac{d\phi_e}{dt} = \frac{dQ_{inside}}{dt}$$

Maxwell's Equations (1864)



$$\oint_S \hat{n} \cdot \vec{E}\, dA = \frac{Q_{inside}}{\epsilon_0}$$

$$\oint_S \hat{n} \cdot \vec{B}\, dA = 0$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I + \mu_0 \varepsilon_0 \frac{d\phi_e}{dt}$$

Maxwell’s Equations in Free Space

In “free space” where there are no electric charges or sources of current, Maxwell’s equations are quite symmetric:

$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{E}}\, d\boldsymbol{A} = \boldsymbol{0}$$

$$\oint_S \hat{\boldsymbol{n}} \cdot \vec{\boldsymbol{B}}\, d\boldsymbol{A} = \boldsymbol{0}$$

$$\oint_C \vec{\boldsymbol{E}} \cdot d\vec{\boldsymbol{\ell}} = -\frac{d\boldsymbol{\phi_m}}{dt}$$

$$\oint_C \vec{\boldsymbol{B}} \cdot d\vec{\boldsymbol{\ell}} = \boldsymbol{\mu_0 \varepsilon_0} \frac{d\boldsymbol{\phi_e}}{dt}$$

Maxwell's Equations in Free Space

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt}$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 \varepsilon_0 \frac{d\phi_e}{dt}$$

A changing magnetic flux induces an electric field.

A changing electric flux induces a magnetic field.

Will this process continue indefinitely?

Light is an Electromagnetic Wave

- Maxwell showed that these equations contain the wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

where $y(x,t)$ is a function of position and time.

- General solution:

$$y(x,t) = f_1(x - vt) + f_2(x + vt)$$

- Harmonic solutions:

$$y(x,t) = y_0 \sin(kx \pm \omega t)$$

- Wavelength, $\lambda = 2\pi/k$ and frequency $f = \omega/2\pi$.
- Velocity, $v = \lambda/f = \omega/k$.

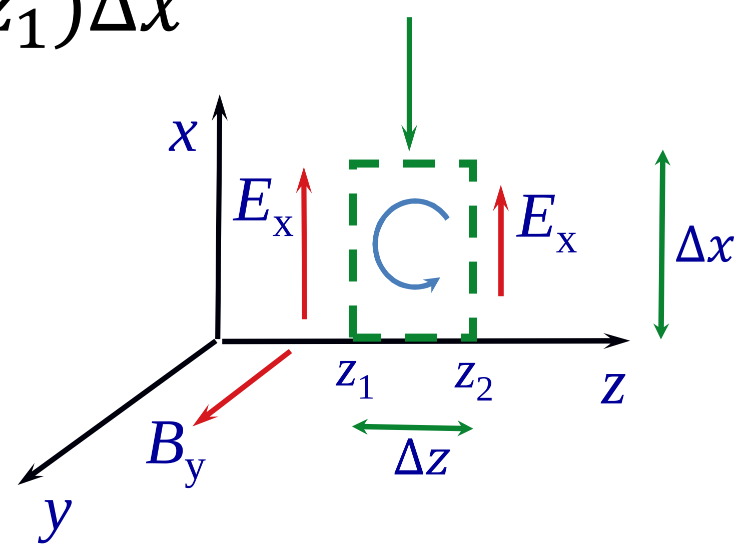
Light is an Electromagnetic Wave

- Faraday's Law:

$$\oint_C \vec{E} \cdot d\vec{\ell} = -\frac{d\phi_m}{dt} = -\frac{\partial B_y}{\partial t}\Delta x \Delta z$$

$$\oint_C \vec{E} \cdot d\vec{\ell} = E_x(z_2)\Delta x - E_x(z_1)\Delta x$$

$$\approx \frac{\partial E_x}{\partial z}\Delta z \Delta x$$



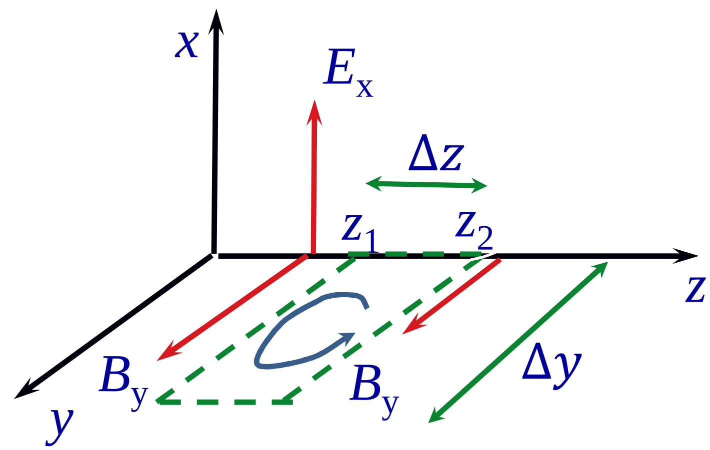
Light is an Electromagnetic Wave

- Ampere's law:

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 \epsilon_0 \frac{d\phi_e}{dt} = \mu_0 \epsilon_0 \frac{\partial E_x}{\partial t} \Delta y \Delta z$$

$$\oint_C \vec{B} \cdot d\vec{\ell} = B_y(z_1)\Delta y - B_y(z_2)\Delta y$$

$$\approx -\frac{\partial B_y}{\partial z} \Delta z \Delta y$$



Putting these together…

$$\frac{\partial E_x}{\partial z} = -\frac{\partial B_y}{\partial t}$$

$$-\frac{\partial B_y}{\partial z} = \mu_0 \epsilon_0 \frac{\partial E_x}{\partial t}$$

Differentiate the first with respect to z:

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial^2 B_y}{\partial z \partial t}$$

Differentiate the second with respect to t:

$$-\frac{\partial^2 B_y}{\partial z \partial t} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}$$

$$\boldsymbol{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

Velocity of Electromagnetic Waves

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$$

$$\boldsymbol{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

Speed of wave propagation is

$$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

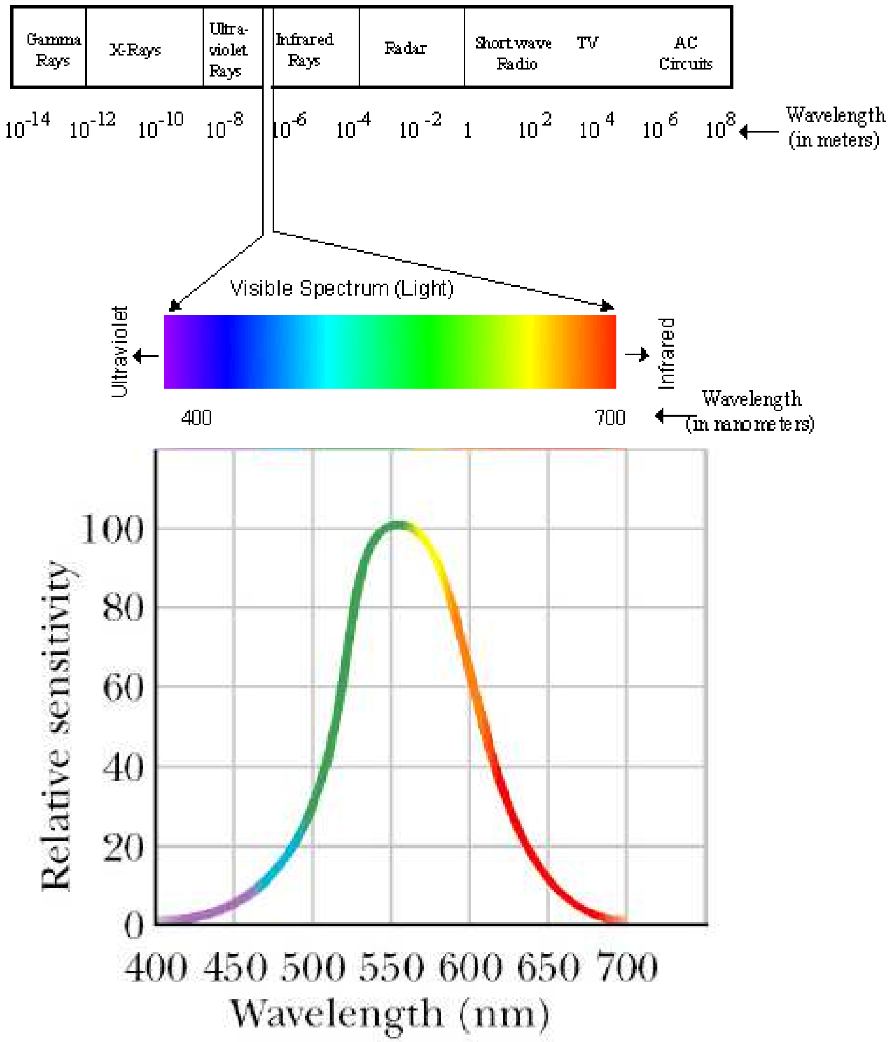
$$= \frac{1}{\sqrt{(4\pi \times 10^{-7} N/A^2)(8.854 \times 10^{-12}\ C^2/N \cdot m)}}$$

$$= \mathbf{2.998 \times 10^8\ m/s}$$

(speed of light)

The electromagnetic spectrum

- In 1850, the only known forms electromagnetic waves were ultraviolet, visible and infrared.
- The human eye is only sensitive to a very narrow range of wavelengths:



Discovery of Radio Waves

ELECTRIC WAVES

BEING

RESEARCHES ON THE PROPAGATION OF ELECTRIC ACTION WITH FINITE VELOCITY THROUGH SPACE

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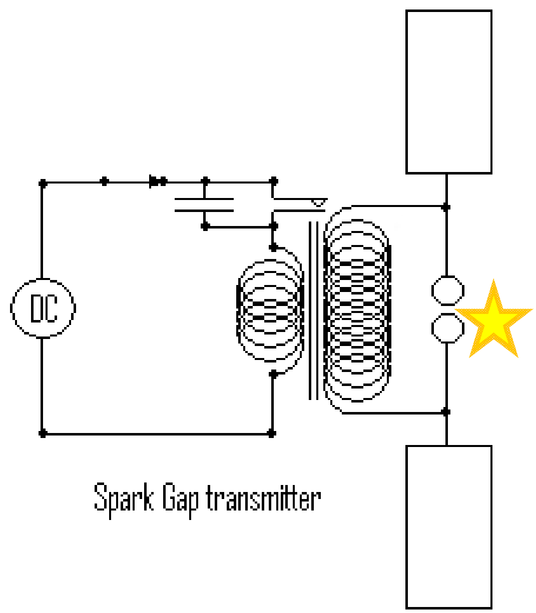
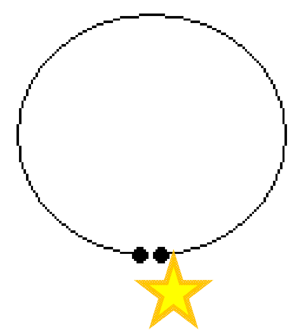
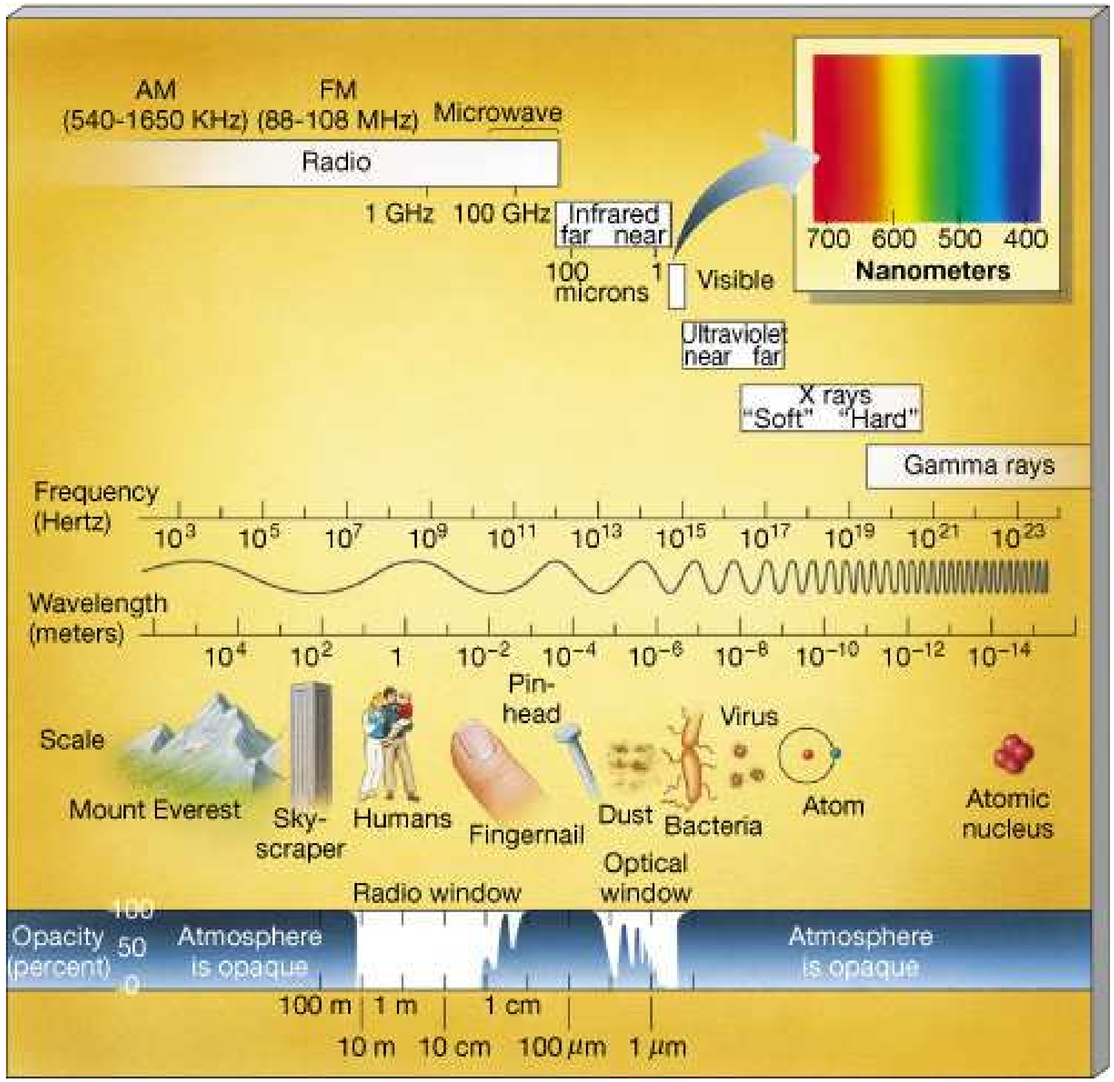




The Electromagnetic Spectrum



Electromagnetic Waves

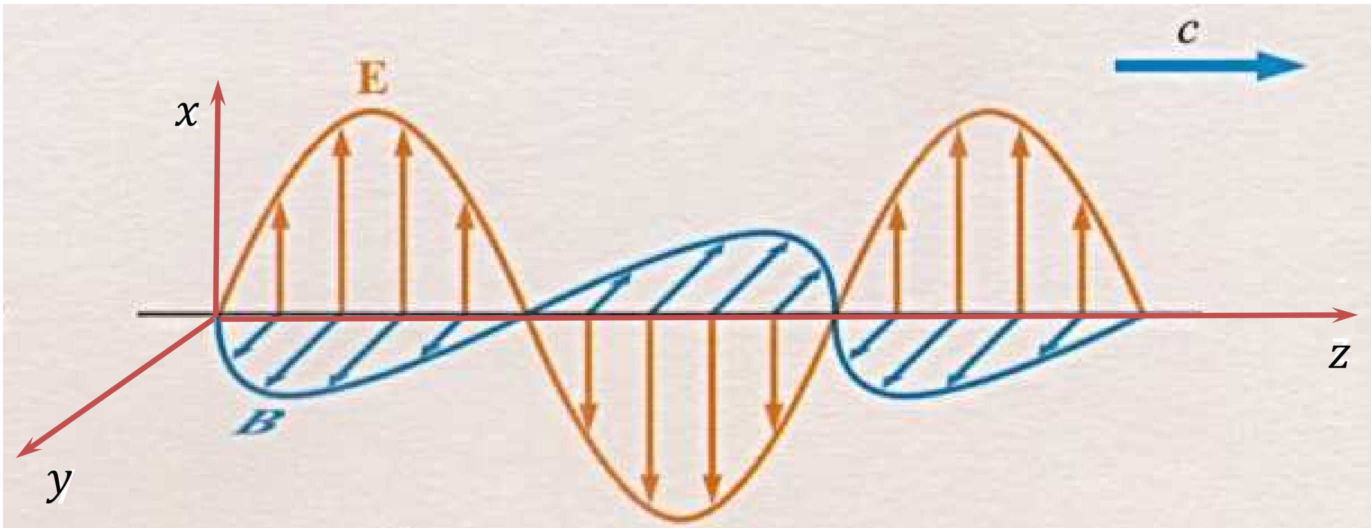
$$\boldsymbol{\frac{\partial^2 E_x}{\partial z^2} = \mu_0 \epsilon_0 \frac{\partial^2 E_x}{\partial t^2}}$$

- A solution is $E_x(z,t) = E_0 \sin(kz - \omega t)$ where $\omega = kc = 2\pi c/\lambda$
- What is the magnetic field?

$$\frac{\partial B_y}{\partial t} = -\frac{\partial E_x}{\partial z} = -kE_0 \cos(kz - \omega t)$$

$$B_y(x,t) = \frac{k}{\omega} E_0 \sin(kx - \omega t)$$

Electromagnetic Waves



- $\vec{E}$, $\vec{B}$ and $\vec{v}$ are mutually perpendicular.
- In general, the direction is

$$\hat{s} = \hat{E} \times \hat{B}$$

Energy in Electromagnetic Waves

Energy stored in electric and magnetic fields (lecture 17):

$$u_e = \frac{1}{2}\epsilon_0 E^2 \qquad\qquad u_m = \frac{1}{2\mu_0}B^2$$

For an electromagnetic wave, $B = E/c = E\sqrt{\mu_0\epsilon_0}$

$$u_m = \frac{1}{2\mu_0}B^2 = \frac{1}{2}\epsilon_0 E^2 = u_e$$

The total energy density is

$$u = u_m + u_e = \epsilon_0 E^2$$

Intensity of Electromagnetic Waves

- Intensity is defined as the average power transmitted per unit area.

Intensity = Energy density × wave velocity

$$I = \epsilon_0 c \langle E^2 \rangle = \frac{\langle E^2 \rangle}{\mu_0 c}$$

$$\mu_0 c = 377\ \Omega \equiv Z_0$$

(Impedance of free space)

Poynting Vector

- We can construct a vector from the intensity and the direction $\hat{s} = \hat{E} \times \hat{B}$:

$$\vec{S} = \frac{\vec{E} \times \vec{B}}{\mu_0}$$

$$\langle \vec{S} \rangle = \frac{\langle E^2 \rangle}{Z_0} = I$$

- This represents the flow of power in the direction $\hat{s}$
- Average electric field: $E_{rms} = E_0/\sqrt{2}$

$$\langle \vec{S} \rangle = \frac{(E_0)^2}{2Z_0}$$

- Units: Watts/m^2

Question

- The magnetic field component of a radio wave is expressed:

$$B_y = B_0 \sin(kz - \omega t)$$

- In which directions do the Poynting vector and the $\vec{E}$-field point?

(a) +z,+x

(b) +z,-x

(c) -z,+x

(d) -z,-x

(e) +y,+x