PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 24100

Electricity & Optics

Lecture 15 – Chapter 27 sec. 3-5

Fall 2012 Semester

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Clicker Question

- Treat a lightning bolt like a long, straight wire.
- If the current in a lightning bolt is 100 kA, how would the magnetic field 1 km away compare with the Earth's magnetic field ($5 \times 10^{-5}\ T$)?

(a) Much less

(b) Much greater

(c) About the same

$$|\vec{B}| = \frac{\mu_0}{4\pi}\frac{2I}{R}$$

$$= \frac{2 \times 10^{-7} \times 10^5}{10^3} \sim 10^{-5}\ T$$

$$(\mu_0 = 4\pi \times 10^{-7}\ \ T \cdot m/A)$$

Physics Help Center Survey

PHYS Building, Rms. 11- 12

How often do you use the Help Center on average?

A. Never
B. Once or at most twice a semester
C. Several times in a semester
D. Around once a week
E. More than once a week

Physics Help Center Survey

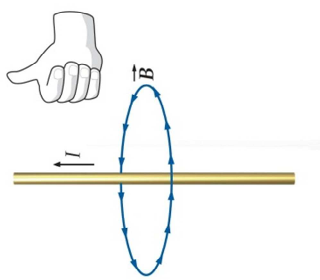
PHYS Building, Rms. 11- 12

How useful was your Help Center visit?

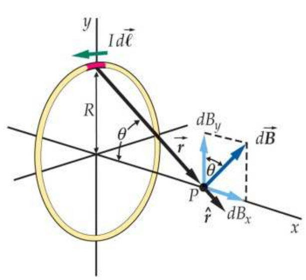
A. I did not use the Help Center
B. Useless
C. Not very useful
D. Useful
E. Very useful

Magnetic Fields

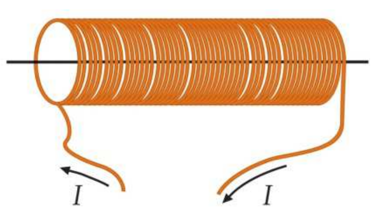
$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{\ell} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\int \frac{I\,d\vec{\ell} \times \vec{r}}{r^3}$$



$$|\vec{B}| = \frac{\mu_0}{4\pi}\frac{2I}{R}$$



$$|\vec{B}| = \frac{\mu_0}{2}\frac{IR^2}{(R^2 + z^2)^{3/2}}$$

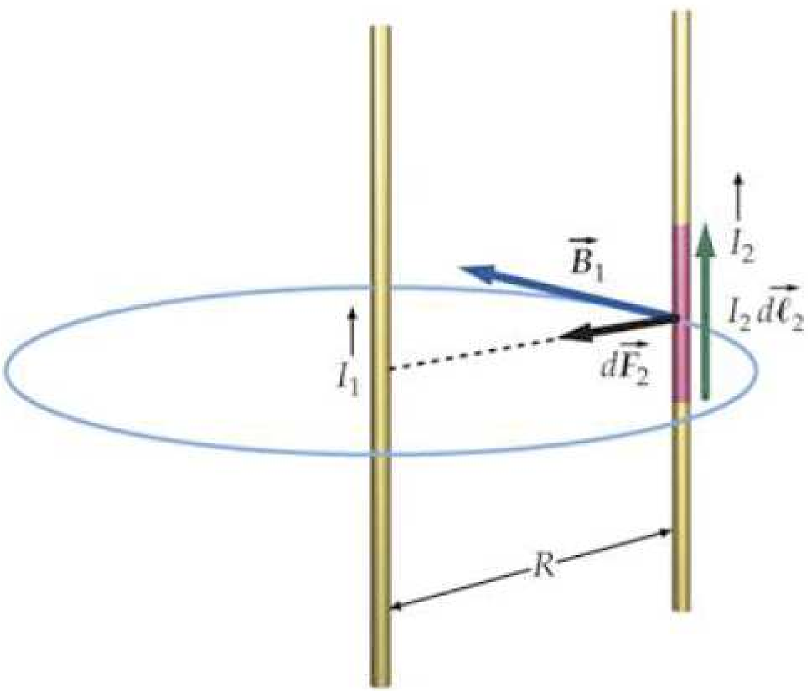


$$|\vec{B}| = \mu_0 n\, I$$

Forces on Current Carrying Wires

- Two wires carrying currents I_1 and I_2 will exert forces on each other:
  - Magnetic field from I_1 is $\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I_1\, d\vec{\ell}_1 \times \hat{r}}{r^2}$
  - Force on I_2 is $d\vec{F}_{12} = I_2\, d\vec{\ell}_2 \times \vec{B}$
- Conversely
  - Magnetic field from I_2 is $\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I_2\, d\vec{\ell}_2 \times \hat{r}}{r^2}$
  - Force on I_1 is $d\vec{F}_{21} = I_1\, d\vec{\ell}_1 \times \vec{B}$

Forces on Parallel Wires

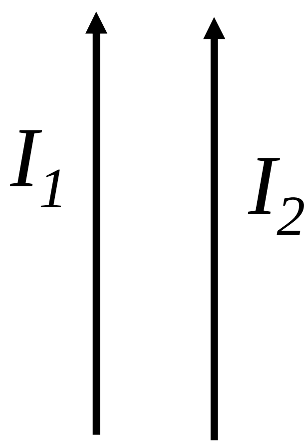


$$\left|\vec{B}\right| = \frac{\mu_0}{4\pi}\frac{2I_1}{R}$$

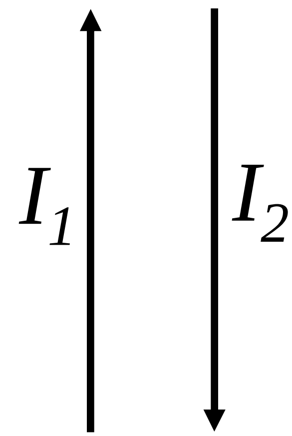
$$d\vec{F}_{12} = I_2\, d\vec{\ell}_2 \times \vec{B}$$

Force per unit length: $$\frac{dF_{12}}{d\ell_2} = \frac{\mu_0}{4\pi}\frac{2I_1\, I_2}{R}$$

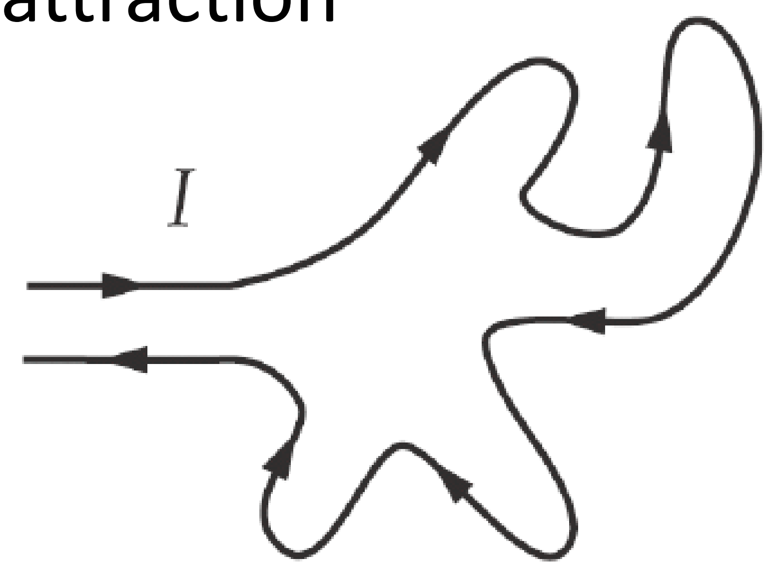
Force on Parallel Wires



attraction



repulsion

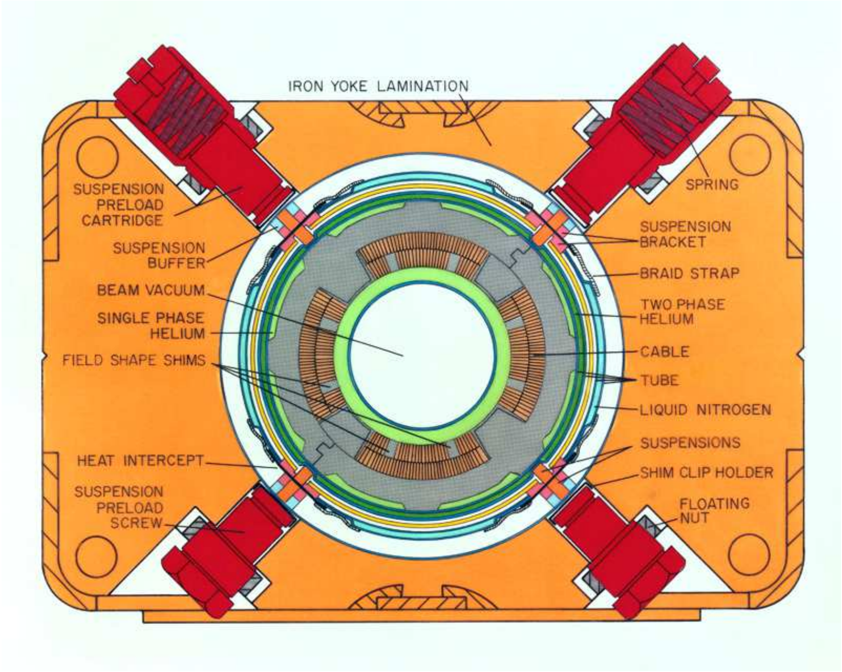


Will this loop expand or contract?

Magnetic "Pressure"

Normal dipole magnets
$B \sim 1\ T$

Superconducting dipole magnets
$B \sim 5\ T$



Magnets need to withstand about 5 tons of internal forces without distortion.

Remember Gauss's Law?

- Electric field:

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\hat{r}}{r^2}\,dQ$$

- Gauss's Law:

$$\oint_S \hat{n}\cdot\vec{E}\,dA = \frac{Q_{inside}}{\epsilon_0}$$

- If $\vec{E}$ is constant over the surface then we can bring it outside the integral
  - The integral is just the surface area
  - This works only when there is sufficient symmetry

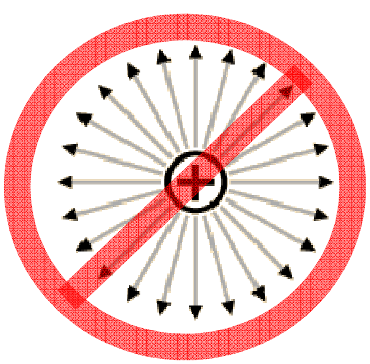
Gauss's Law Applied Magnetism

- In magnetism we can have dipoles or currents but ***no magnetic monopoles***
- Gauss's law:

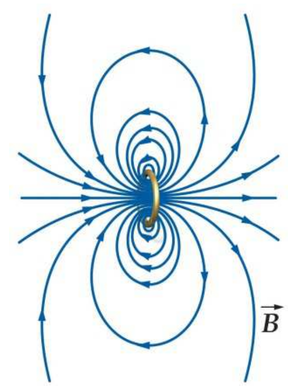
Always zero!

$$\oint_S \hat{n} \cdot \vec{B}\, dA = \frac{Q_{inside}}{\epsilon_0} = 0$$

- One of Maxwell's Equations:



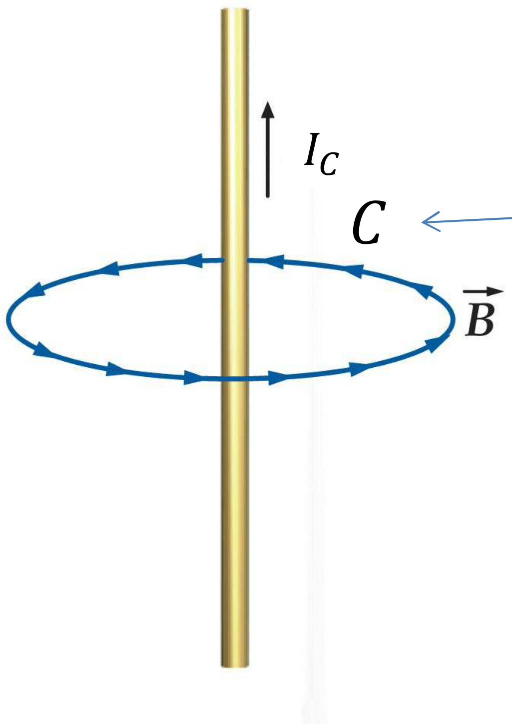
$$\nabla \cdot \vec{B} = 0$$



Ampere's Law

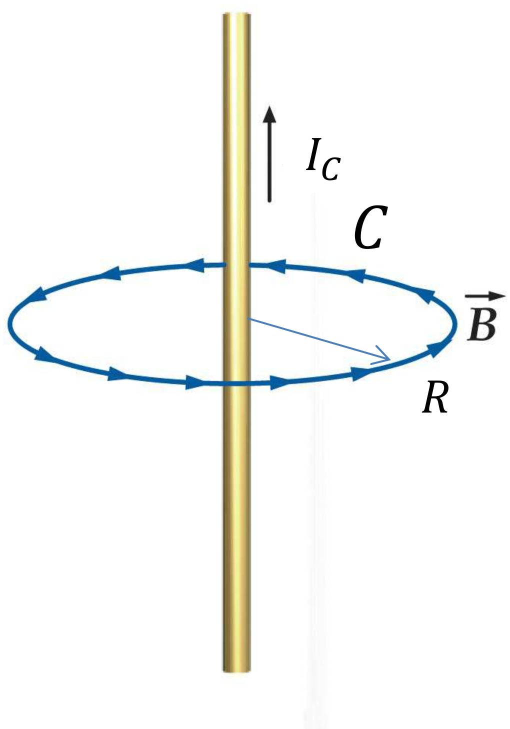
- But can we do something similar to calculate the magnetic field in cases with lots of symmetry?
- Yes:

$$\oint_C \vec{B} \cdot d\vec{\ell} = \mu_0 I_C$$



Example

- What is the magnetic field around a long, straight wire?
- From symmetry, we expect that the magnetic field is always azimuthal: $\vec{B} = B\hat{\varphi}$

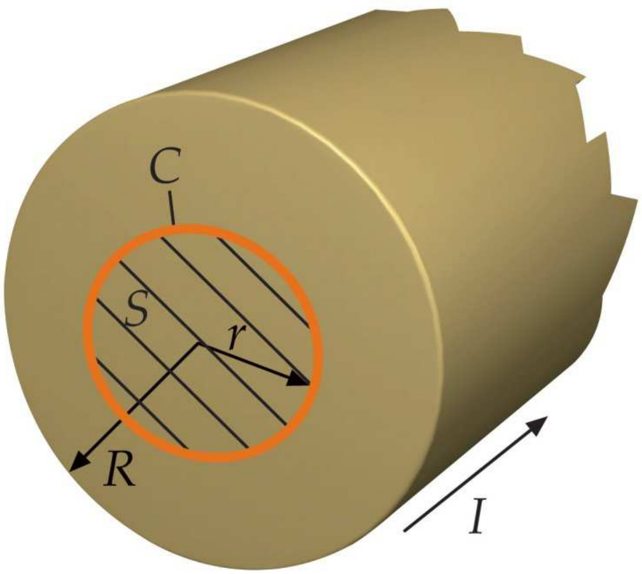


- The path length element is also azimuthal: $d\vec{\ell} = d\ell\hat{\varphi}$

$$\oint_C \vec{B} \cdot d\vec{\ell} = B \oint_C d\ell = \mu_0 I_C$$

$$2\pi BR = \mu_0 I_C \Rightarrow B = \frac{\mu_0 I_C}{2\pi R}$$

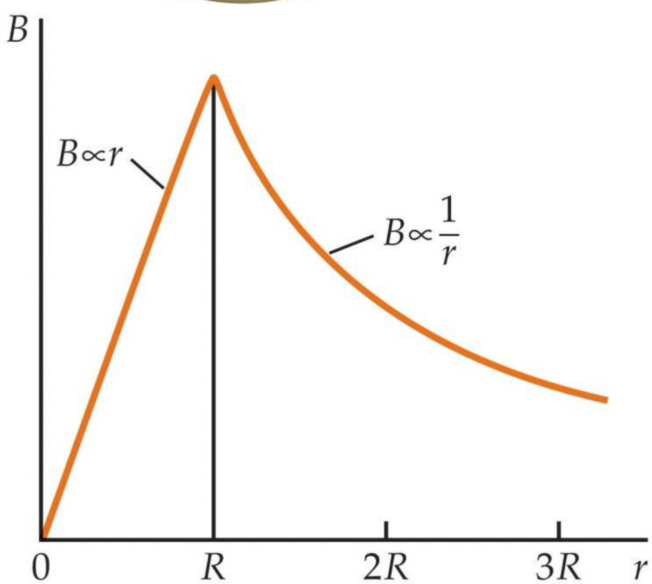
Magnetic Field Inside a Long Straight Wire



$$I_C = I\frac{r^2}{R^2}$$

Ratio of areas inside the wire

$$B(r) = \frac{\mu_0 I_C}{2\pi R}$$

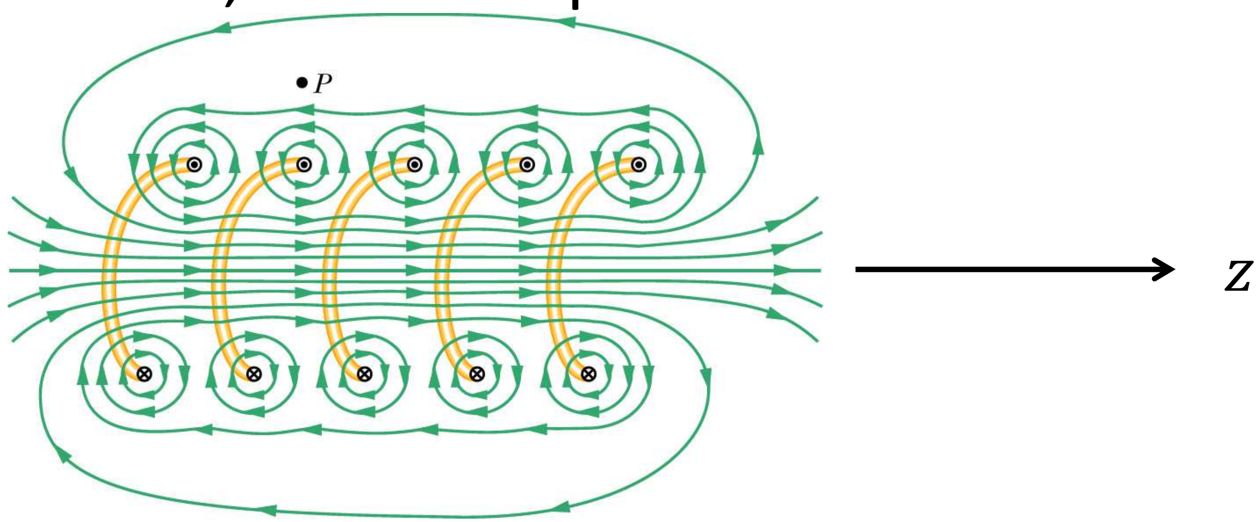


$$B = \frac{\mu_0 I r}{2\pi R^2}$$ (Inside)

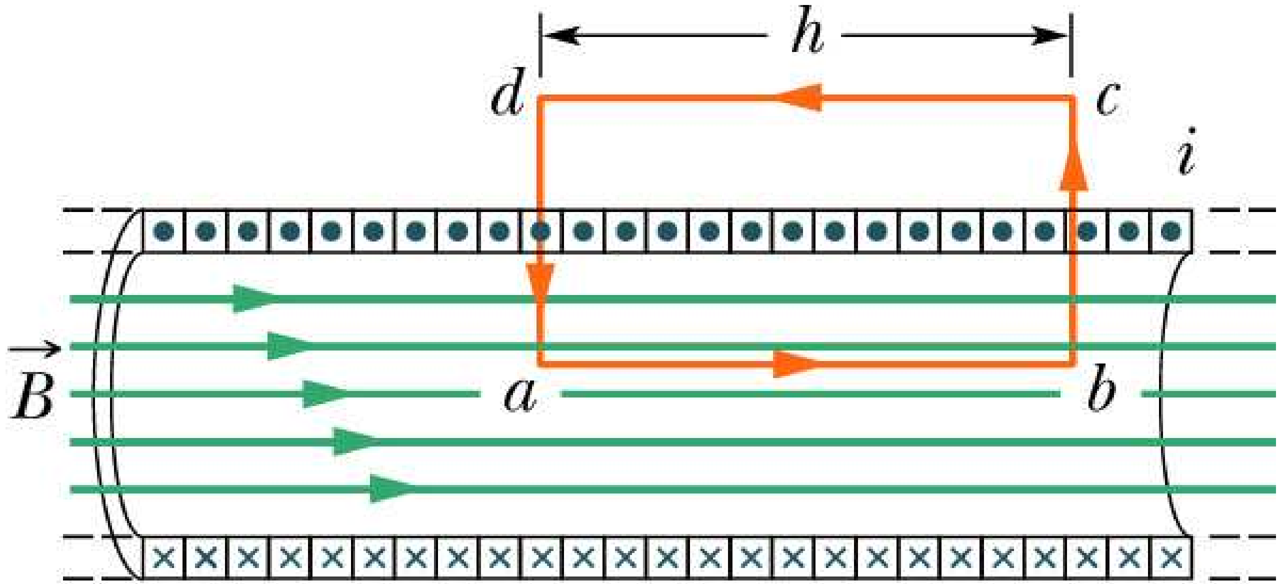
$$B = \frac{\mu_0 I}{2\pi R}$$ (Outside)

Magnetic Field Inside a Solenoid

- Symmetry principles:
  - The magnetic field always points along the axis of the solenoid: $\vec{B} = B\hat{k}$
  - It is independent of z, except at the ends.
- Outside the solenoid, we expect $\vec{B} \to 0$ as $r \to \infty$
- Inside the solenoid, does $\vec{B}$ depend on r?

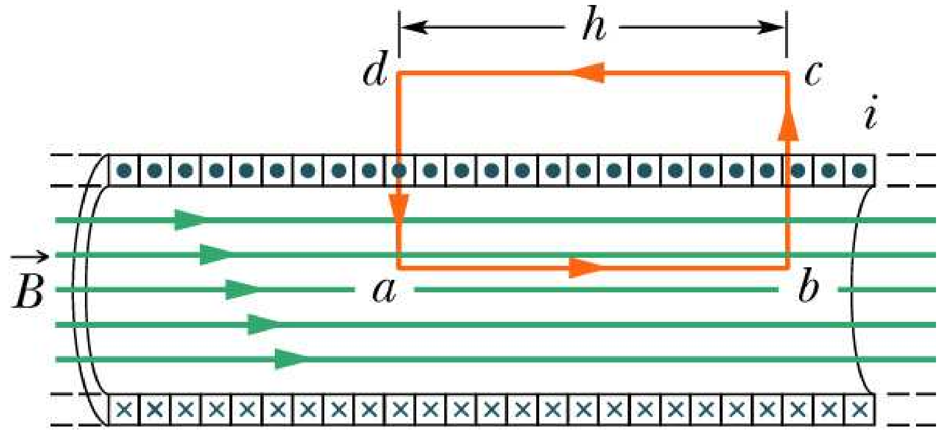


Magnetic Field Inside a Solenoid



$$\oint_C \vec{B} \cdot d\vec{\ell} = \int_a^b \vec{B} \cdot d\vec{\ell} + \underbrace{\int_b^c \vec{B} \cdot d\vec{\ell} + \int_c^d \vec{B} \cdot d\vec{\ell} + \int_d^a \vec{B} \cdot d\vec{\ell}}_{\text{These will all be zero!}} = \mu_0 I_C$$

Magnetic Field Inside a Solenoid



n is the number of turns per unit length.

Make the path cd very far away, where $\vec{B} \approx 0$.

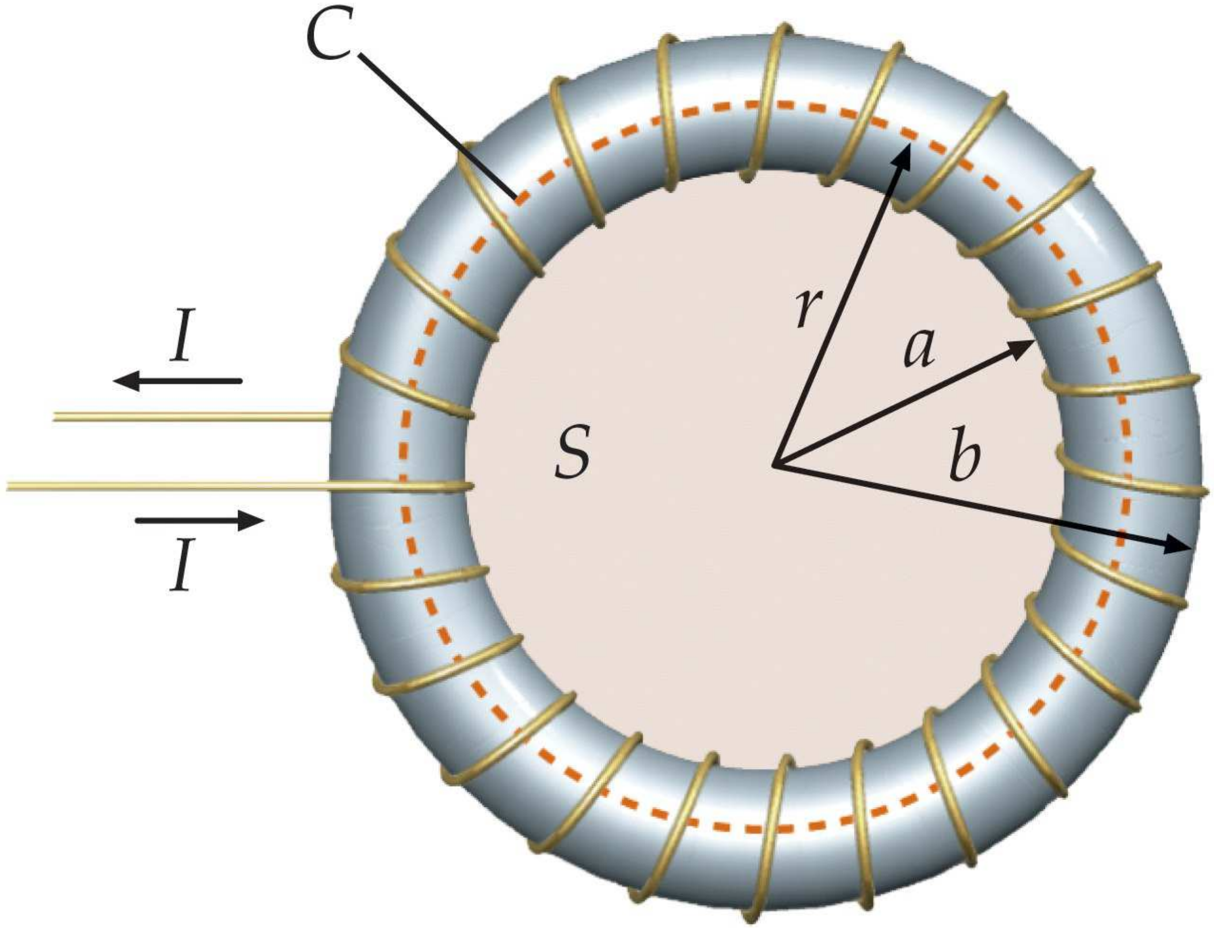
- Enclosed current: $I_C = n\,I\,h$

$$\oint_C \vec{B} \cdot d\vec{\ell} = B\,h = \mu_0 I_C$$

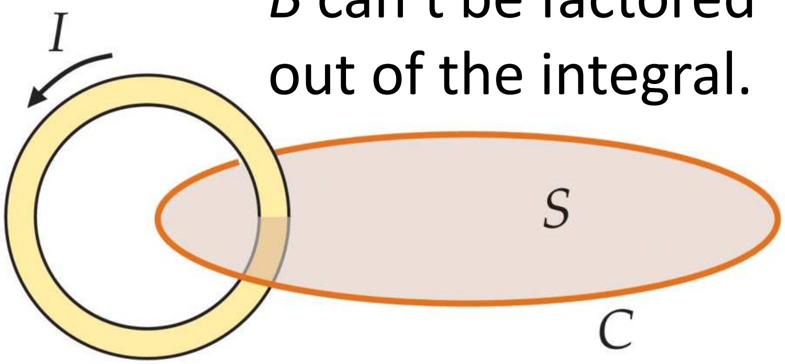
$$B = \mu_0 n\,I$$

Independent of r inside the solenoid.

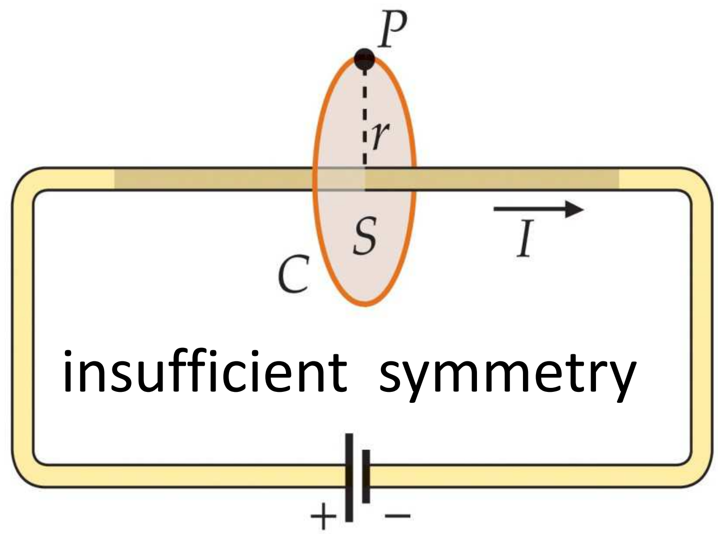
Magnetic Field Inside a Toroid



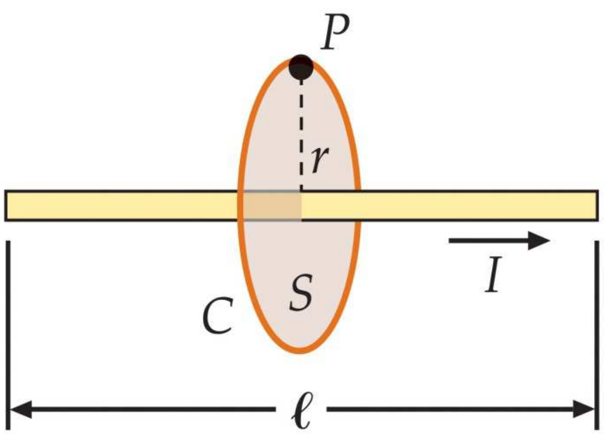
When Ampere's Law doesn't Help



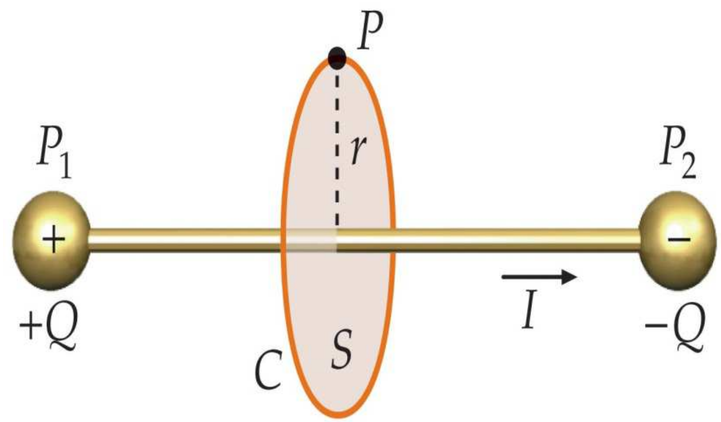
B can't be factored out of the integral.



insufficient symmetry



finite length current segment is (unphysical)



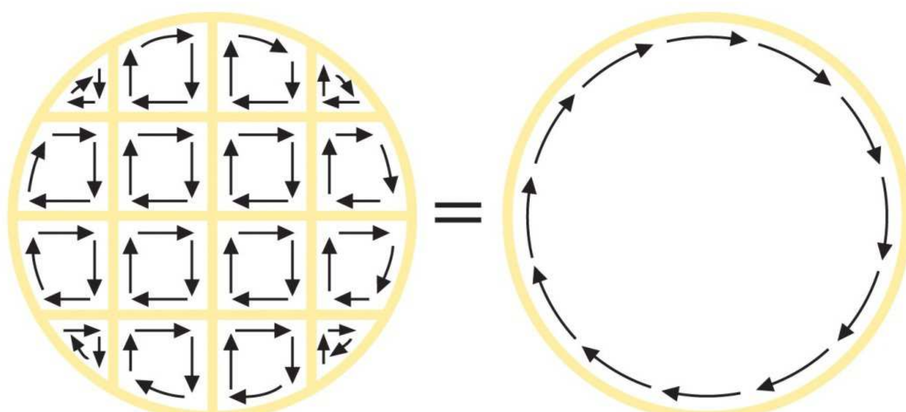
current is not continuous (time dependent)

Magnetic Properties of Materials

- Atoms in many materials act like magnetic dipoles.
- Magnetization is the net dipole moment per unit volume:

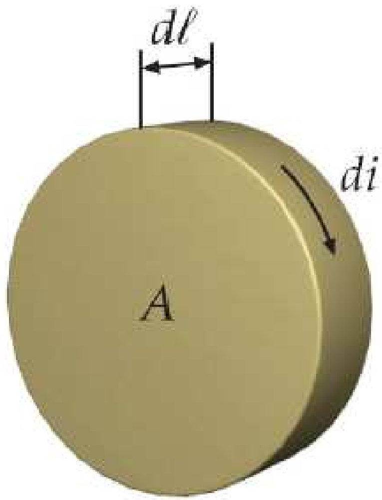
$$\vec{M} = \frac{d\vec{\mu}}{dV}$$

- In the presence of an external magnetic field, these dipoles can start to line up with the field:



Net current inside the material is zero. We are left with a surface current and therefore a magnetic moment

Magnetization and “Bound Current”



Magnetic dipole for a current loop: $\vec{\mu} = A\,I\,\hat{n}$

Magnetic moment per unit length:

$$d\mu = A\,di$$

Magnetization:

$$M = \frac{d\mu}{dV} = \frac{d\mu}{A\,d\ell} = \frac{di}{d\ell}$$

This is the “surface current” per unit length.

Magnetic field due to the surface current is the same as in a solenoid:

$$B = \mu_0 \underbrace{n\,I}_{\text{current per unit length}} = \mu_0\,M$$

Magnetization and Magnetic Susceptibility

- How well do the microscopic magnetic dipoles align with an external applied magnetic field?
- Simplest model: linear dependence on $\vec{B}_{app}$
  - Magnetization: $\vec{M} \propto \vec{B}_{app}$
  - Magnetic field due to surface current:
    $$\vec{B}_m = \mu_0 \vec{M} \equiv \chi_m \vec{B}_{app}$$
  - ***Magnetic susceptibility***: χ_m
- Total magnetic field:
  $$\vec{B} = \vec{B}_{app} + \vec{B}_m = (1 + \chi_m)\vec{B}_{app} \equiv K_m \vec{B}_{app}$$
  - ***Relative permeability***: K_m

Magnetic Susceptibility

- Different materials react differently to external magnetic fields:

$\boldsymbol{\chi_m > 0}$ **small** $\boldsymbol{\chi_m}$	**Paramagnetism**	aluminum, tungsten
$\boldsymbol{\chi_m < 0}$ small $\lvert\boldsymbol{\chi_m}\rvert$	**Diamagnetism**	bismuth, copper, silver
$\boldsymbol{\chi_m > 0}$ large $\boldsymbol{\chi_m}$	**Ferromagnetism**	iron, cobalt, nickel

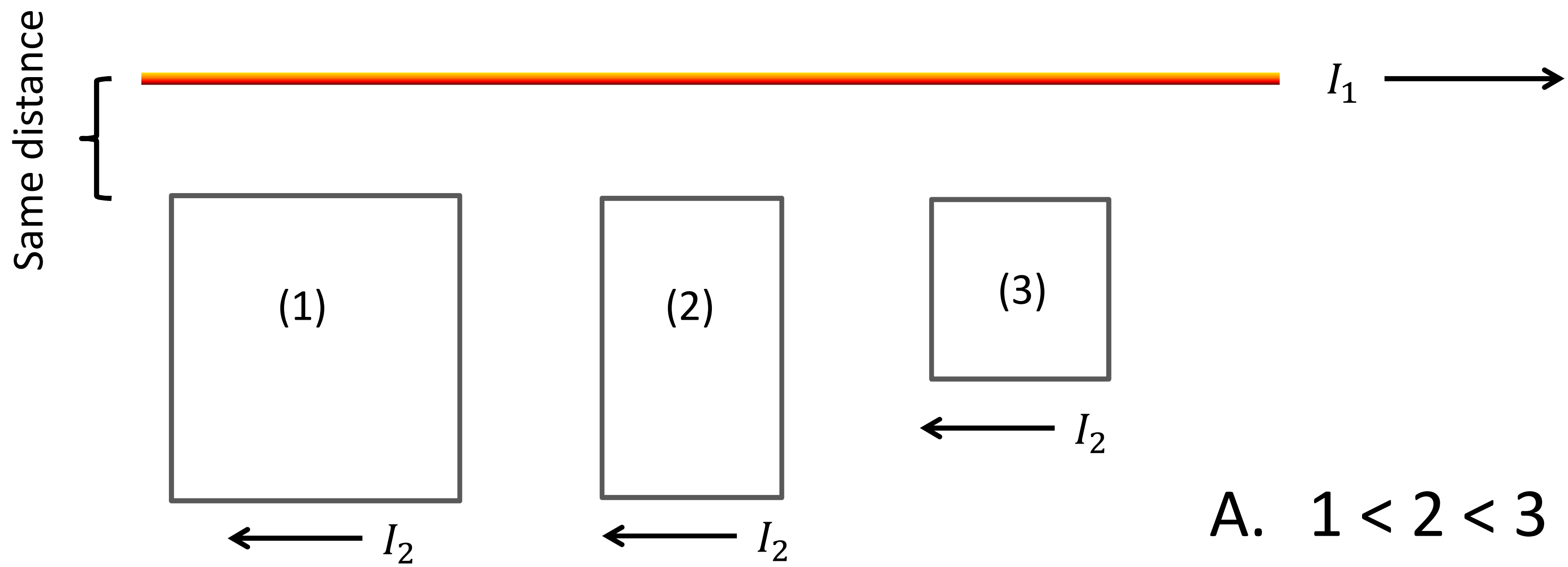
- Dipoles in paramagnetic materials align with $\vec{B}_{app}$
- Dipoles in diamagnetic materials align opposite $\vec{B}_{app}$
- Ferromagnetic materials align strongly even in weak $\vec{B}_{app}$

Magnetic Susceptibility

Material	χ_m	Type
Bi	-1.66×10^{-5}	diamagnetic
Ag	-2.6×10^{-5}	diamagnetic
Al	2.3×10^{-5}	paramagnetic
Fe (annealed)	5,500	ferromagnetic
Permalloy	25,000	ferromagnetic
mu-metal	100,000	ferromagnetic
superconductor	-1	diamagnetic (perfect)

Clicker Question

- Rank the current loops in order of increasing force:



A. 1 < 2 < 3
B. 2 < 3 < 1
C. 3 < 1 < 2
D. 3 < 2 < 1
E. 3 = 2 = 1