PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 24100

Electricity & Optics

Lecture 13 – Chapter 26 sec. 2-4

Fall 2012 Semester

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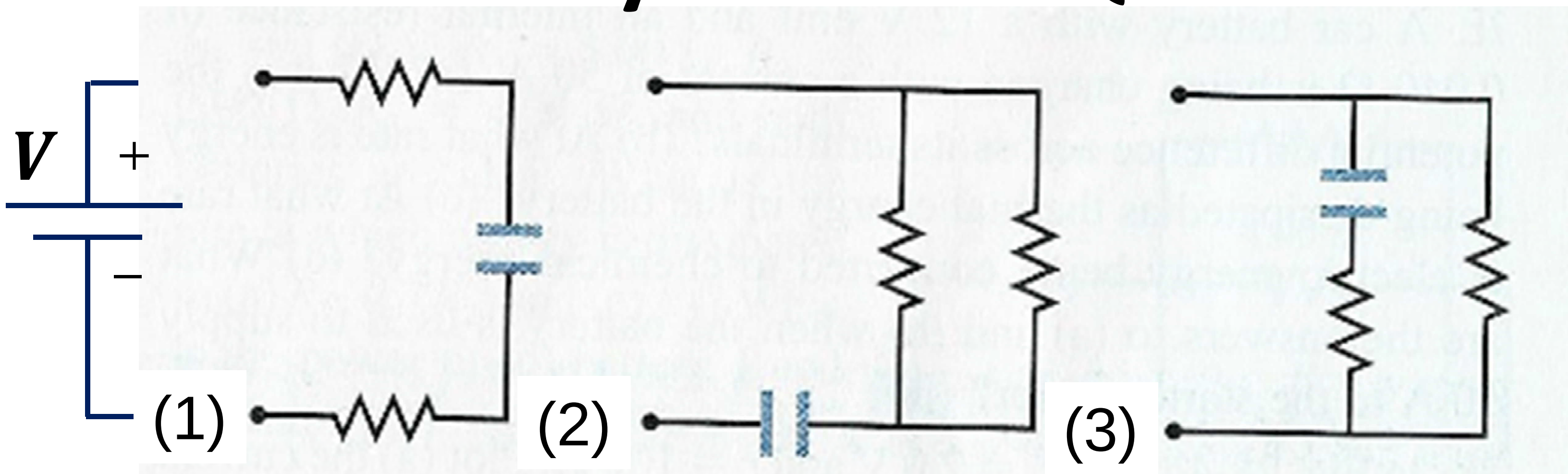
Thursday's Clicker Question



The figure shows three section of circuit that are to be connected in turn to the same battery via a switch. The resistors are identical, as are the capacitors. Rank the sections according to the final charge ($t \to \infty$) on the capacitor.

(a) 2 = 3 > 1

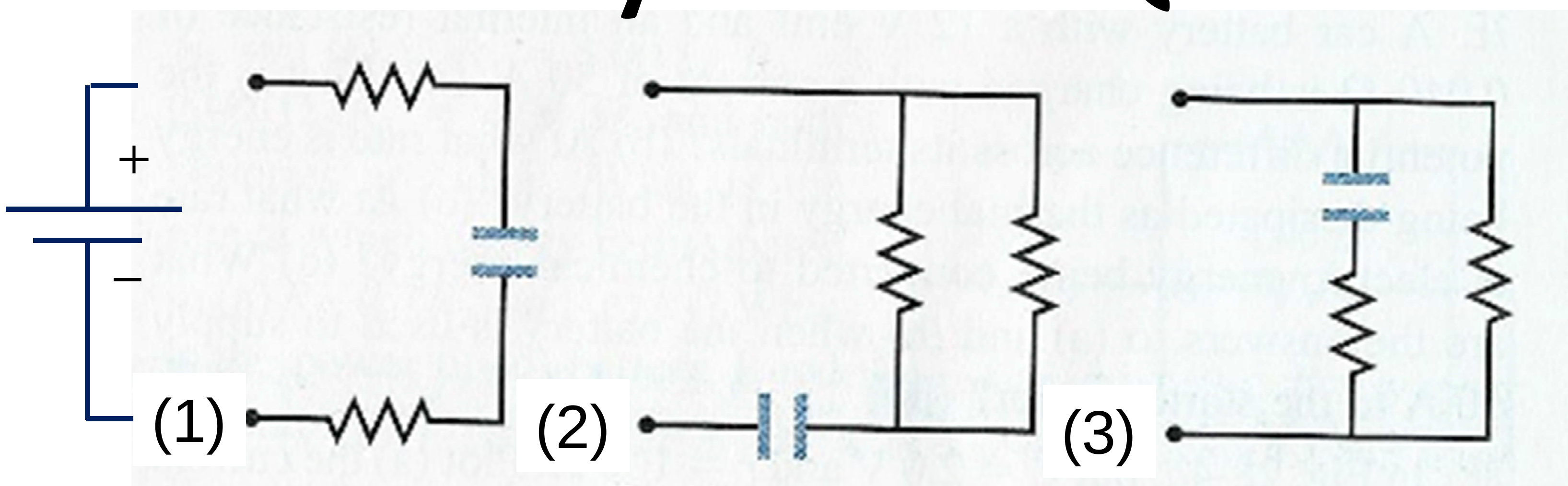
(b) 1 = 2 = 3

(c) 3 > 1 > 2

(d) 1 > 3> 2

(e) 2 > 3 > 1

Thursday's Clicker Question

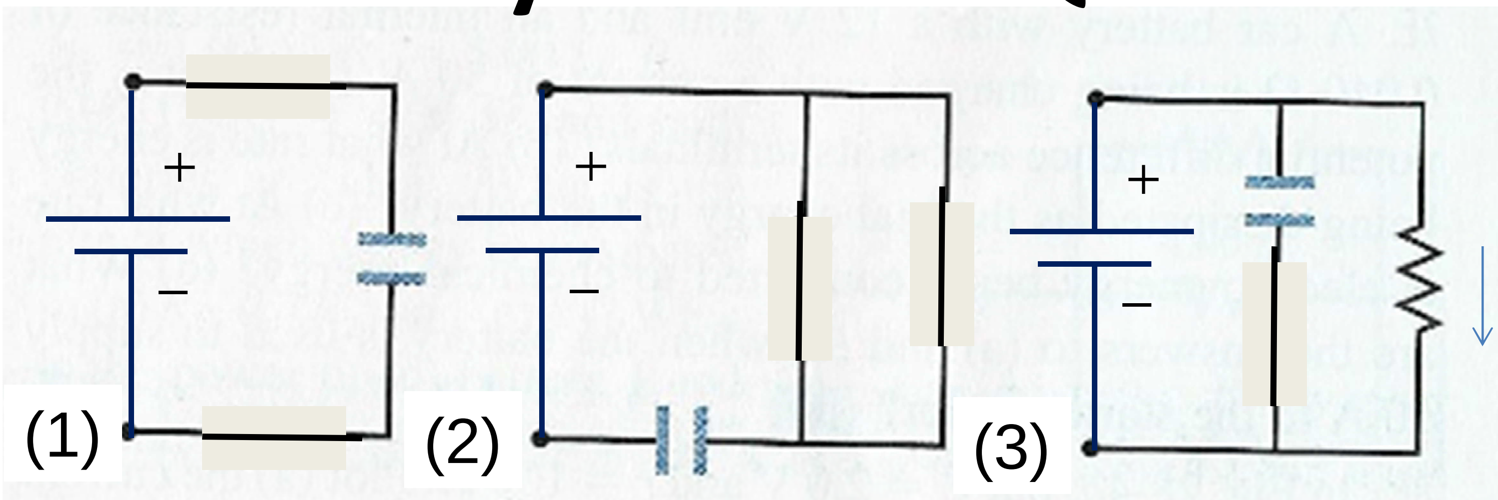


As $t \rightarrow \infty$, the capacitor acts like an open circuit: no current will flow through it.

When no current flows through a resistor, there is no potential difference across it.

Consider the effective circuits as $t \rightarrow \infty$...

Thursday’s Clicker Question



In circuit (3), current still flows through the resistor, but this doesn’t change the potential across the capacitor.

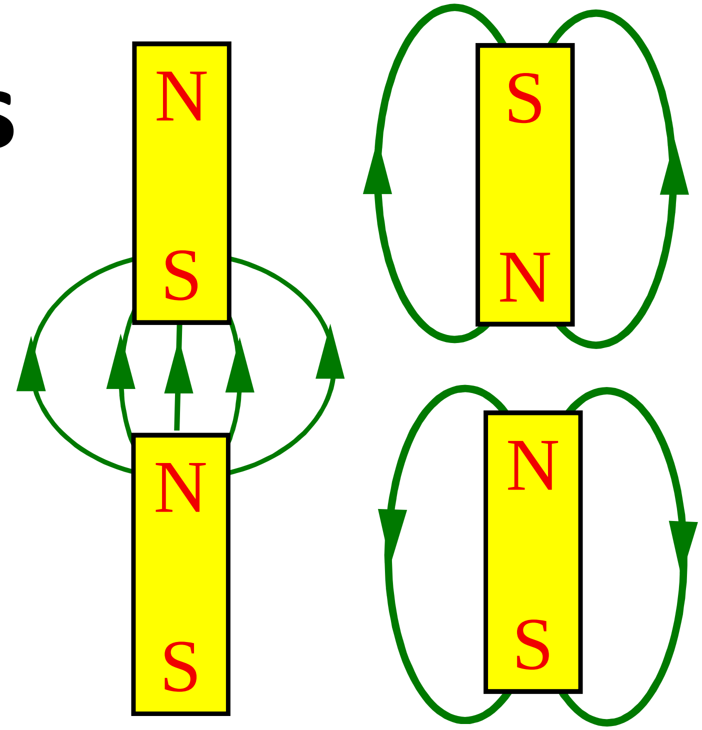
The same potential is reached across all three capacitors. Capacitance is equal so the charge is also equal.

Bar Magnets

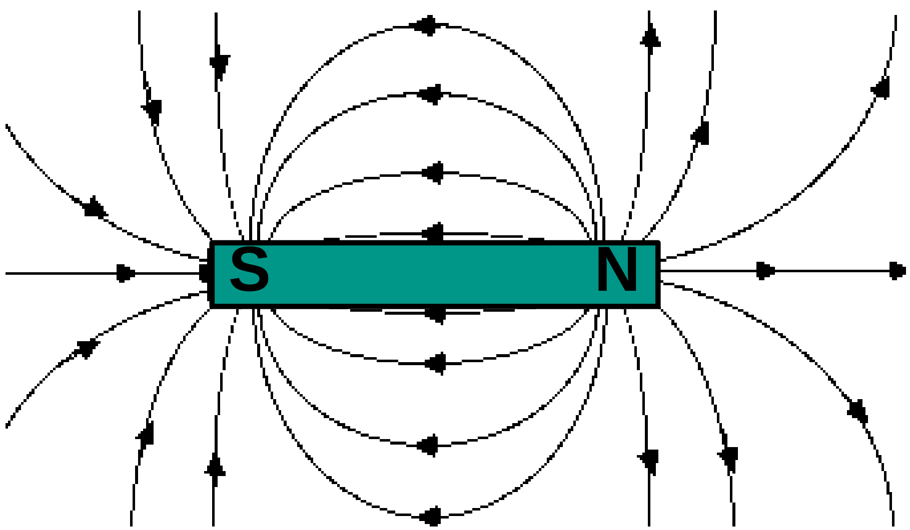
- Bar magnet

  Like poles repel; Unlike poles attract.

- Magnetic Field lines: (defined in same way as electric field lines, direction and density)



Attraction Repulsion

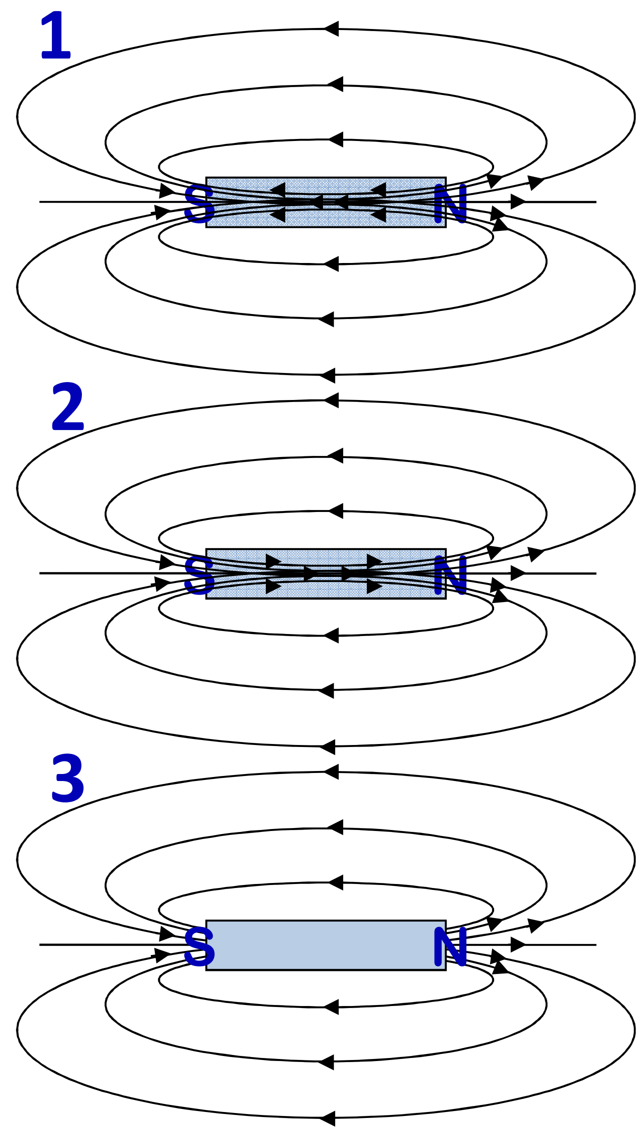


From North to South

Magnetic Fields

We draw the magnetic field the same way we drew electric field lines:

- Density of lines indicates the field strength
- Lines point from N to S *outside* the magnet
- If we cut the magnet in half, we get two little magnets.
- *Inside* the magnet, the lines point from S to N



Lorentz Force

- The force, $\vec{F}$, on a charge q, moving with velocity $\vec{v}$ in a magnetic field $\vec{B}$ is given by:

$$\vec{F} = q\,\vec{v} \times \vec{B}$$

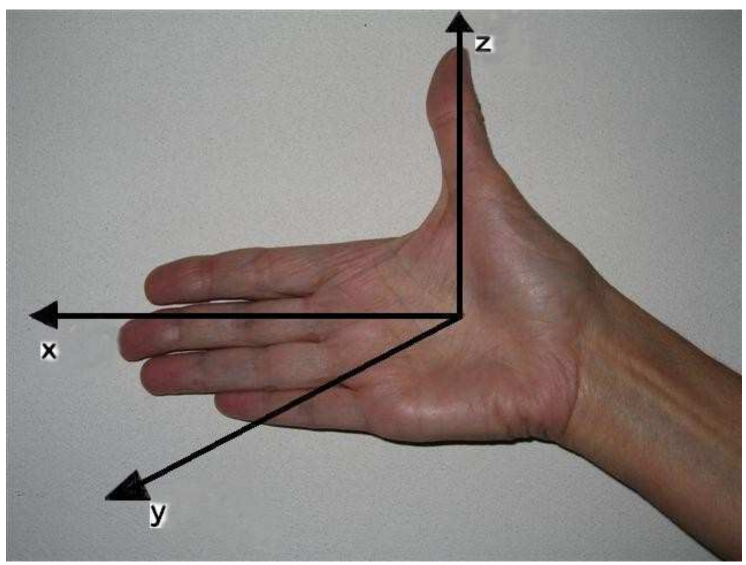
- If there is also an electric field $\vec{E}$, then the force is

$$\vec{F} = q\,\vec{E} + q\,\vec{v} \times \vec{B}$$

(The “×” is the vector cross-product…)

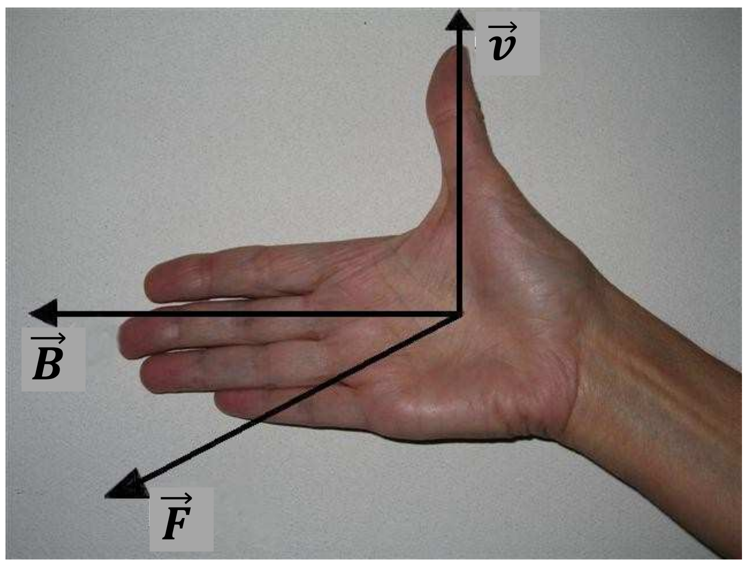
Vector Cross Products

- General considerations:
  - $\vec{v} \times \vec{B}$ is always perpendicular to both $\vec{v}$ and $\vec{B}$
  - If $\vec{v}$ and $\vec{B}$ are parallel, then $\vec{v} \times \vec{B}$ is zero.
- The "right-hand rule" gives you the direction:

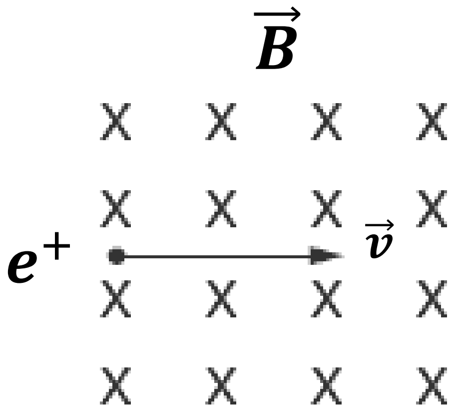


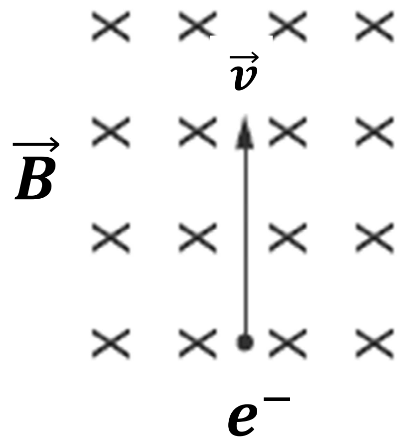
Right Hand Rule

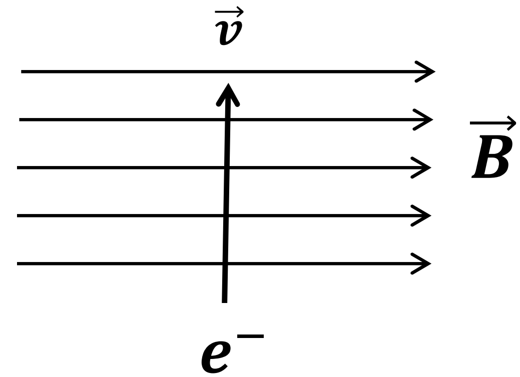
- Fingers go in the direction of $\vec{B}$
- Thumb points in the direction of $\vec{v}$
- Then your hand pushes in the direction of $\vec{F}$ (for a positive charge).

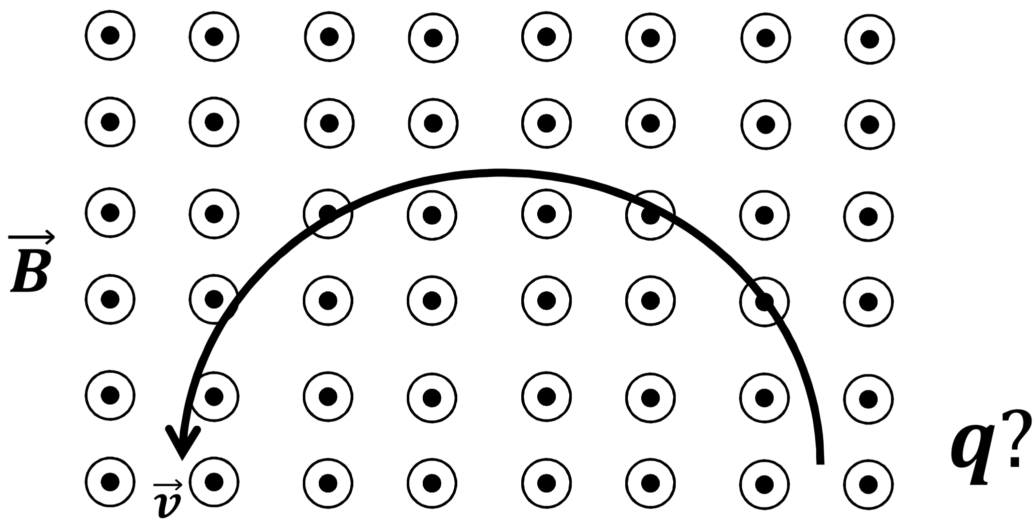


Direction of $\overrightarrow{F}$

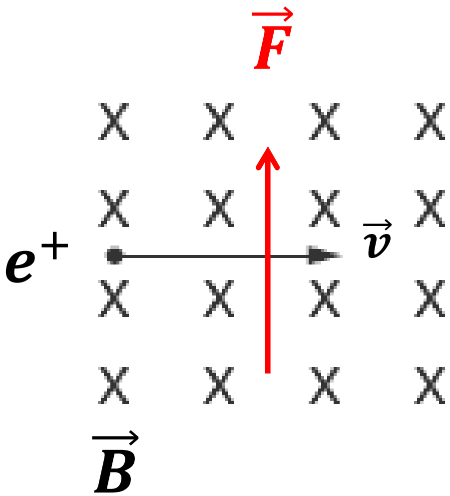


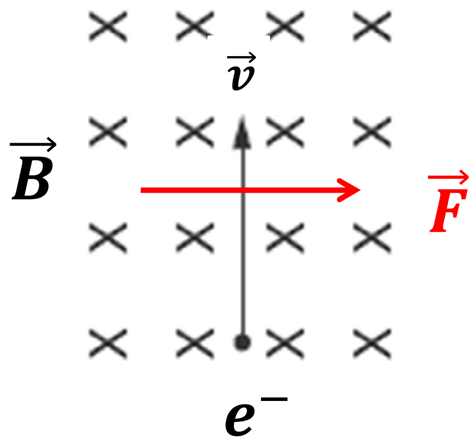


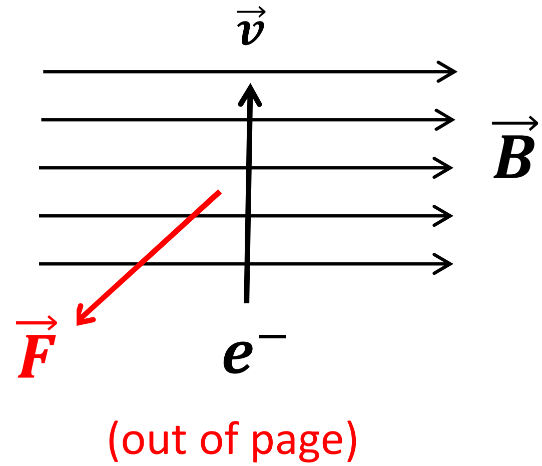


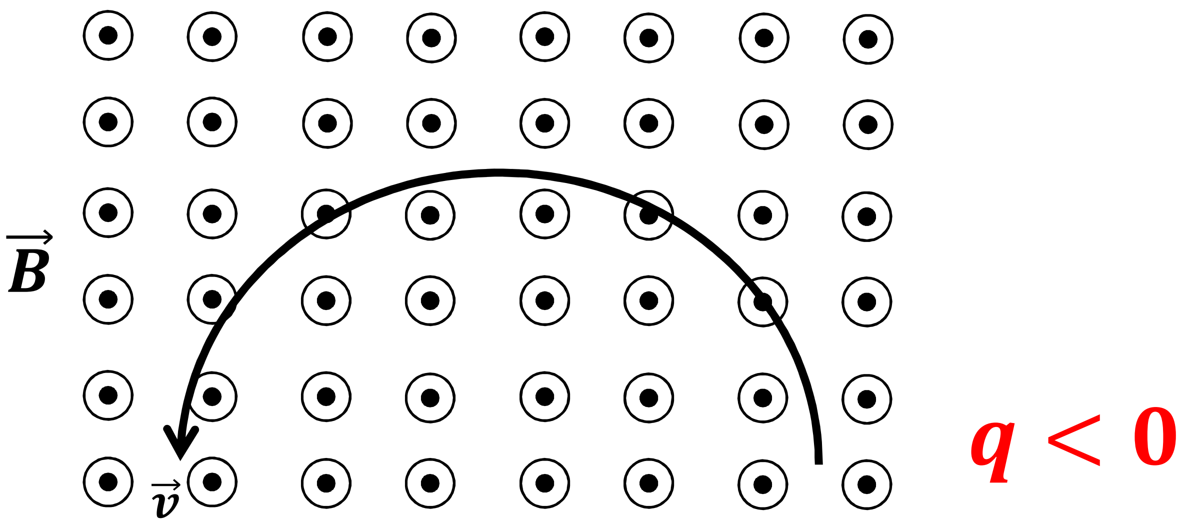


Direction of $\vec{F}$









Vector Cross Products

- Cross products of unit vectors:

$$\hat{\imath} \times \hat{\jmath} = \hat{k}$$
$$\hat{\jmath} \times \hat{k} = \hat{\imath}$$
$$\hat{k} \times \hat{\imath} = \hat{\jmath}$$

- Calculate using determinants:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$
$$= \hat{\imath}\left(a_y b_z - a_z b_y\right) - \hat{\jmath}(a_x b_z - a_z b_x) + \hat{k}\left(a_x b_y - a_y b_x\right)$$

- Important relation:

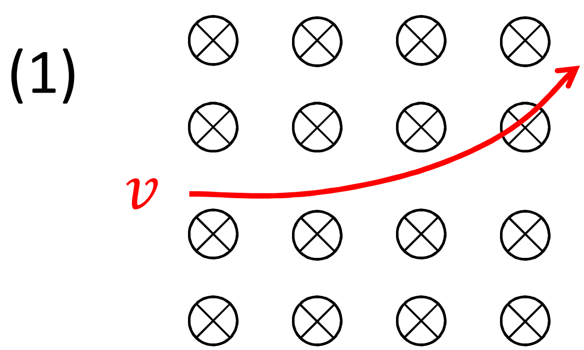
$$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$$

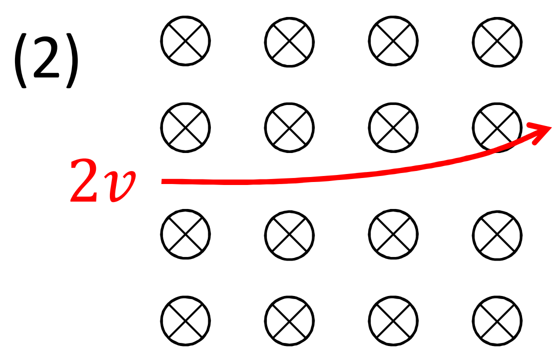
Lorentz Force

- Properties:
  - $\vec{F}$ is always perpendicular to $\vec{v}$ and $\vec{B}$
  - The force can change the direction but not the speed of the charge
  - If q is positive, $\vec{F}$ is in the direction of $\vec{v} \times \vec{B}$
  - If q is negative, $\vec{F}$ is opposite the direction of $\vec{v} \times \vec{B}$
  - $\vec{F}$ is never parallel to $\vec{v}$…

Question

- Two ***protons*** move through a region of uniform magnetic field
- One has twice the initial velocity of the other
- Compare the work done by the magnetic field in deflecting the electrons





(a) $W_1 < W_2$ **(b)** $W_1 = W_2$ **(c)** $W_1 > W_2$

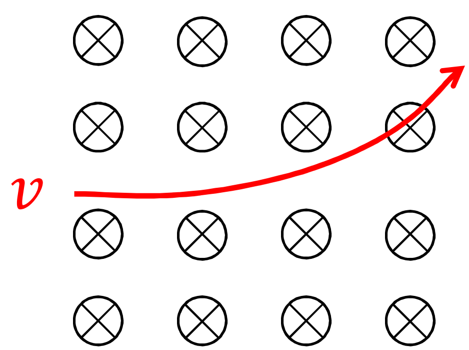
Question

- The force is always perpendicular to the velocity
- Work done on the charge:

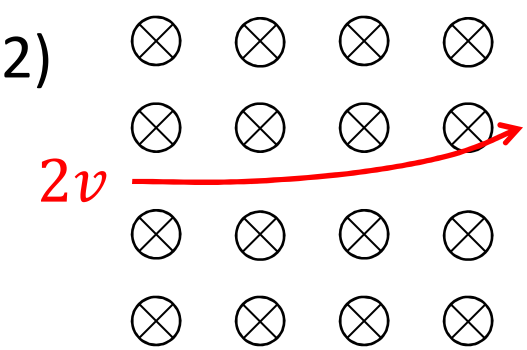
$$dW = -\vec{F} \cdot d\vec{\ell} = 0$$

- The direction changes but the speed stays the same.

(1)

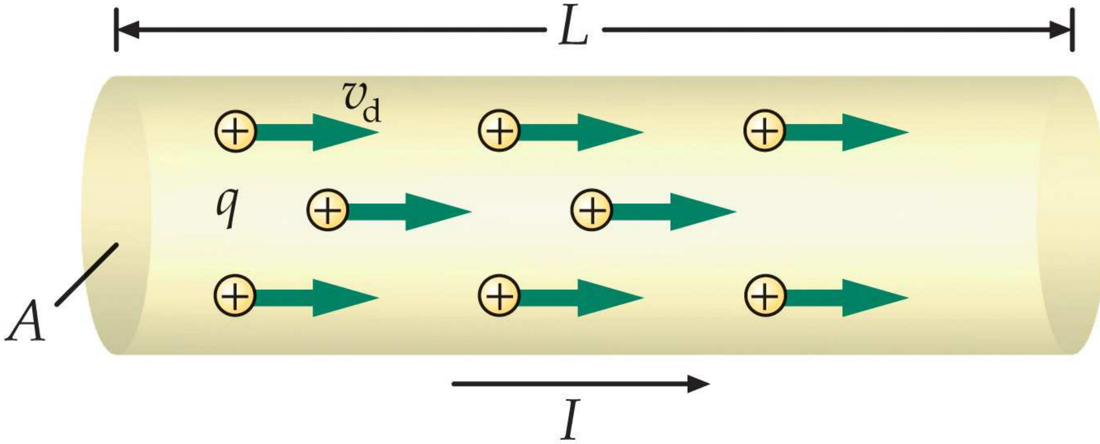


(2)



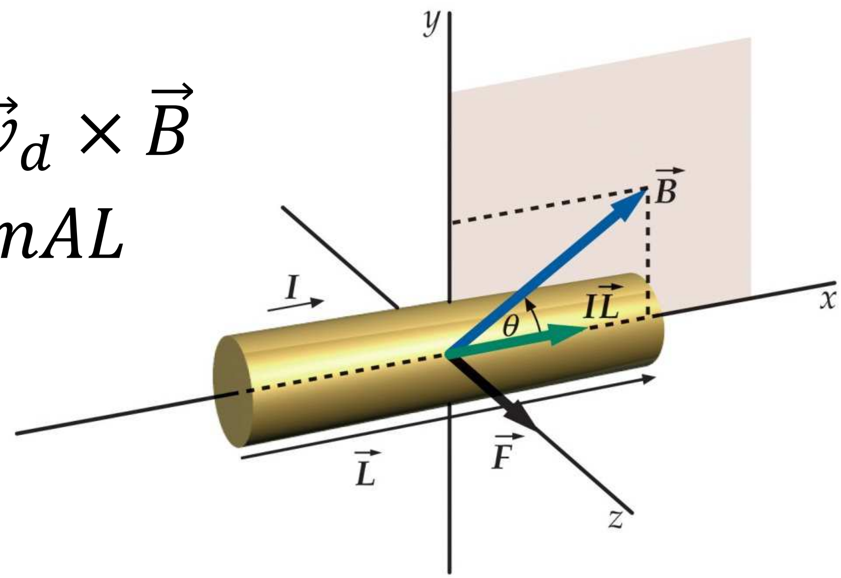
(a) $W_1 < W_2$ **(b)** $\boldsymbol{W_1 = W_2}$ **(c)** $W_1 > W_2$

Force on a Straight Wire



Force on one charge: $\vec{F} = e\,\vec{v}_d \times \vec{B}$

Total force: $\sum \vec{F} = e\left(\vec{v}_d \times \vec{B}\right) nAL$



Current is $I = nqv_dA$

So the force can be written

$$\vec{F} = I\,\vec{L} \times \vec{B}$$

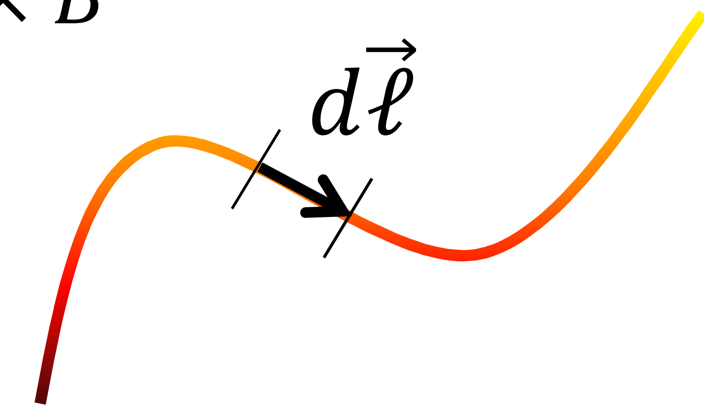
Force on a Wire Segment

- Think of an arbitrary wire as being composed as many very short, straight pieces:

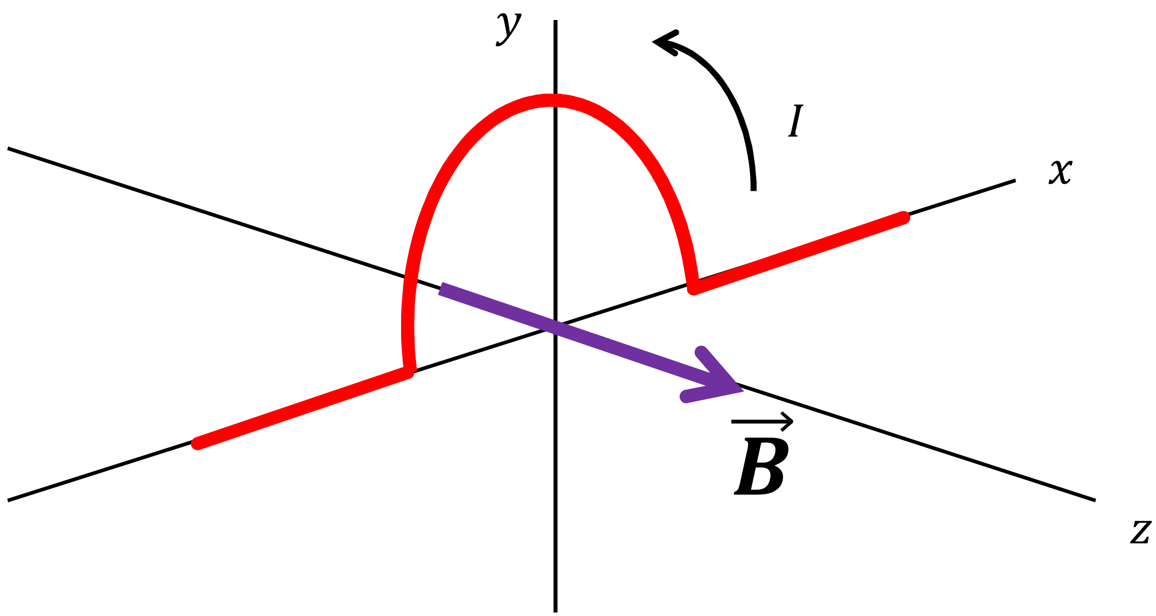
$$d\vec{F} = I\, d\vec{\ell} \times \vec{B}$$

- Integrate over the wire:

$$\vec{F} = I \int d\vec{\ell} \times \vec{B}$$



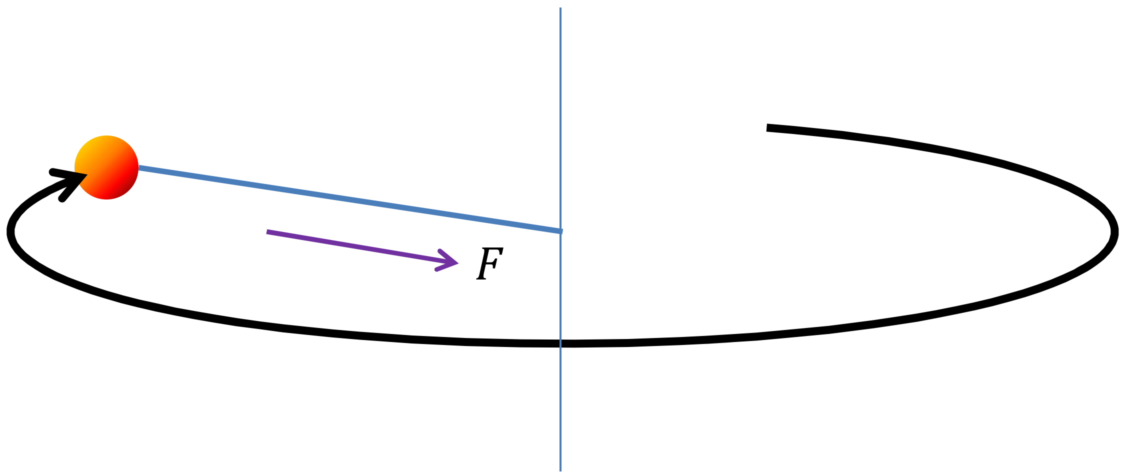
Force on a Bent Wire



$$d\vec{F} = I\, d\vec{\ell} \times \vec{B}$$

Mechanics Problem

- What is the tension in a string used to keep a mass in uniform circular motion?



Acceleration: $a = v^2/r$

Force: $F = ma = m\,v^2/r$

Motion in a Uniform Magnetic Field

- For a point charge, $\vec{F} = q\,\vec{v} \times \vec{B}$
- Force is perpendicular to $\vec{v}$ and $\vec{B}$
- Magnitude is $F = qvB$
- This provides the centripetal acceleration:

$$\frac{mv^2}{r} = qvB$$

- Radius of circular motion:

$$r = \frac{mv}{qB} = \frac{p}{qB}$$

Circular Motion

- Radius of curvature:

$$r = \frac{mv}{qB} = \frac{p}{qB}$$

- Cyclotron period:

$$T = \frac{2\pi r}{v} = 2\pi \frac{m}{qB}$$

- Cyclotron frequency:

$$f = \frac{1}{T} = \frac{qB}{2\pi m}$$

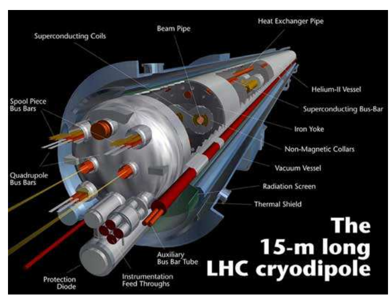
- Angular frequency:

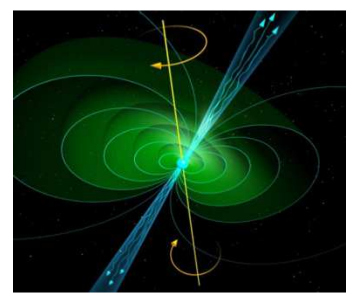
$$\omega = 2\pi f = \frac{qB}{m}$$

All of these depend on the "charge-to-mass" ratio, q/m.

Magnetic Field Strength

Source	Typical B field (Tesla)
Interstellar magnetic field	10^{-9}
Earth’s magnetic field	5×10^{-5}
Fridge magnet	5×10^{-3}
Electromagnet	10^{-2}
Rare earth magnet	1
Magnetic Resonance Imaging (MRI) machine	2
Superconducting magnets	10
Neutron star	10^{6}

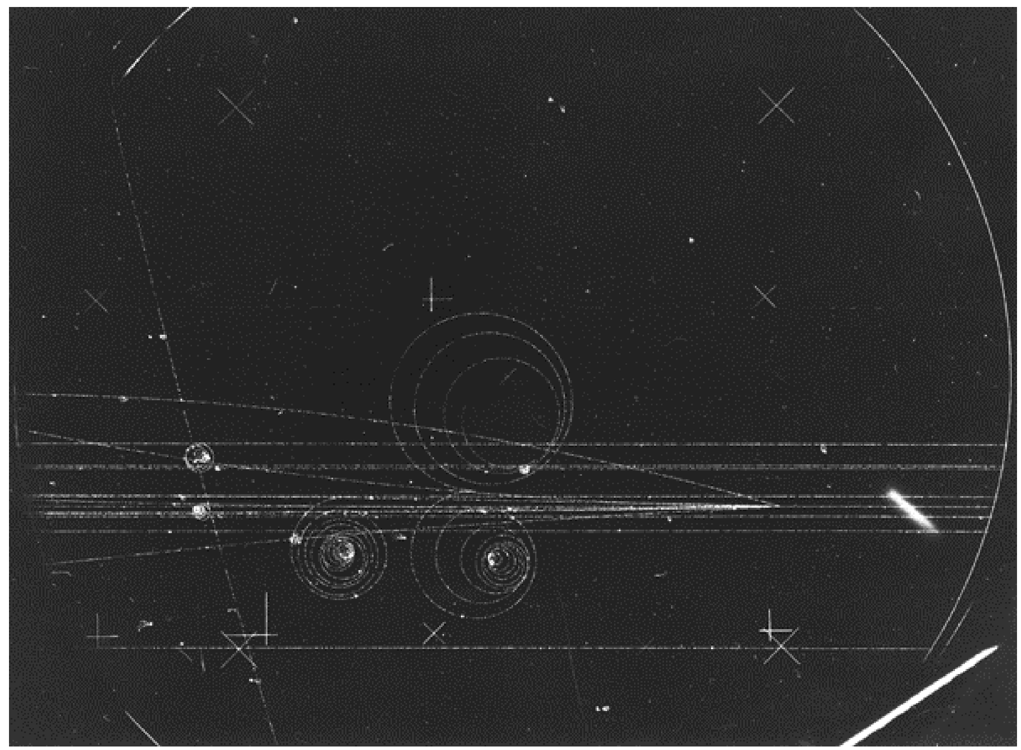




Applications

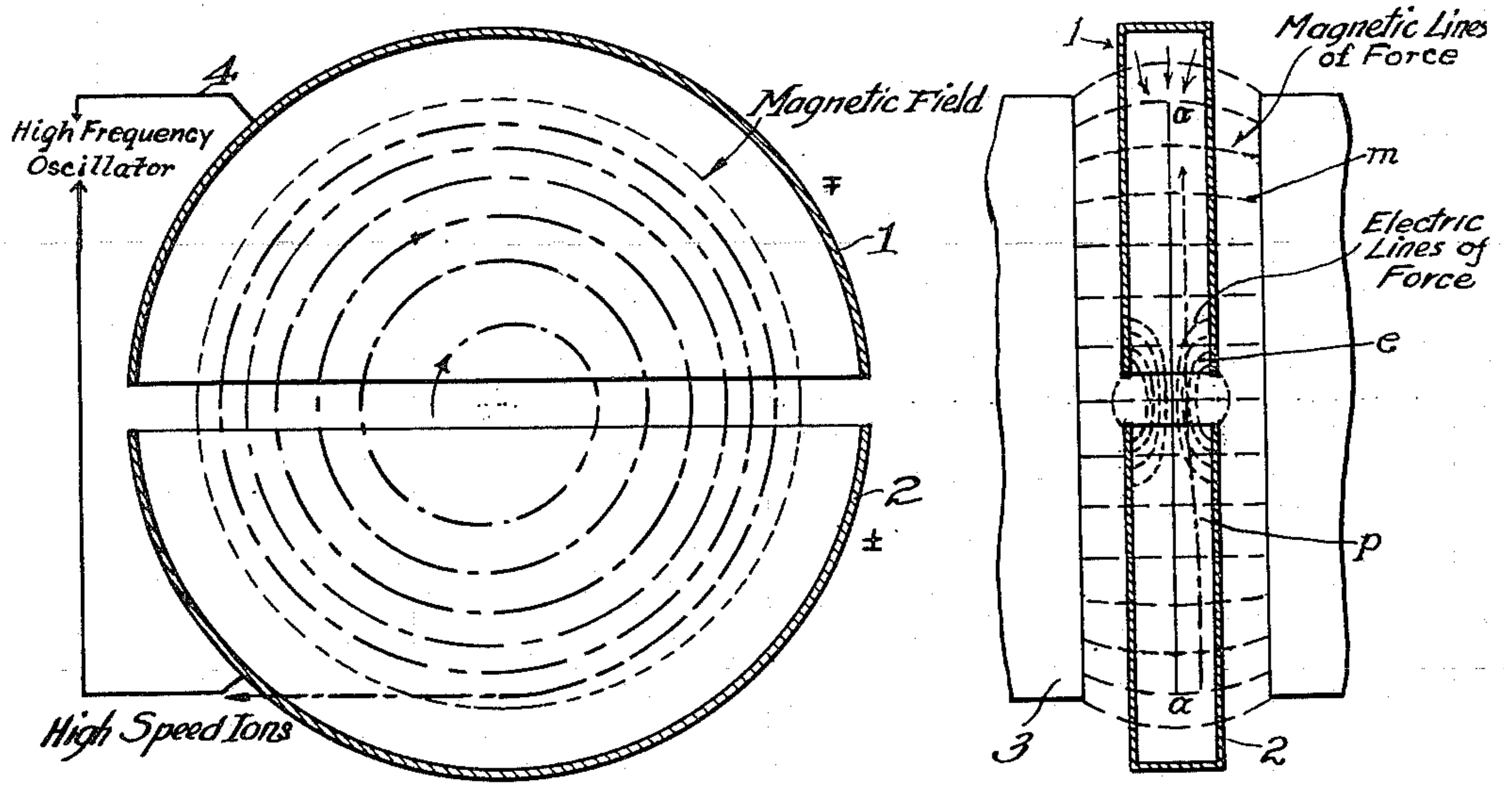
Bending charged particles in magnetic fields has many applications in nuclear and particle physics.

- Measuring momentum: $p = q\,B\,r$

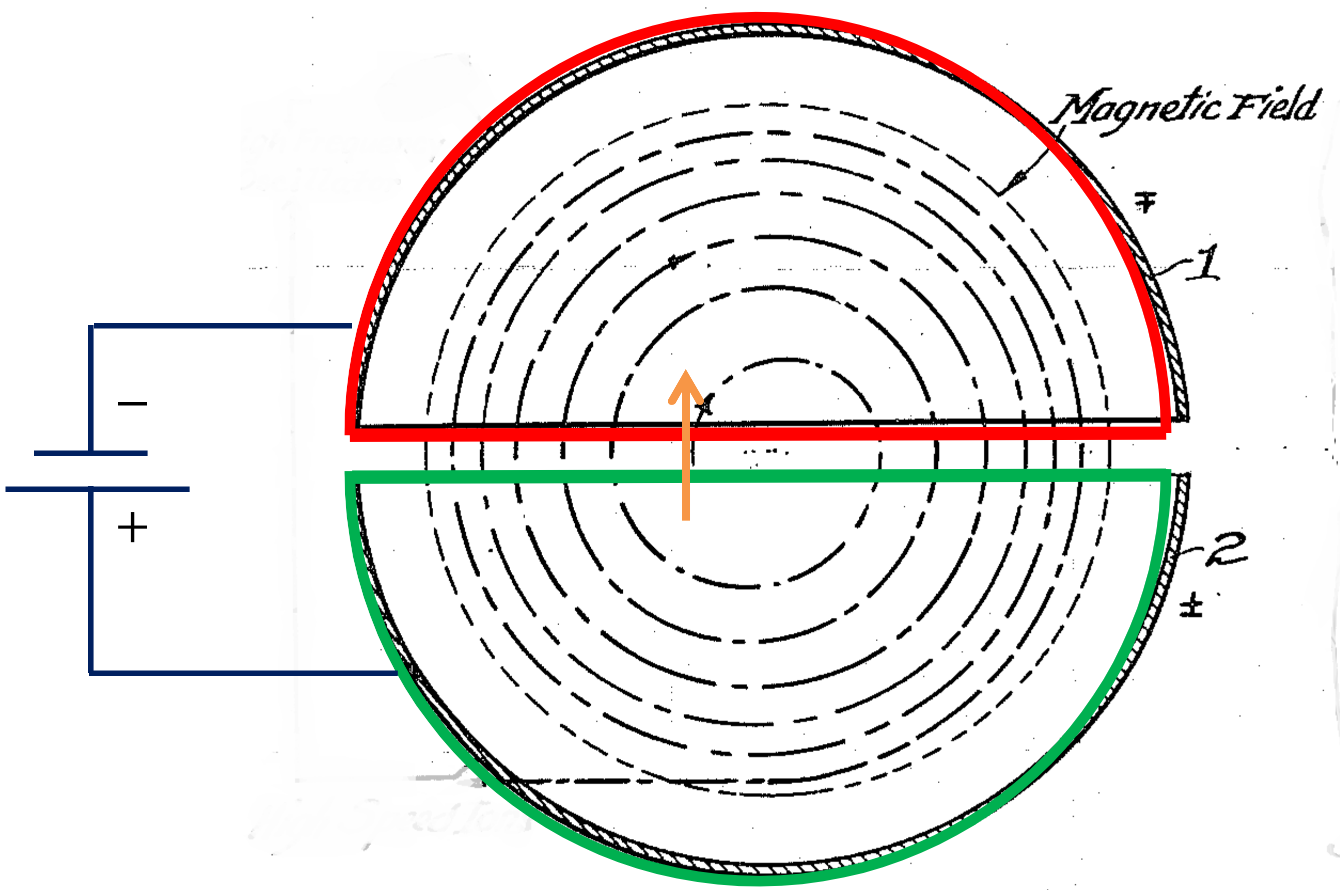


Applications

Cyclotrons are relatively small particle accelerators used for cancer treatment, radioactive isotope production for cancer therapy and medical research



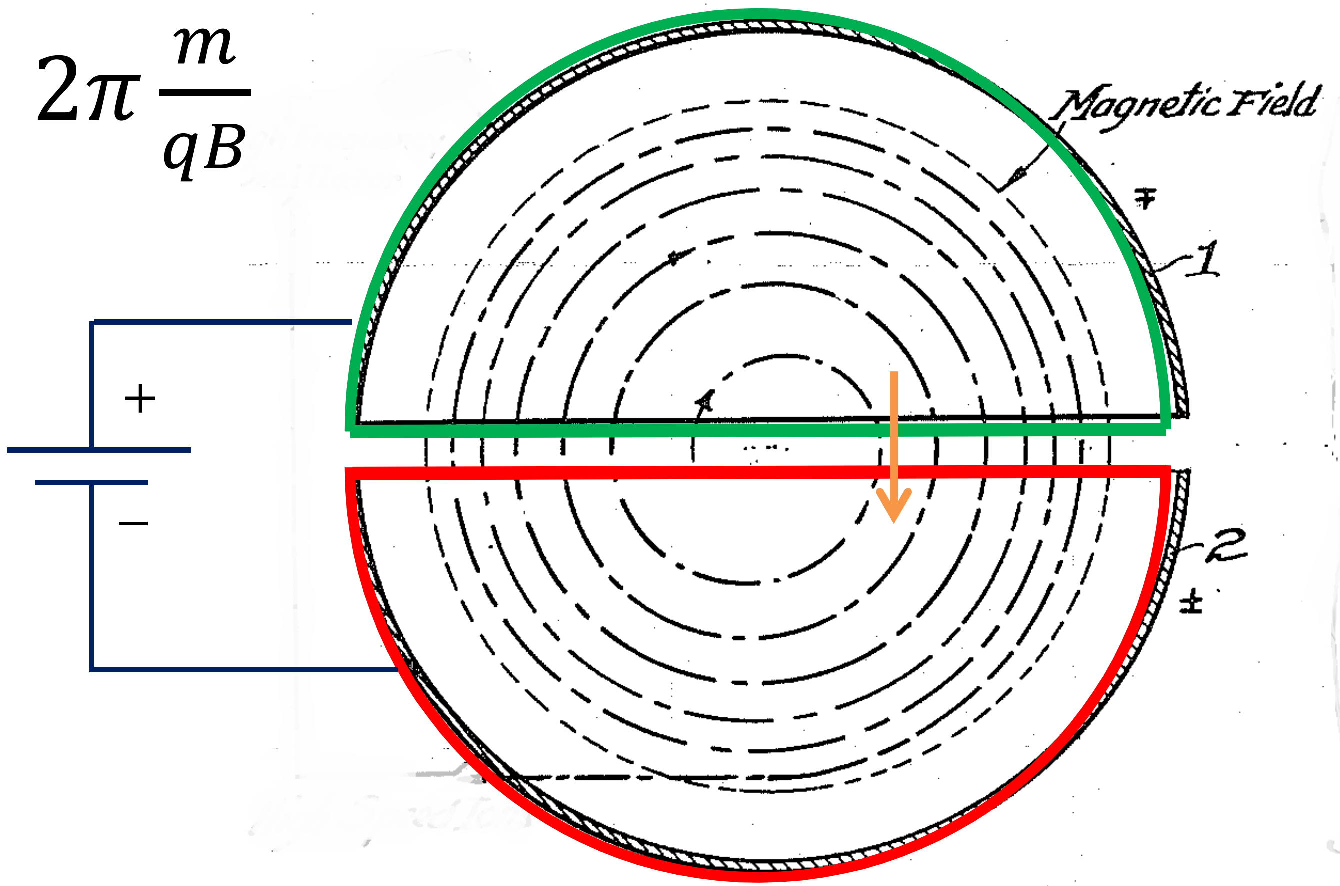
Applications



Applications

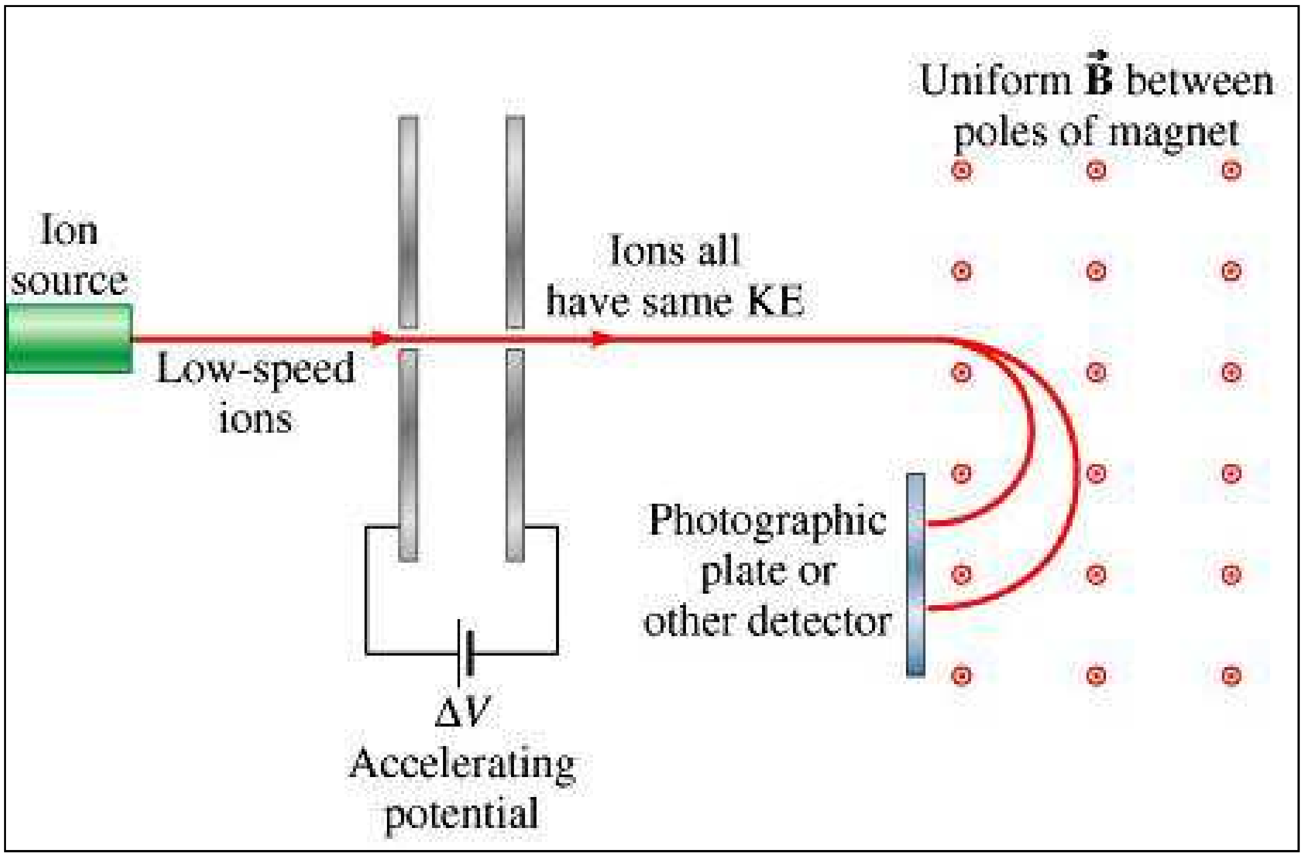
- Period is does not depend on velocity:

$$T = 2\pi \frac{m}{qB}$$



Applications

- Mass spectrometer: measures q/m



Example

- Separating ^{6}Li and ^{7}Li ➔ $q = +3e$
- Accelerate across a 10 MV potential:

$$T = \frac{1}{2}mv^2 = q\,V$$

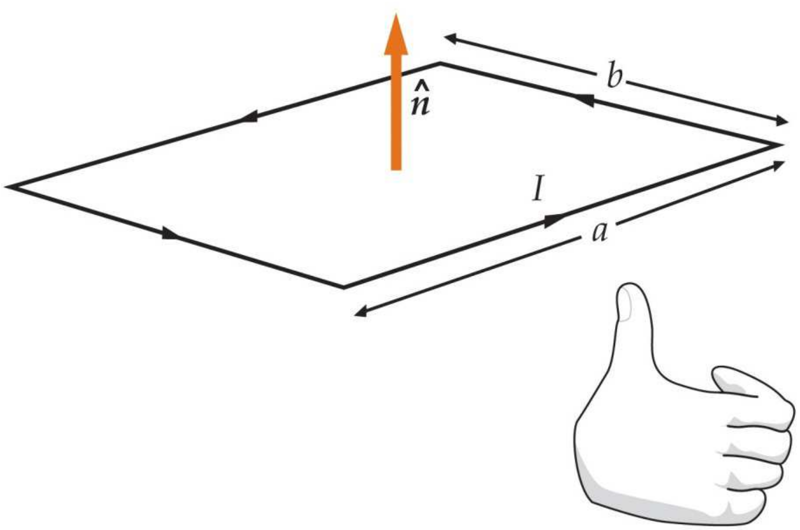
$$v = \sqrt{\frac{2qV}{m}}$$

- Bend in a 1 Tesla magnetic field:

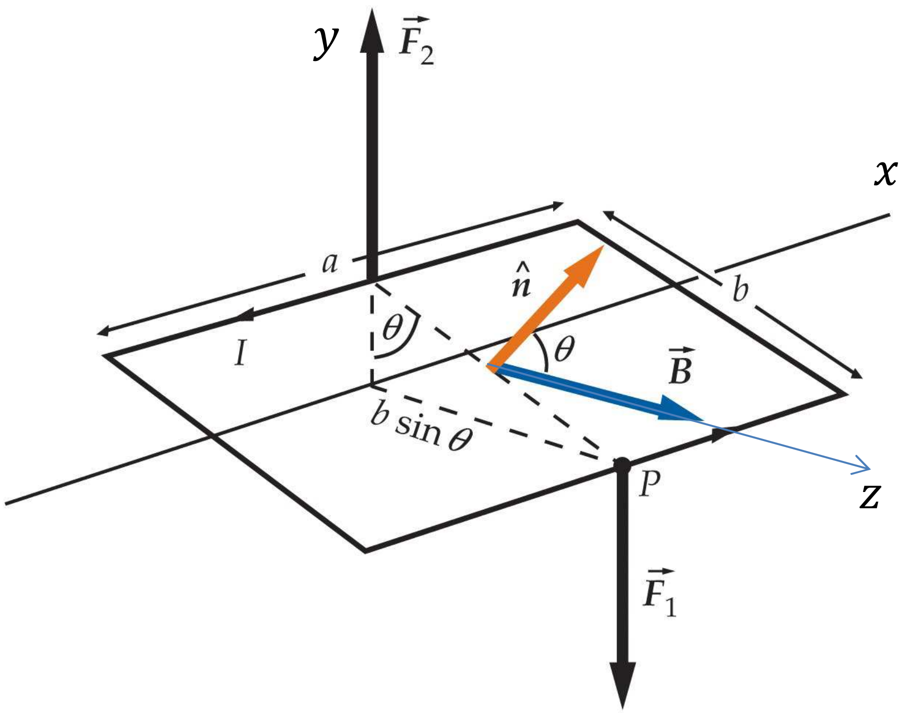
$$r = \frac{mv}{qB} = \frac{\sqrt{2V\ m/q}}{B}$$

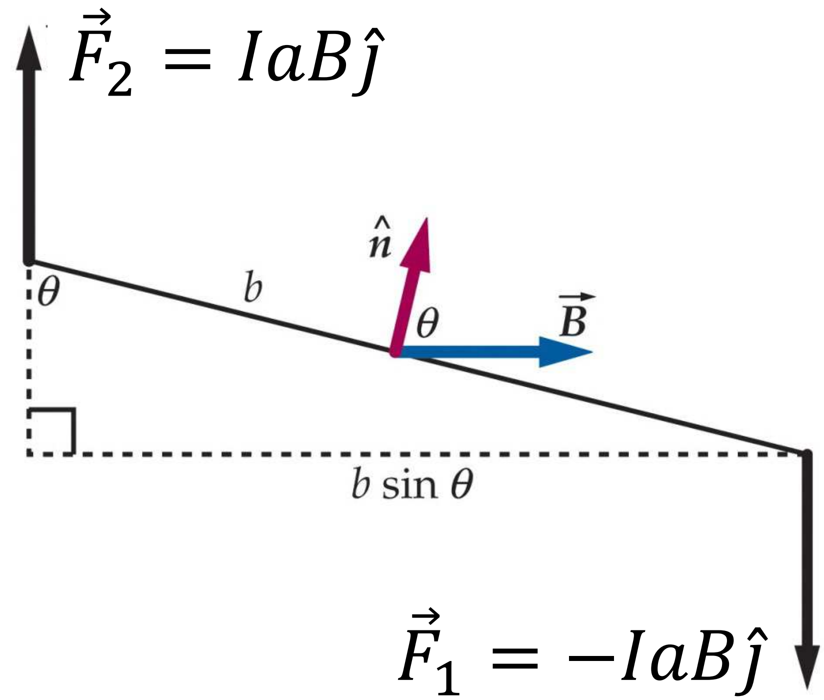
Torque on a Current Loop

- Consider a rectangular loop of wire carrying current I in a magnetic field.
- The orientation of the loop is given by the unit vector $\hat{n}$ perpendicular to the plane of the loop.



Torque on a Current Loop





- Magnitude of torque is $\tau = IabB\sin\theta$
- Direction is perpendicular to $\vec{B}$ and $\hat{n}$:

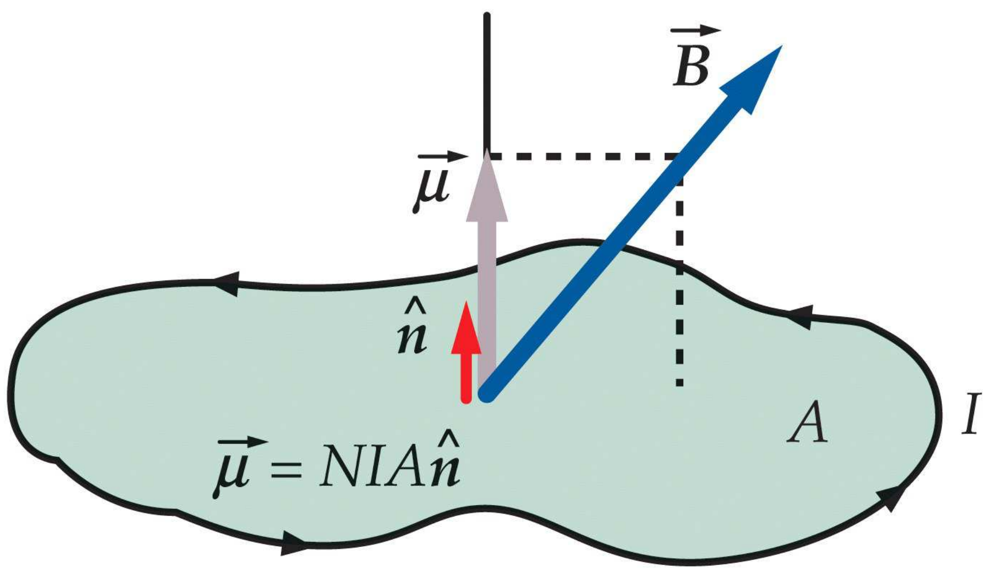
$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

$$\vec{\mu} = Iab\hat{n}$$

Torque on a Current Loop

- In general, the torque does not depend on the shape, just the area.
- With N turns of wire in the loop, multiply by N.

$$\vec{\mu} = NIA\,\hat{n}$$



$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

Potential Energy

- Work done by the magnetic field:

$$dW = -\tau\, d\theta$$

- Loss of potential energy:

$$dU = -dW = \mu B \sin\theta\, d\theta$$

- Total change in potential energy:

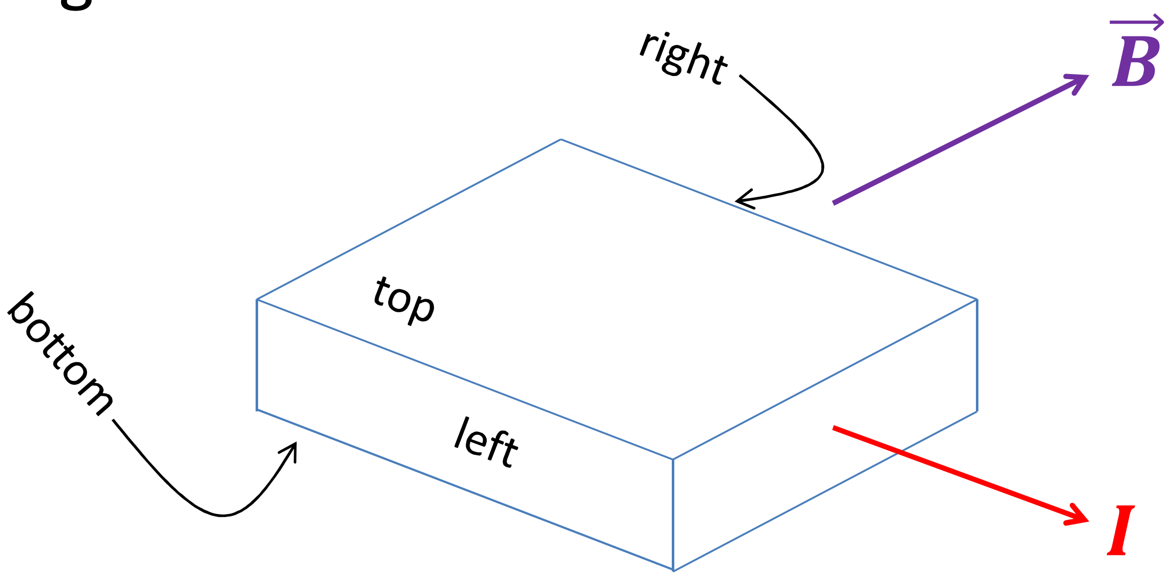
$$\Delta U = \int_{\theta_0}^{\theta} \mu B \sin\theta\, d\theta = -\mu B \cos\theta$$

- Potential energy of a dipole in a magnetic field:

$$U = -\vec{\mu} \cdot \vec{B}$$

Question

- A rectangular conductor carries current I in a magnetic field as shown:



- If the charge carriers are electrons, on which surface will a negative charge accumulate?

(a) Top (b) Bottom (c) Left (d) Right