ANNOUNCEMENT

***Exam 2**: Monday, November 5, 2012, 8 PM - 10 PM

***Location:** Elliot Hall of Music

*Covers all readings, lectures, homework from Chapters 25 through 28.

*The exam will be multiple choice (15-18 questions).

Be sure to bring your student ID card and your own two-page (two-side) crib sheet, one from exam 1 and a new one.

NOTE THAT FEW EQUATIONS WILL BE GIVEN – YOU ARE REMINDED THAT IT IS YOUR RESPONSIBILITY TO CREATE WHATEVER TWO-SIDED CRIB SHEET YOU WANT TO BRING TO THIS EXAM.

The equation sheet that will be given with the exam is posted on the course homepage. Click on the link on the left labeled "EquationSheet"

PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 24100

Electricity & Optics

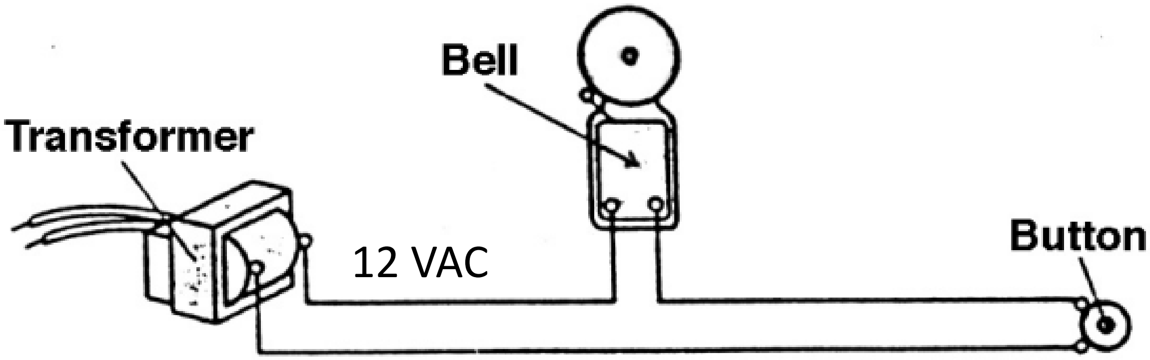
Lecture 20 – Chapter 29 sec. 4,6,3

Fall 2012 Semester

Matthew Jones

Clicker Question

110 VAC



- A door-bell uses a transformer to produce an AC voltage of 12 volts (RMS).
- What is the peak voltage?

(a) 12 Volts
(b) 24 Volts
(c) 17 Volts
(d) 8.5 Volts

Clicker Question

- For a sinusoidal voltage source,

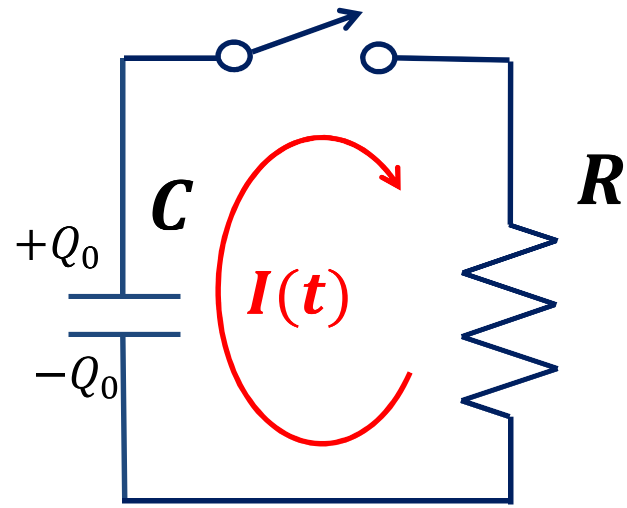
$$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$

- Therefore, the peak voltage is 1.414 times the RMS voltage.

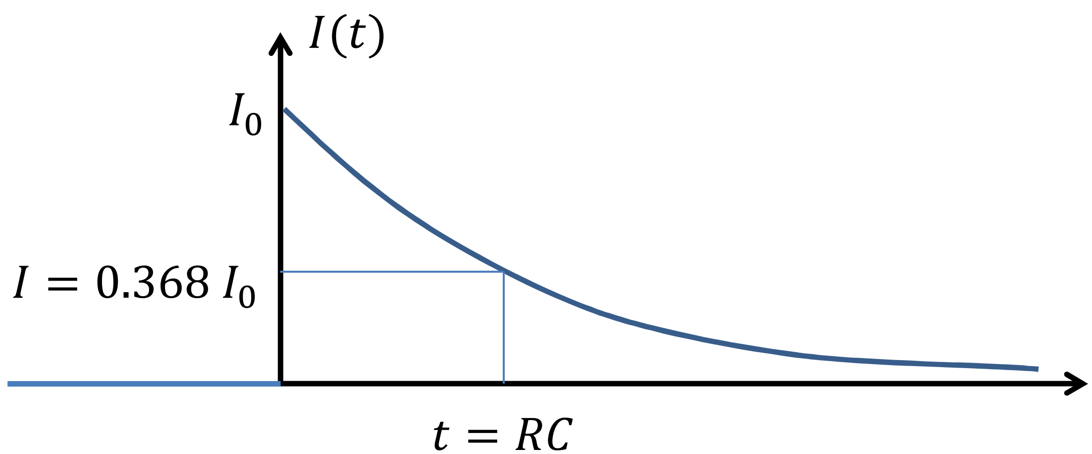
$$(12\ V) \times 1.414 = 16.97\ V$$

RC Circuits

$$I(t) = \frac{Q_0}{RC} e^{-t/RC}$$

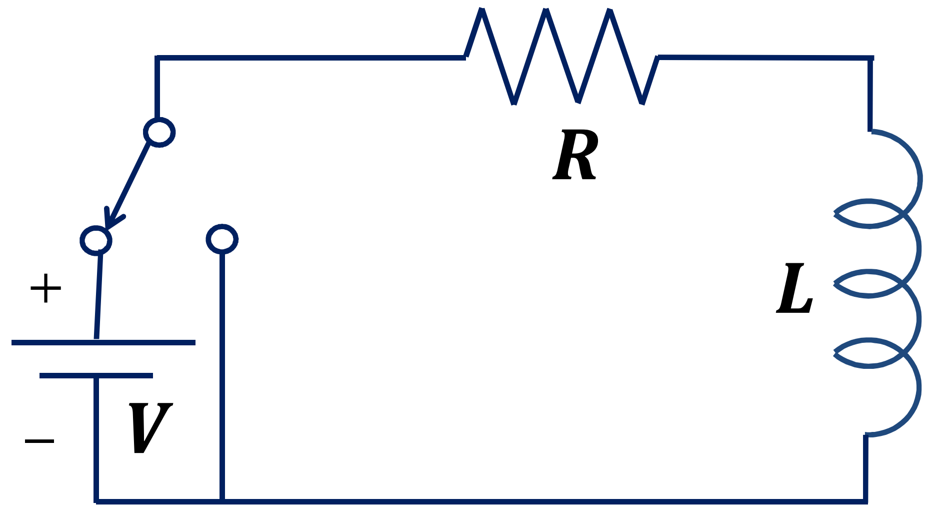


- The capacitor stores energy in the electric field
- The resistor dissipates energy

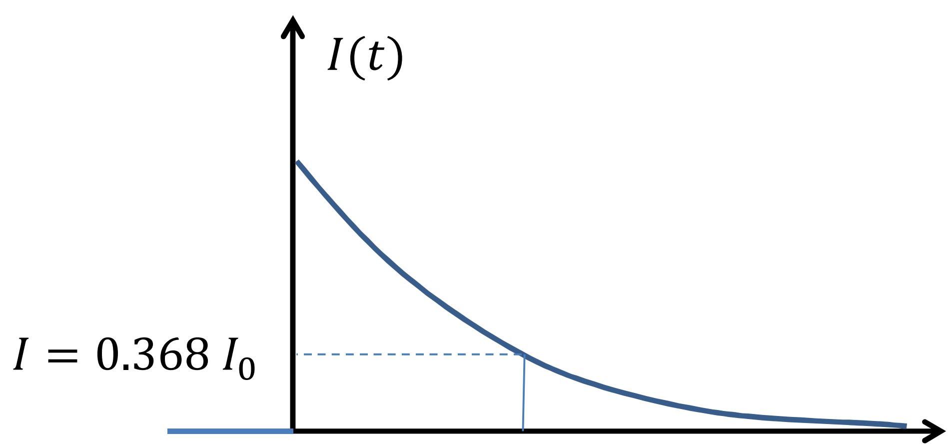


LR Circuits

$$I(t) = \frac{V}{R}\, e^{-tR/L}$$

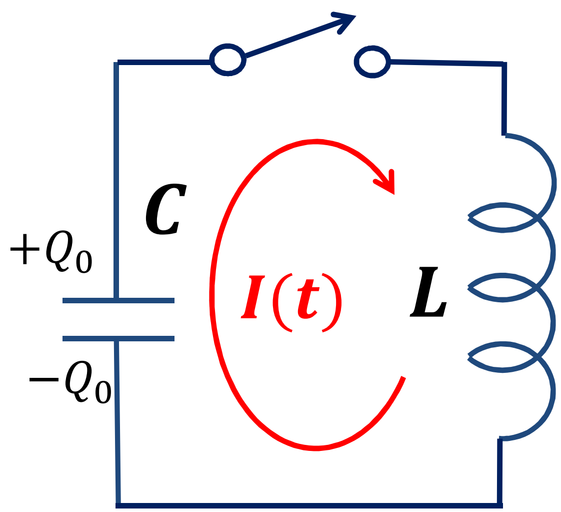


- The inductor stores energy in its magnetic field.
- The resistor dissipates energy



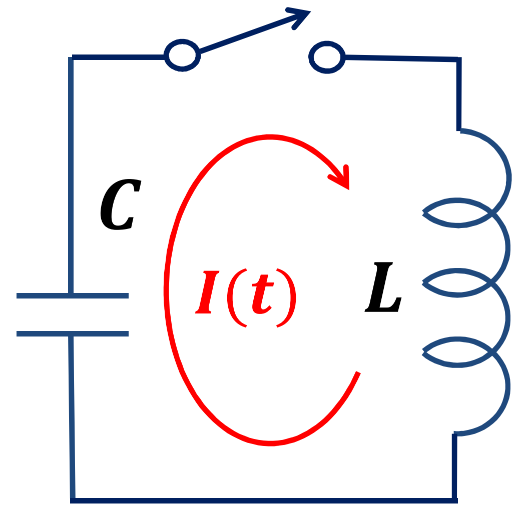
LC Circuits

- What if we had a capacitor and an inductor?



- When the switch is closed, will current flow? (yes)
- Will energy be dissipated? (no)

LC Circuits



- Kirchhoff's Loop Rule:

$$V_C + V_L = 0$$

$$\frac{1}{C}\int_0^t I(t)dt + L\frac{dI}{dt} = 0$$

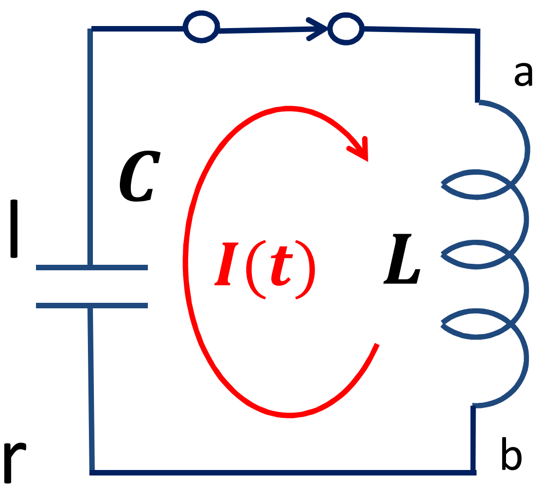
- Differentiate once and multiply by C:

$$I(t) + LC\frac{d^2I}{dt^2} = 0$$

- A solution is

$$I(t) = I_0 \sin \omega t \text{ where } \omega = \frac{1}{\sqrt{LC}}$$

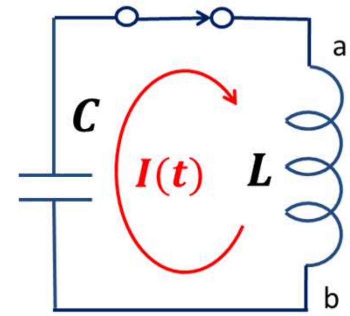
Question 1



- At $t = 0$, when the switch is closed, the capacitor has a total charge of Q_0
- At a later time, t_1, the capacitor is uncharged.
- What is the voltage across the inductor?

(a) $V_{ab} < 0$ (b) $V_{ab} = 0$ (c) $V_{ab} > 0$

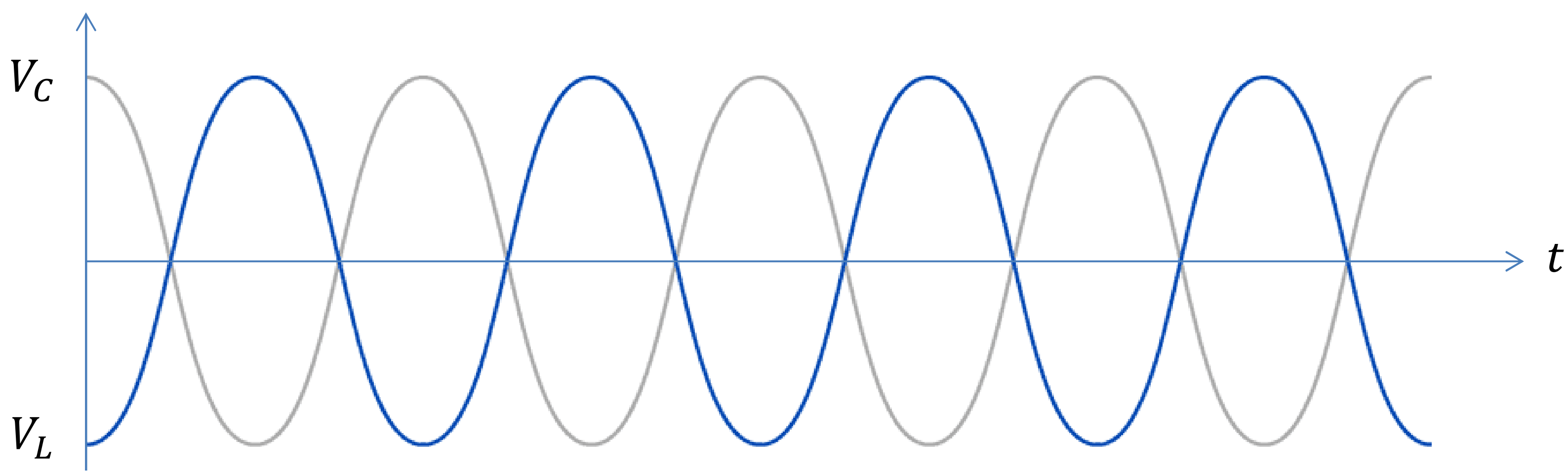
Question 1



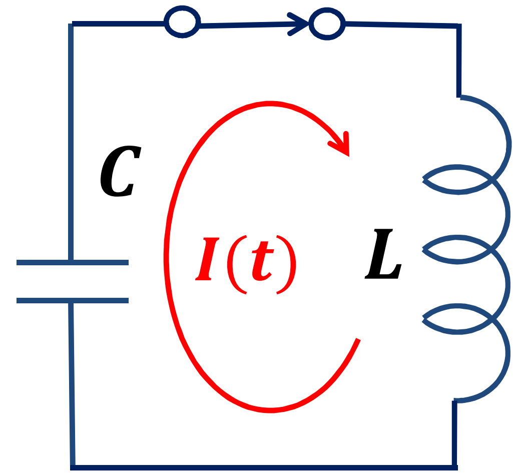
- Kirchhoff's loop rule:

$$V_C + V_L = 0$$

- When the capacitor is uncharged, $V_C = 0$.
- Therefore, $V_L = V_{ab} = 0$ at the time t_1.



Question 2



- At $t = 0$, the capacitor has charge Q_0 and the circuit oscillates with frequency ω_1
- Suppose the circuit started with an initial charge of $2Q_0$… what would the oscillation frequency, ω_2 be?

(a) $\omega_2 < \omega_1$ (b) $\omega_2 = \omega_1$ (c) $\omega_2 > \omega_1$

Question 2

- The solution to the differential equation was

$$I(t) = I_0 \sin \omega t \text{ where } \omega = \frac{1}{\sqrt{LC}}$$

- Doubling the initial charge will double the current, but will not change the frequency.

Conservation of Energy

- The current flowing in the circuit is

$$I(t) = I_0 \sin \omega t$$

- Charge on the capacitor:

$$Q(t) = -\int_0^t I(t)dt = \frac{I_0}{\omega}\cos \omega t$$

The negative sign is because we have drawn the direction of the current such that positive current means charge is leaving the capacitor.

- Energy stored in a capacitor:

$$U_e = \frac{1}{2C}Q^2 = \frac{1}{2}CV^2$$

- Energy stored in an inductor:

$$U_m = \frac{1}{2}LI^2$$

Conservation of Energy

- Current: $I(t) = I_0 \sin \omega t$
- Charge: $Q(t) = \frac{I_0}{\omega} \cos \omega t = Q_0 \cos \omega t$
- Total energy:

$$U_e + U_m = \frac{1}{2C}\left(\frac{I_0}{\omega}\right)^2 \cos^2 \omega t + \frac{L}{2}(I_0)^2 \sin^2 \omega t$$

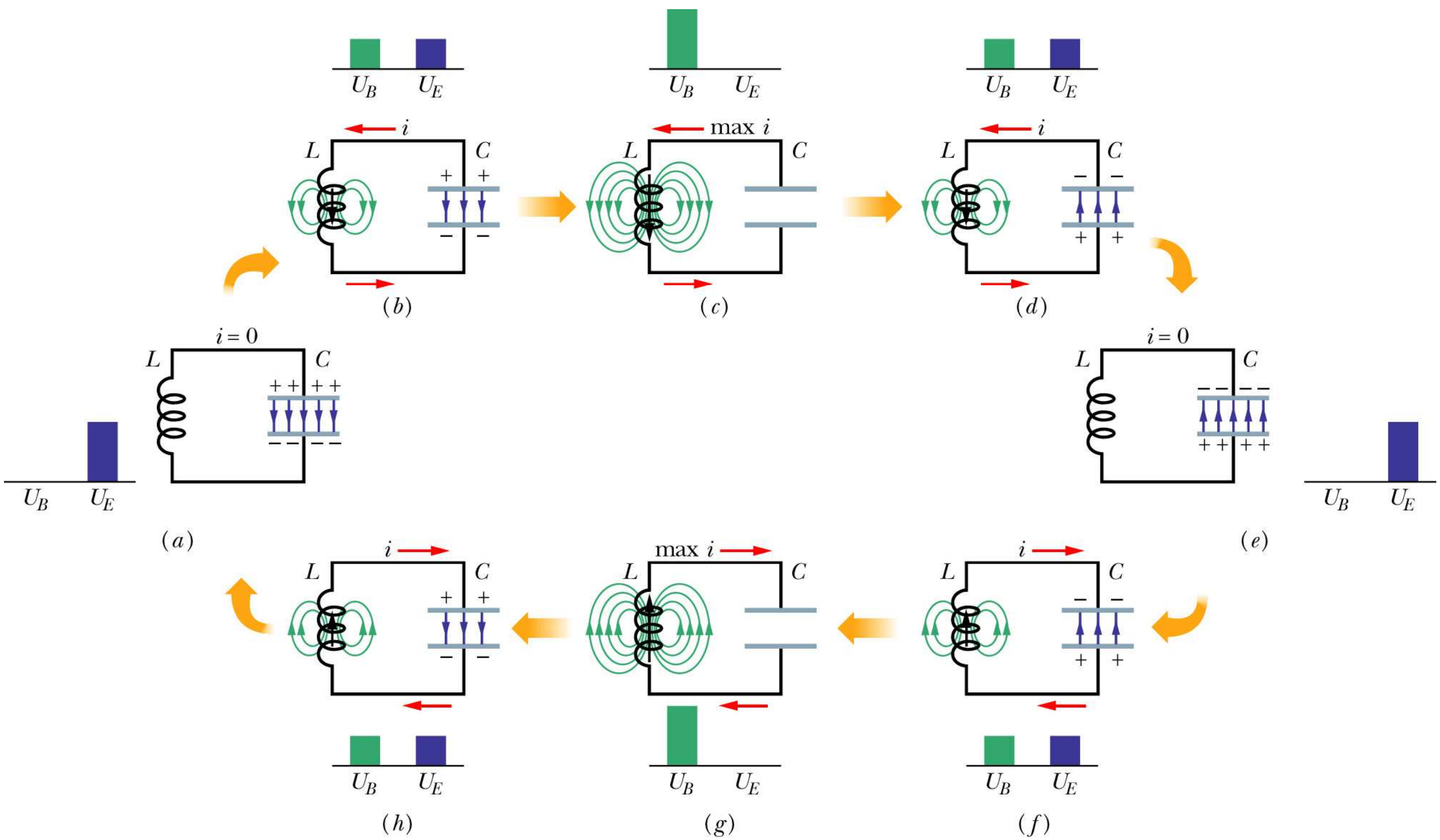
- But $\omega^2 = 1/LC$ so

$$U_e + U_m = \frac{L}{2}(I_0)^2 \cos^2 \omega t + \frac{L}{2}(I_0)^2 \sin^2 \omega t$$

$$= \frac{L(I_0)^2}{2} = \frac{(Q_0)^2}{2C}$$

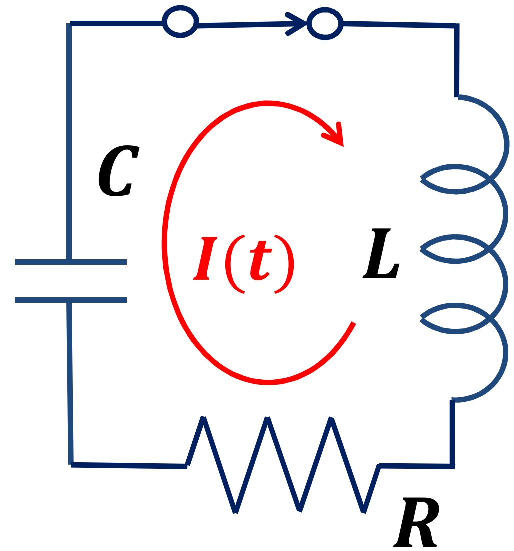
This is independent of time, so the total energy remains constant.

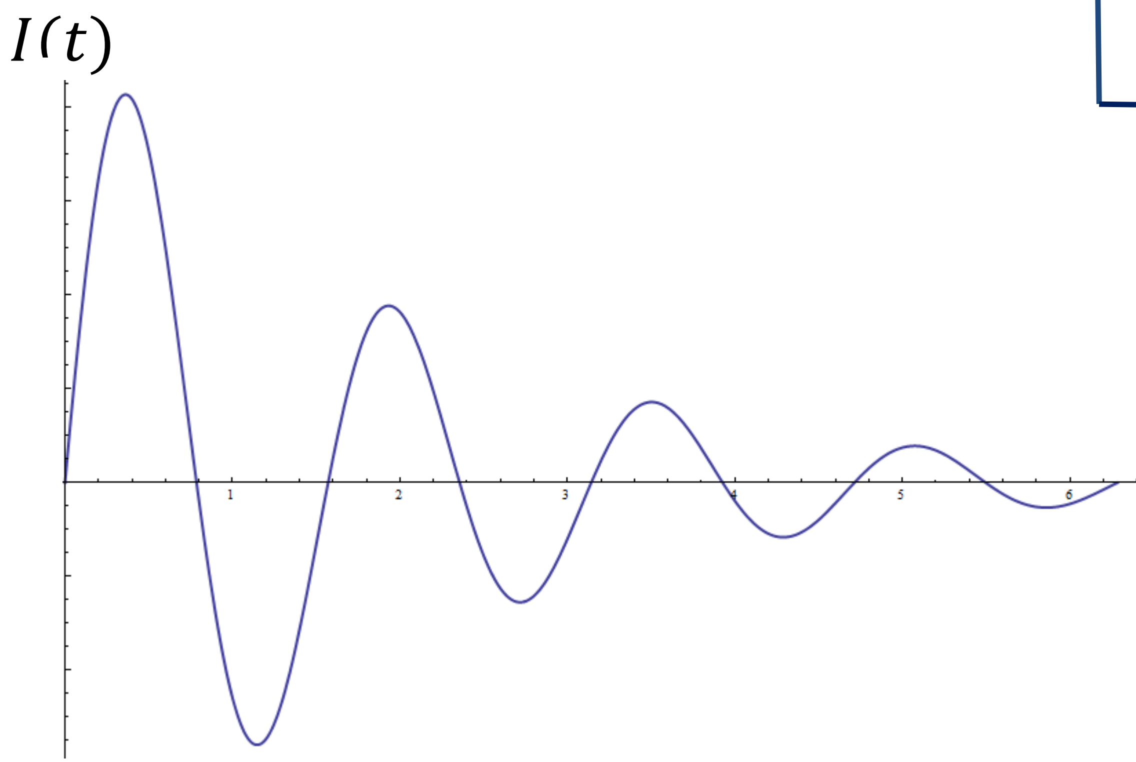
U_m and U_e



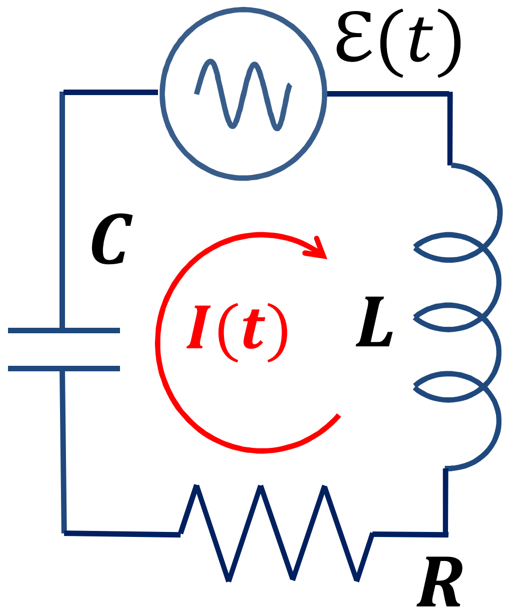
RLC Circuit

- The circuit will oscillate, but energy will be dissipated in the resistor:





Driven RLC Circuit

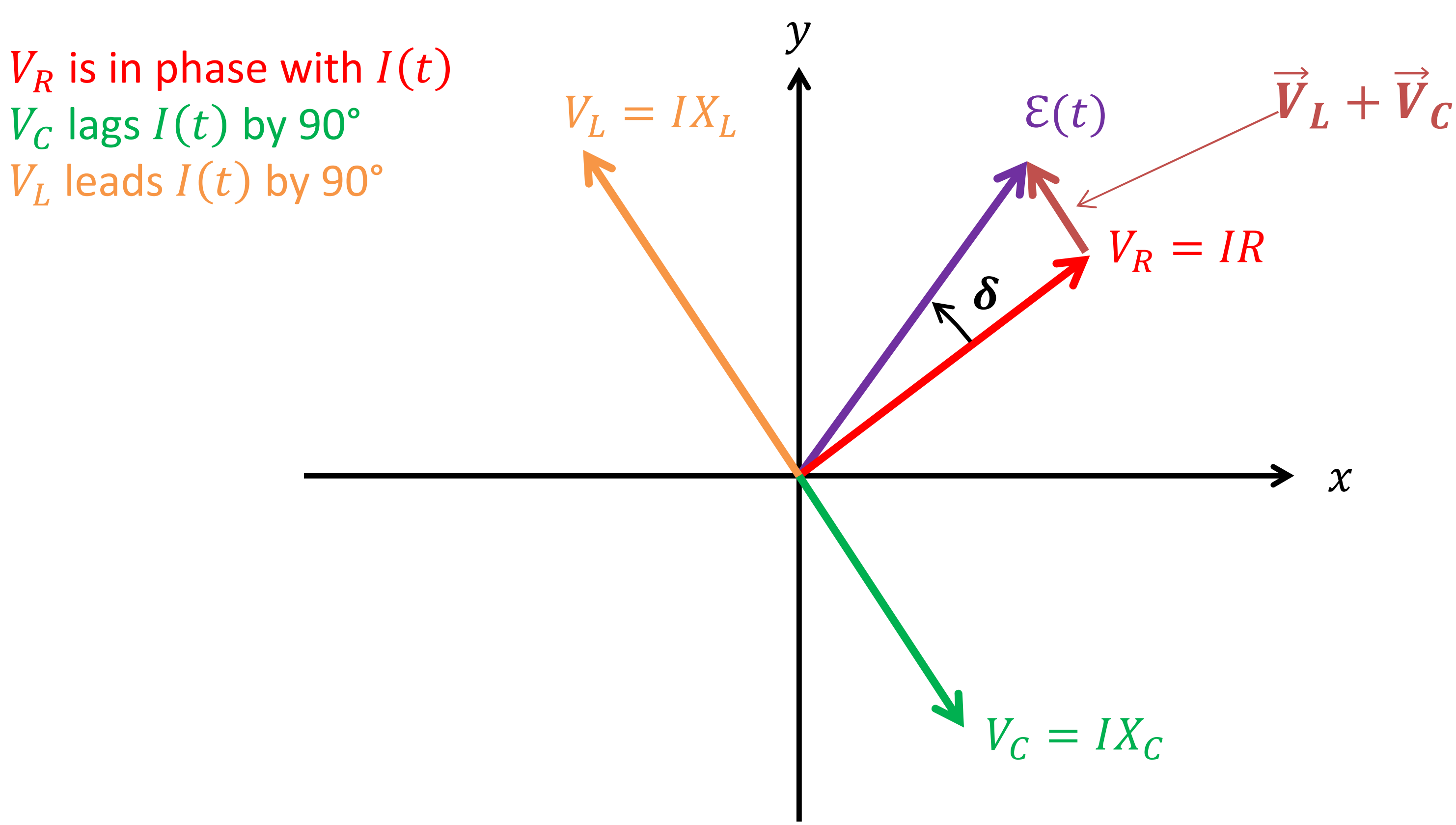


- We need to calculate $I(t)$ in terms of $\mathcal{E}(t)$
  - There might be a phase difference, δ
- We will analyze this using phasors.

Phasors

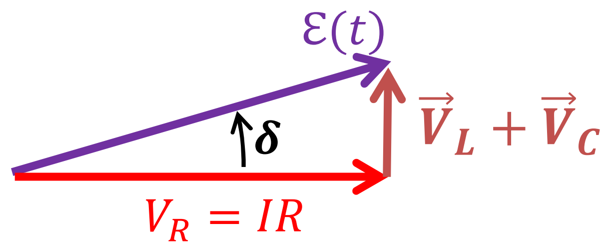
- Resistance: $V_R = I\ R$
  - Voltage and current are in phase
- Inductive reactance: $V_L = I\ X_L$
  - Voltage leads the current by 90^o
- Reactive capacitance: $V_L = I\ X_C$
  - Voltage lags the current by 90^o

Phasors



Kirchhoff's loop rule: $\mathcal{E}(t) = V_R + V_C + V_L$

Phasors



- Reactance: $V_L + V_C = I(t)(X_L - X_C)$
- Resistance: $V_R = I(t)R$

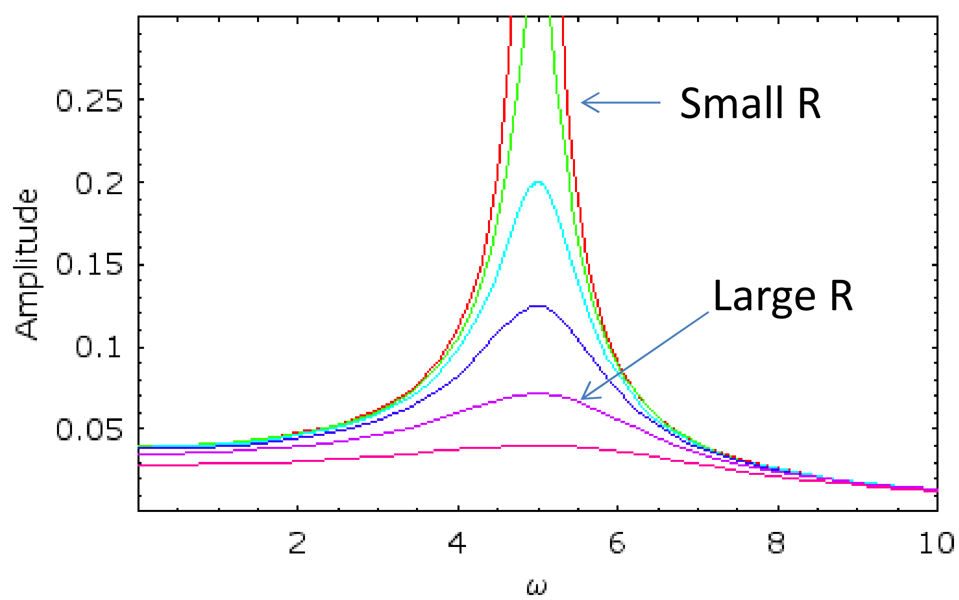
$$\mathcal{E}(t) = I(t)\sqrt{R^2 + (X_L - X_C)^2}$$

$$I(t) = \frac{\mathcal{E}(t)}{\sqrt{R^2 + (X_L - X_C)^2}}$$

- Current is maximum when $X_L = X_C$
  - Resonance

Resonance

- Remember, $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$
- These are equal when $\omega = \frac{1}{\sqrt{LC}}$
  - The natural frequency at which the circuit would oscillate



Question

- An LC circuit has $C = 100\ pF$ and $L = 100\ \mu H$.
- If it oscillates with an amplitude of 100 mV, what is the amplitude of the current?

(a) $100\ mA$

(b) $100\ \mu A$

(c) $10\ mA$

(d) $10\ \mu A$