PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 24100

Electricity & Optics

Lecture 14 – Chapter 27 sec. 1-2

Fall 2012 Semester

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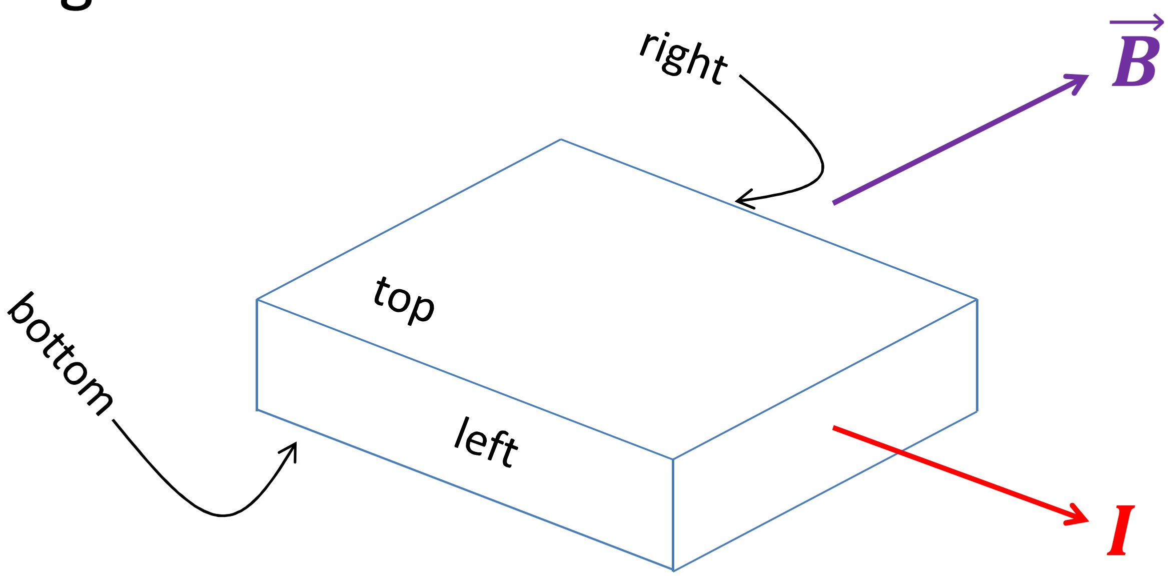
Question

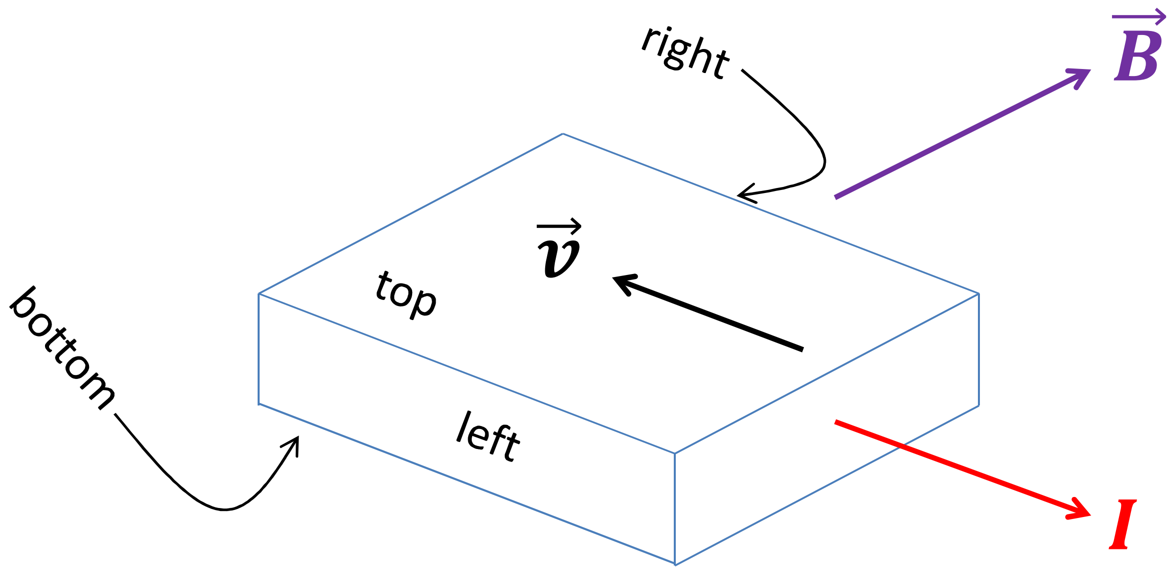
- A rectangular conductor carries current I in a magnetic field as shown:



- If the charge carriers are electrons, on which surface will a negative charge accumulate?

(a) Top (b) Bottom (c) Left (d) Right

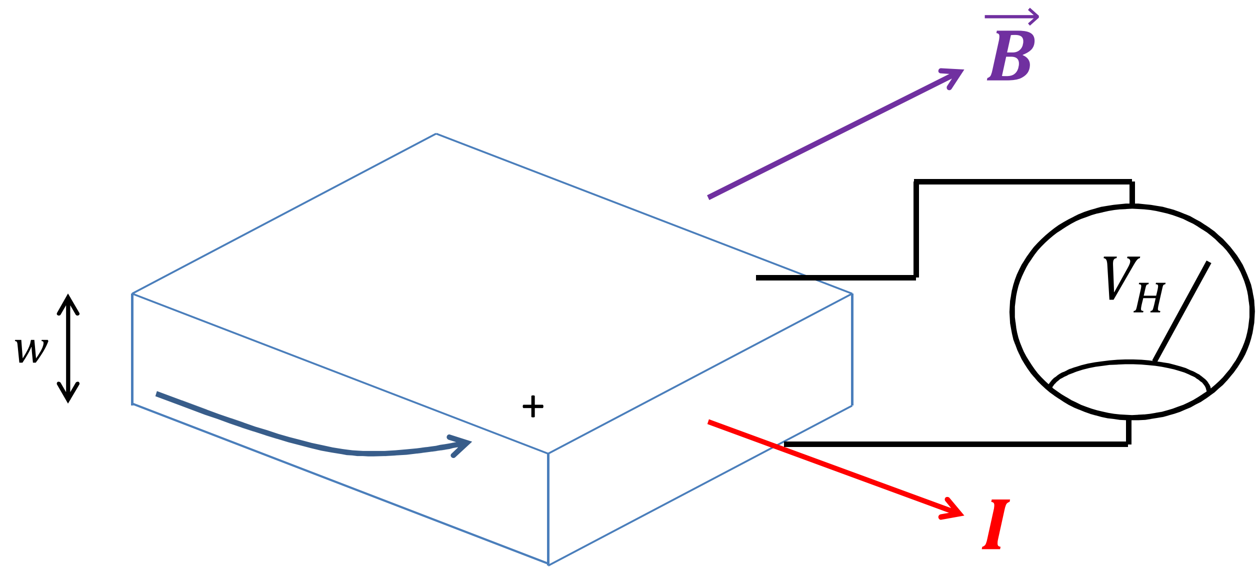
Question



$$\vec{F} = q\,\vec{v} \times \vec{B}$$

- The force on the charge carriers will be perpendicular to both $\vec{B}$ and $\vec{v}$ so it must be either top or bottom.
- Electrons move in the direction opposite the current.
- The right-hand-rule tells you that $\vec{v} \times \vec{B}$ points down.
- But q is negative, so the negative charge accumulates on the ***top***.

The Hall Effect



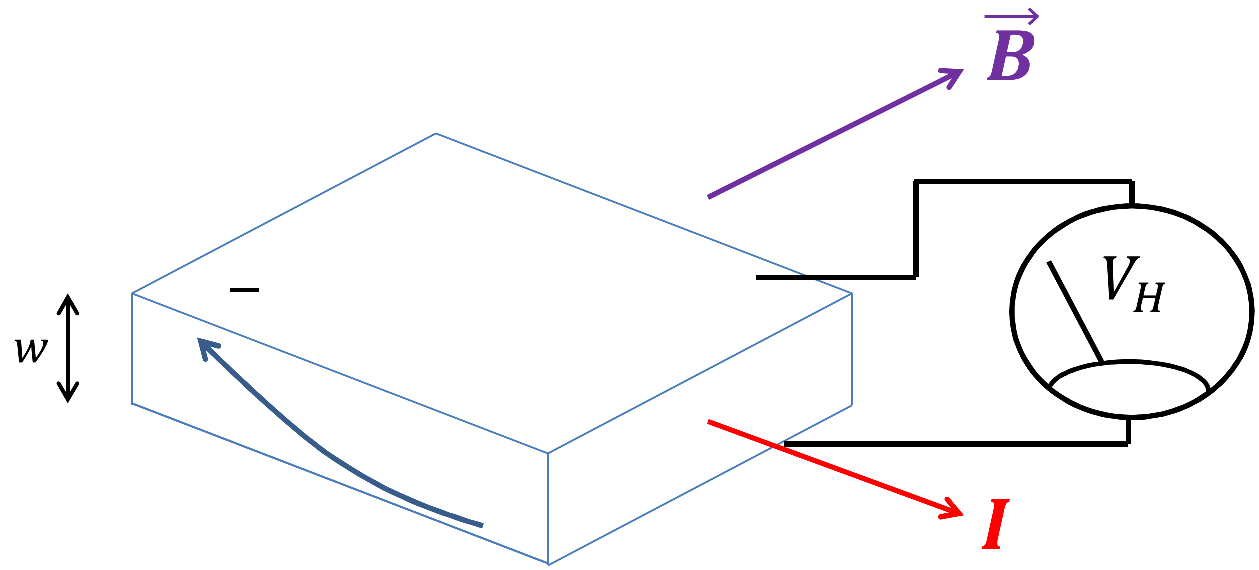
- The Lorentz force is balanced by the electrostatic force:

$$qv_dB = qE_H = q\frac{V_H}{w}$$

$$V_H = v_dBw$$

- This tells us the direction and magnitude of the drift velocity of the charge carriers.

The Hall Effect



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$$qv_dB = qE_H = q\frac{V_H}{w}$$

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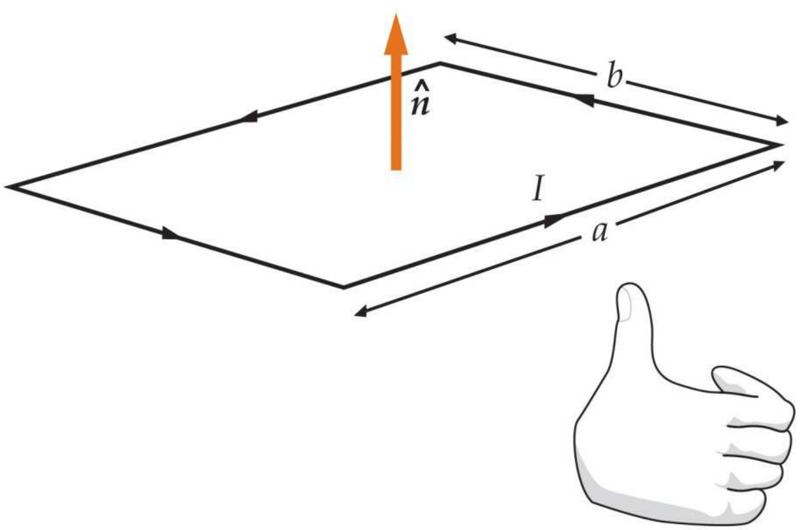
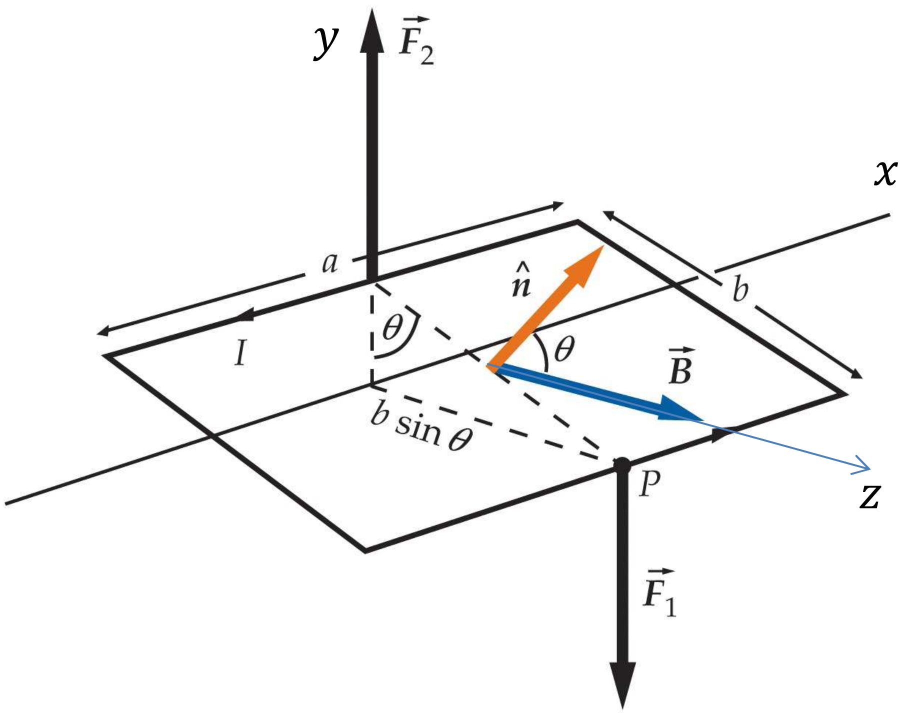
- This tells us the ***direction*** and ***magnitude*** of the drift velocity of the charge carriers.

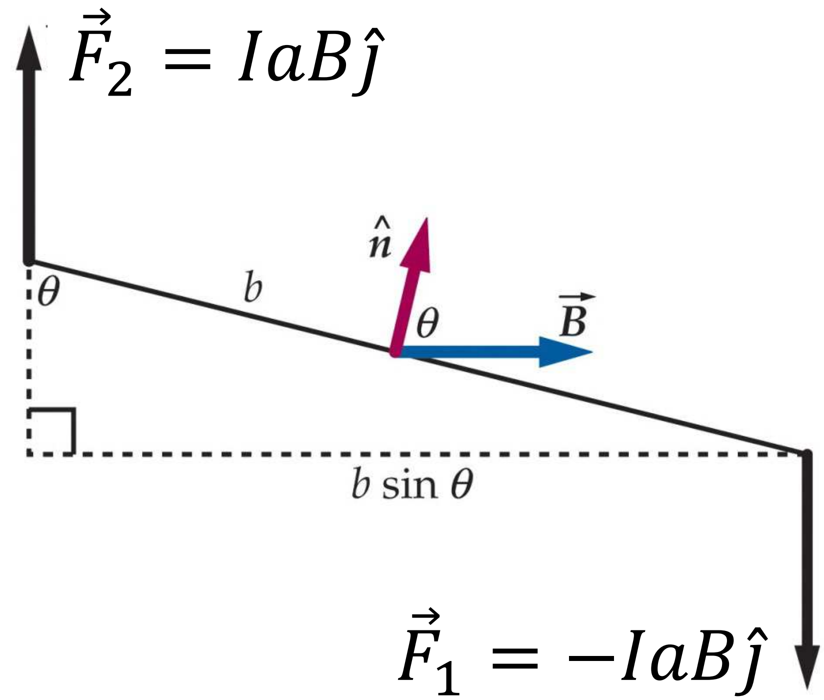
Torque on a Current Loop

- Consider a rectangular loop of wire carrying current I in a magnetic field.
- The orientation of the loop is given by the unit vector $\hat{n}$ perpendicular to the plane of the loop.



Torque on a Current Loop





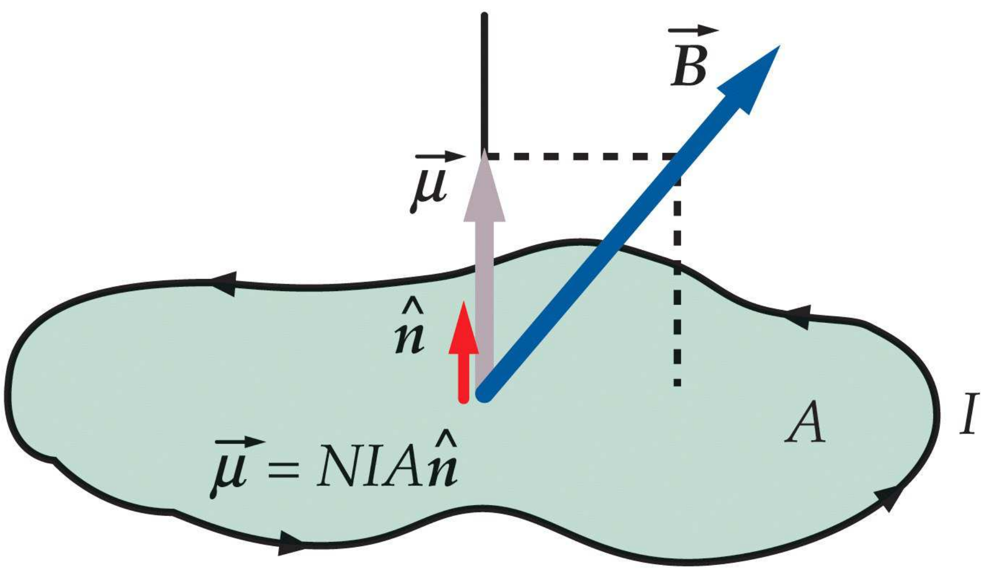
- Magnitude of torque is $\tau = IabB\sin\theta$
- Direction is perpendicular to $\vec{B}$ and $\hat{n}$:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$
$$\vec{\mu} = Iab\hat{n}$$

Torque on a Current Loop

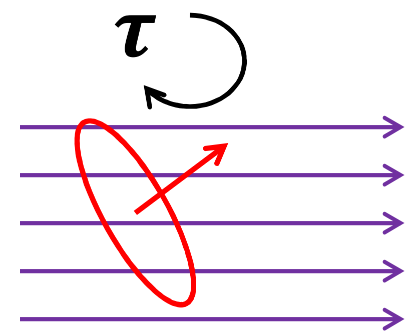
- In general, the torque does not depend on the shape, just the area.
- With N turns of wire in the loop, multiply by N.

$$\vec{\mu} = NIA\,\hat{n}$$



$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Potential Energy



- Work done ***by*** the magnetic field:

$$dW = -\tau\, d\theta$$

- Loss of potential energy:

$$dU = -dW = \mu B \sin\theta\, d\theta$$

- Total change in potential energy:

$$\Delta U = \int_{\theta_0}^{\theta} \mu B \sin\theta\, d\theta = -\mu B \cos\theta$$

- Potential energy of a dipole in a magnetic field:

$$U = -\vec{\mu} \cdot \vec{B}$$

- Minimal potential energy when $\vec{\mu}$ and $\vec{B}$ are aligned.

Magnetic Field

- Electrostatics:
  - An electric field exerts a force on a charge
  - An charge produces an electric field

$$\vec{E} = \frac{Q}{4\pi\epsilon_0}\frac{\hat{r}}{r^2}$$

- Magnetism:
  - A magnetic field exerts a force on a moving charge
  - *A moving charge produces a magnetic field*

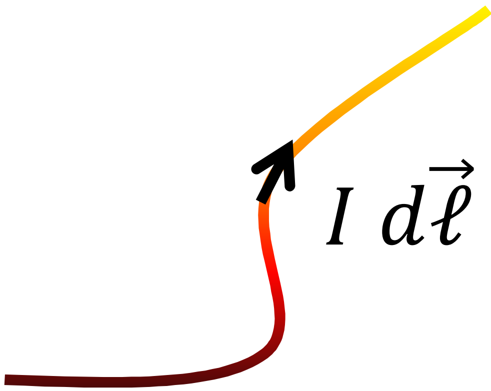
$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v} \times \hat{r}}{r^2}$$

$$\mu_0 = 4\pi \times 10^{-7}\ T \cdot m/A$$

Magnetic Field

- Moving charge:

$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\vec{v} \times \hat{r}}{r^2}$$

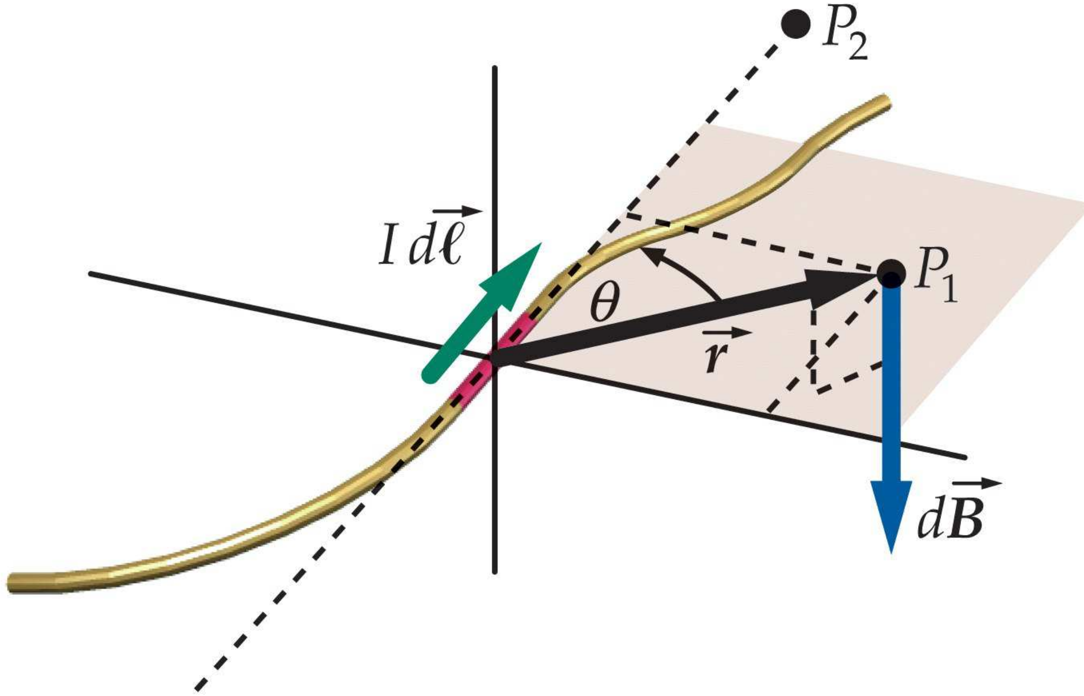


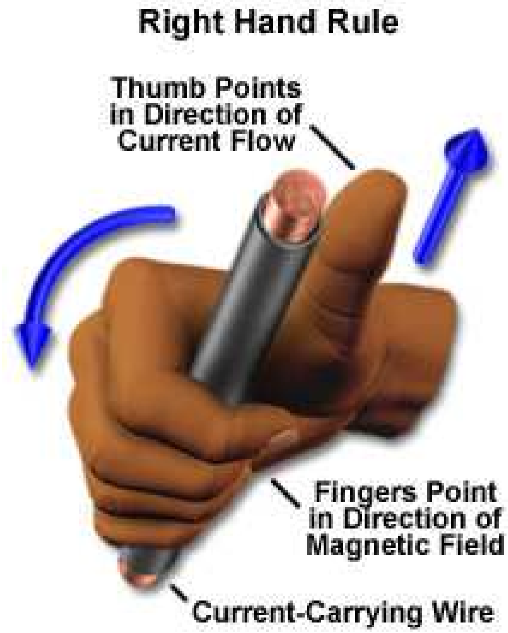
- Current flowing in a wire:

$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{I\, d\vec{\ell} \times \hat{r}}{r^2}$$

(Biot-Savart Law)

Observations

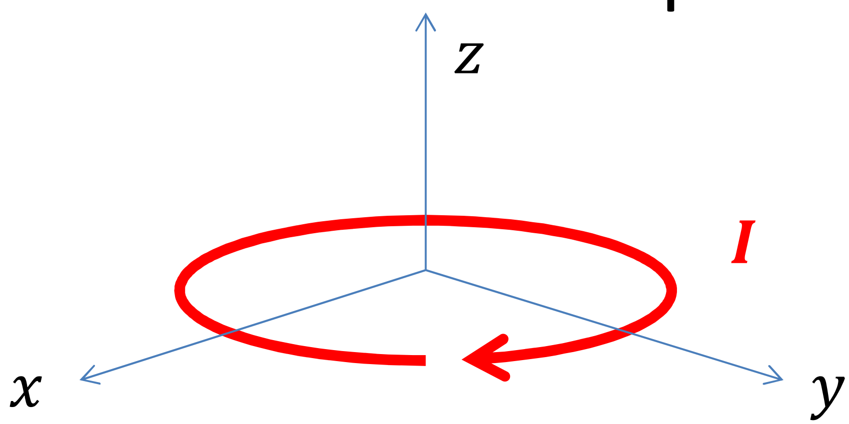




$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{I\,d\vec{\ell} \times \hat{r}}{r^2}$$

Question

- What is the direction of the magnetic field at the center of the current loop:



(a) $\hat{\boldsymbol{k}}$ (b) $-\hat{\boldsymbol{k}}$ (c) $\hat{\boldsymbol{\imath}}$ (d) $\hat{\boldsymbol{\jmath}}$

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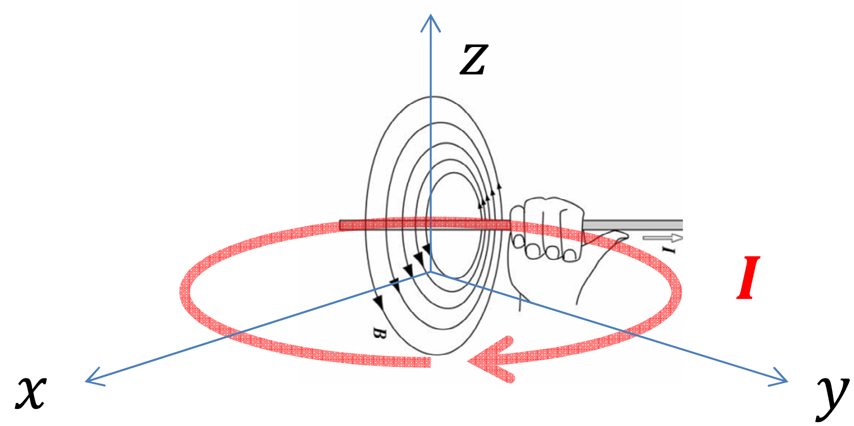


(a) $\hat{\boldsymbol{k}}$ (b) $-\hat{\boldsymbol{k}}$ (c) $\hat{\boldsymbol{\imath}}$ (d) $\hat{\boldsymbol{\jmath}}$

Current Carrying Wires

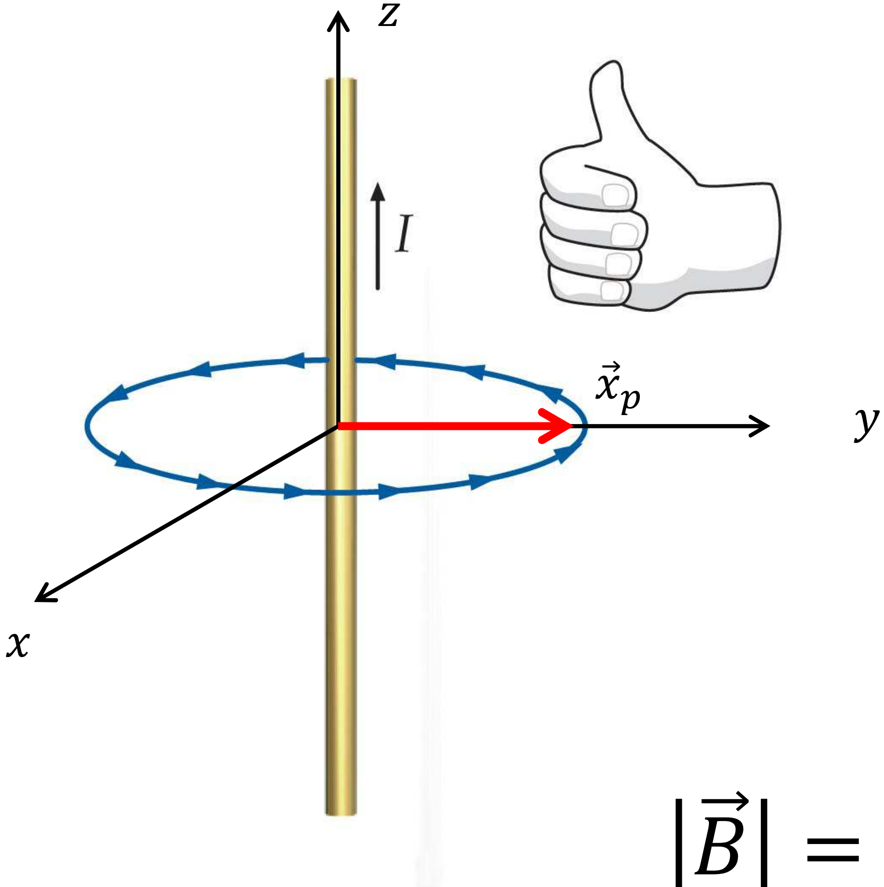
- Use the principle of superposition:

$$\vec{B} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{\ell} \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\int \frac{I\, d\vec{\ell} \times \vec{r}}{r^3}$$

- Very similar to the way we calculated $\vec{E}$:
  1) Pick a coordinate system
  2) Label source and field points
  3) Pick variables to express their components
  4) Express $d\vec{\ell}, \vec{r}$ and r using these variables
  5) Evaluate $d\vec{\ell} \times \vec{r}$ ← New step!
  6) Write out the integral for each component
  7) Evaluate the integrals one way or another.

Example

- Magnetic field around a long, straight wire:



Without loss of generality, pick the field point at R on the y-axis and the source point on the z-axis.

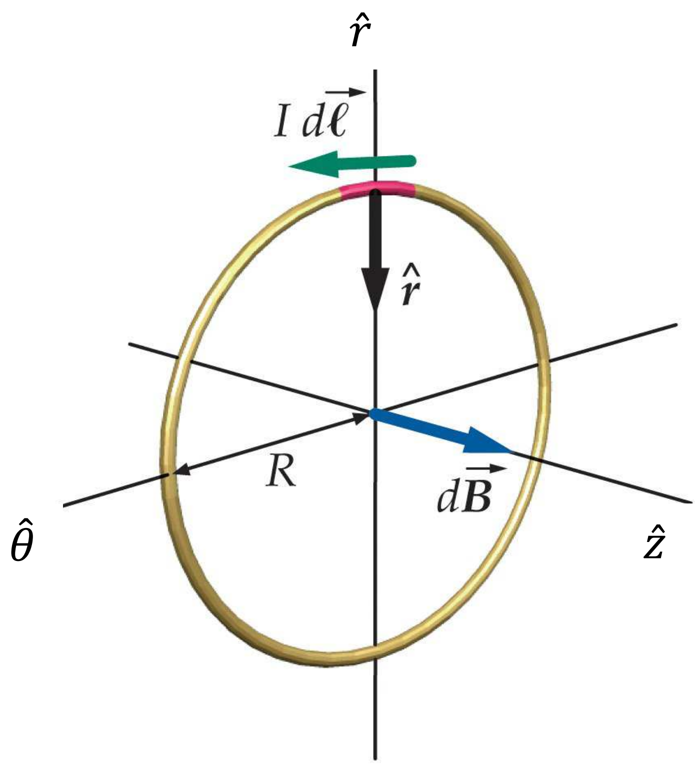
The magnetic field will be in the $-\hat{\imath}$ direction.

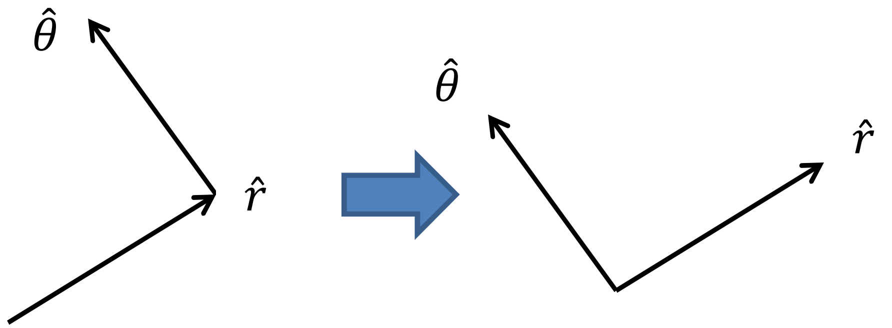
We could also use a cylindrical coordinate system with unit vectors $\hat{\rho}, \hat{\varphi}, \hat{z}$

$$|\vec{B}| = \frac{\mu_0}{4\pi}\frac{2I}{R}$$

Example

- Magnetic field at the center of a current loop

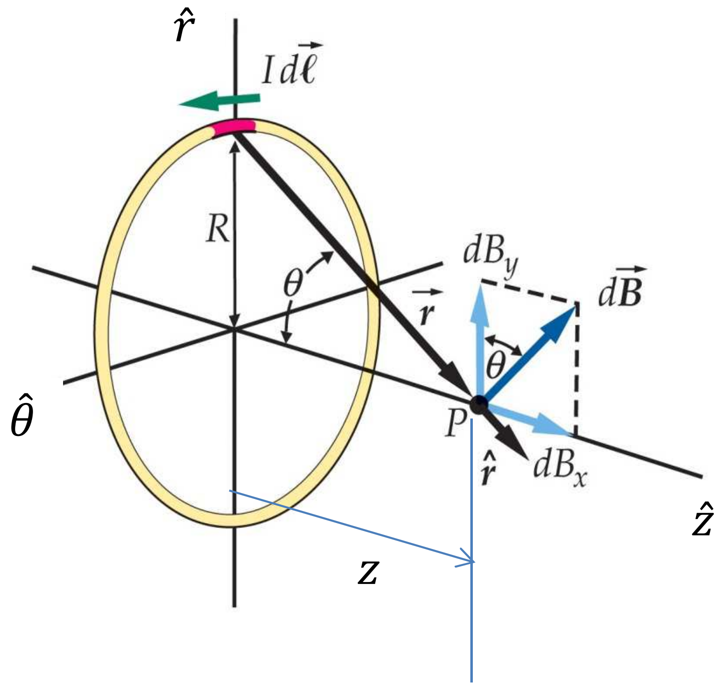




$$\left|\vec{B}\right| = \frac{\mu_0}{2}\frac{I}{R}$$

Example

- Magnetic field on the axis of a current loop



$$|\vec{B}| = \frac{\mu_0}{2} \frac{IR^2}{(R^2 + z^2)^{3/2}}$$

Example

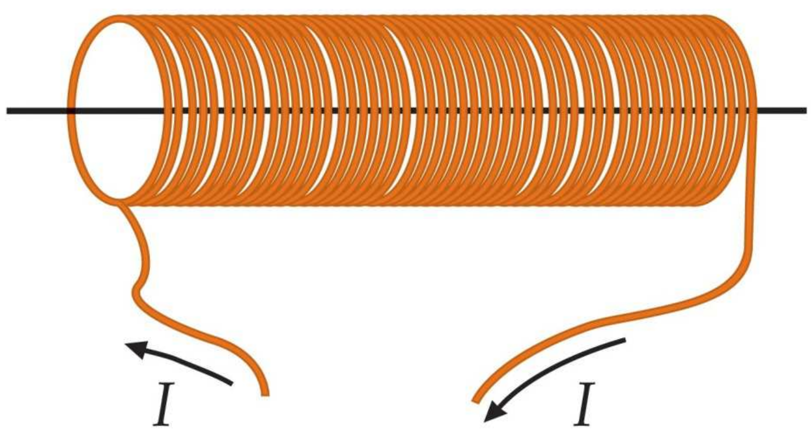
- A solenoid is like a bunch of current loops with $n = N/L$ loops per unit length.

$$dB = \frac{\mu_0}{2}\frac{di\, R^2}{(R^2 + z^2)^{3/2}}$$

where $di = n\, I\, dz$.

- Inside a long solenoid, $L \gg R$:

$$B = \mu_0 n I$$



Clicker Question

- Treat a lightning bolt like a long, straight wire.
- If the current in a lightning bolt is 100 kA, how would the magnetic field 1 km away compare with the Earth's magnetic field ($5 \times 10^{-5}\ T$)?

(a) Much less

(b) Much greater

(c) About the same

$$(\mu_0 = 4\pi \times 10^{-7}\ \ T \cdot m/A)$$