PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

# Physics 24100

# Electricity & Optics

## Lecture 11 – Chapter 25 sec. 4-5

Fall 2012 Semester

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# Thursday's Clicker Question

- The resistance across the human body is approximately $2\ k\Omega$
- If it takes only $50\ mA$ of current to kill a human, what voltage could be lethal?

(a) 0.1 Volts

(b) 1 Volt

(c) 10 Volts

(d) 100 Volts

(e) 1000 Volts

Ohm's law: $V = I\,R = (50\ mA) \times (2\ k\Omega) = 100\ V$

# Kirchhoff's Rules

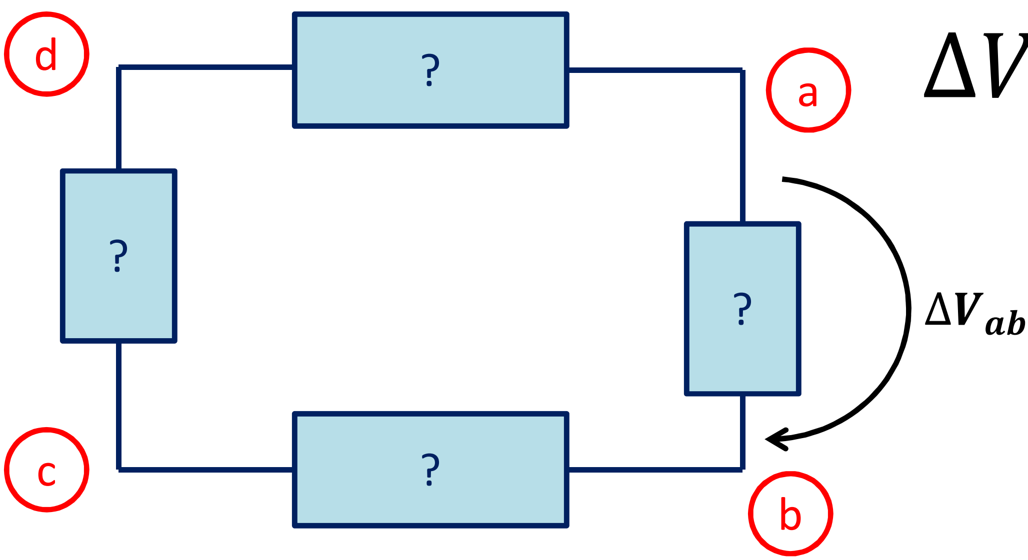
- General problem:
  - Calculate the currents that flow in an electric circuit composed of voltage sources and resistors connected by wires.
  - Recall that work done to move a charge $q$ from point $a$ to point $b$ is $W = -q \int_a^b \vec{E} \cdot d\vec{\ell}$
  - If $a$ and $b$ are the same point then $W = 0$

$$\oint \vec{\mathrm{E}} \cdot \mathrm{d}\vec{\ell} = 0$$

# Kirchhoff's Loop Rule

$$\oint \vec{\mathrm{E}} \cdot \mathrm{d}\vec{\ell} = 0$$

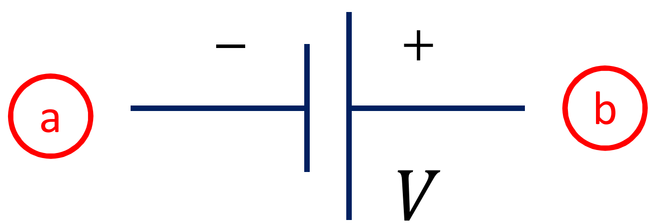
"The sum of the potential differences around a closed loop is zero."




$$\Delta V_{ab} + \Delta V_{bc} + \Delta V_{cd} + \Delta V_{da} = 0$$

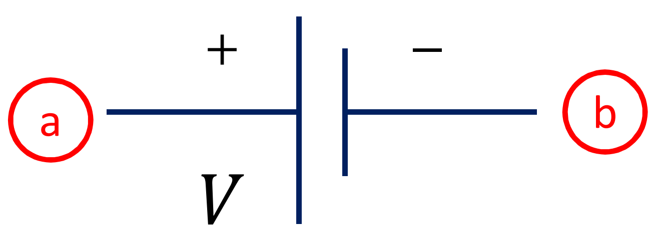
# Circuit Elements

- Voltage sources (like batteries):




$$V_b = V_a + V$$

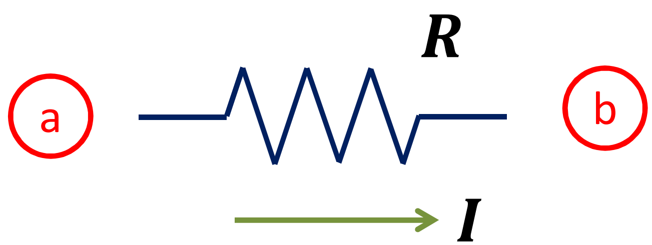
- Make sure you get the sign right!




$$V_b = V_a - V$$

# Circuit Elements

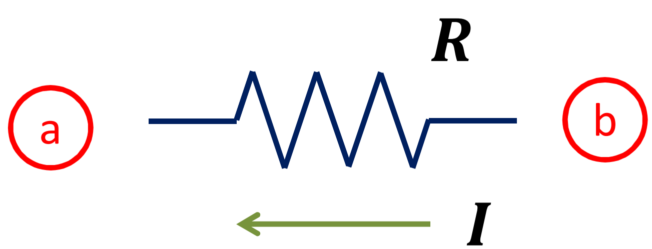
- Resistors:




The charges lose energy as they are pushed through the resistor.

$$V_b = V_a - IR$$

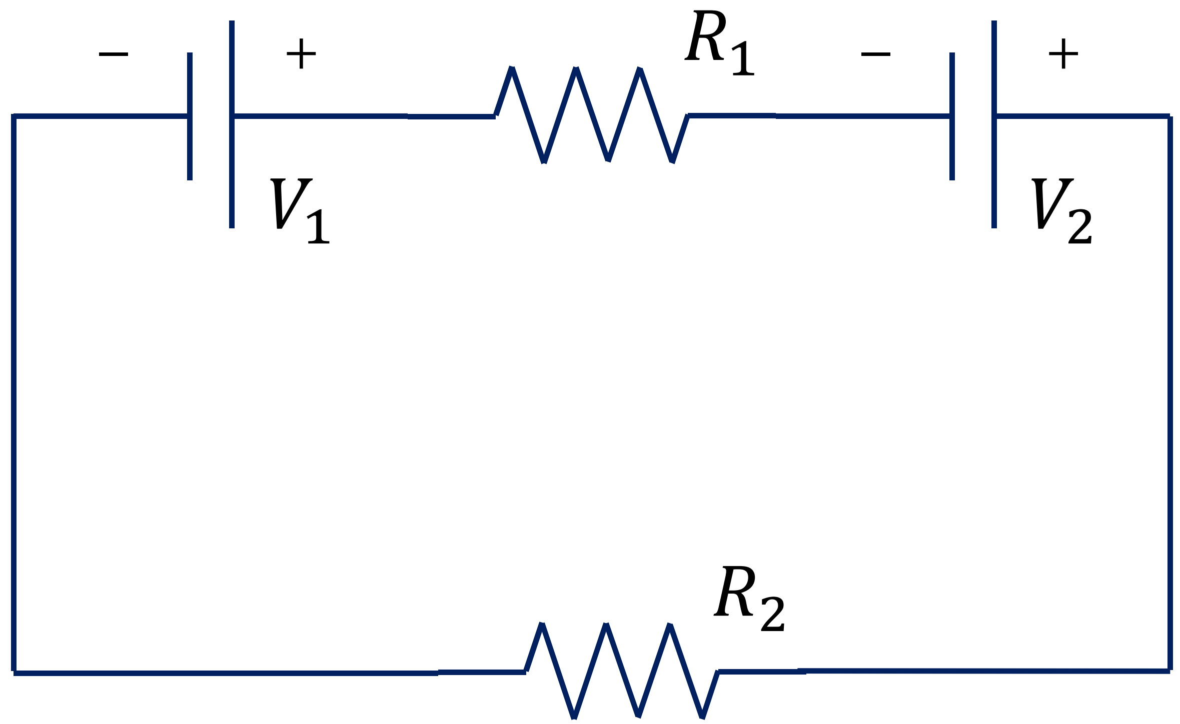
- Make sure you get the sign right!




$$V_b = V_a + IR$$

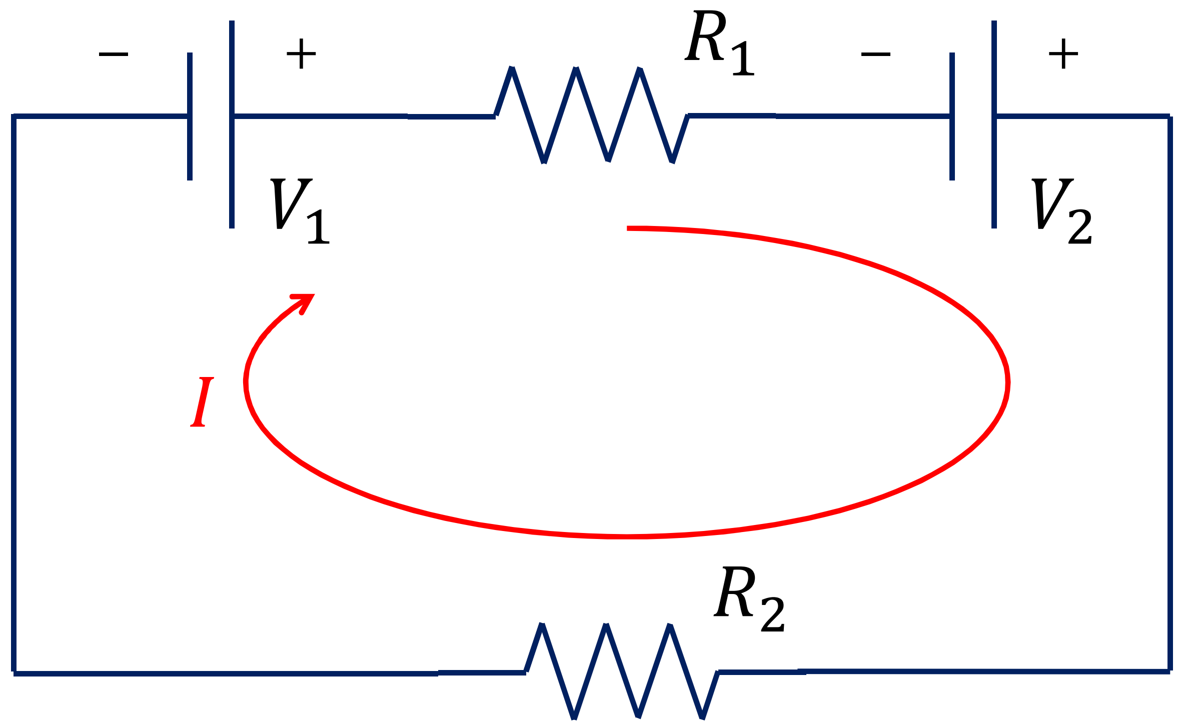
# Circuit Analysis

- Find the current in the following circuit:



# Circuit Analysis

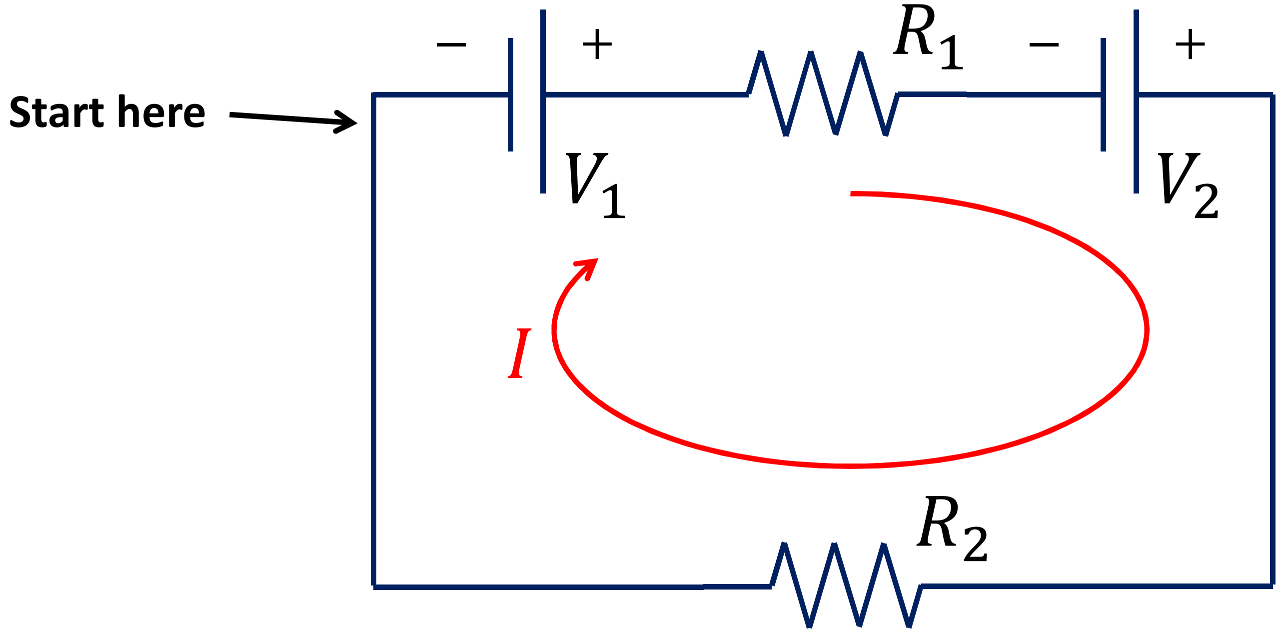
- Step 1: Draw a loop to represent the current.




Which direction? It doesn't matter, but let's ALWAYS pick clockwise to avoid confusion.

# Circuit Analysis

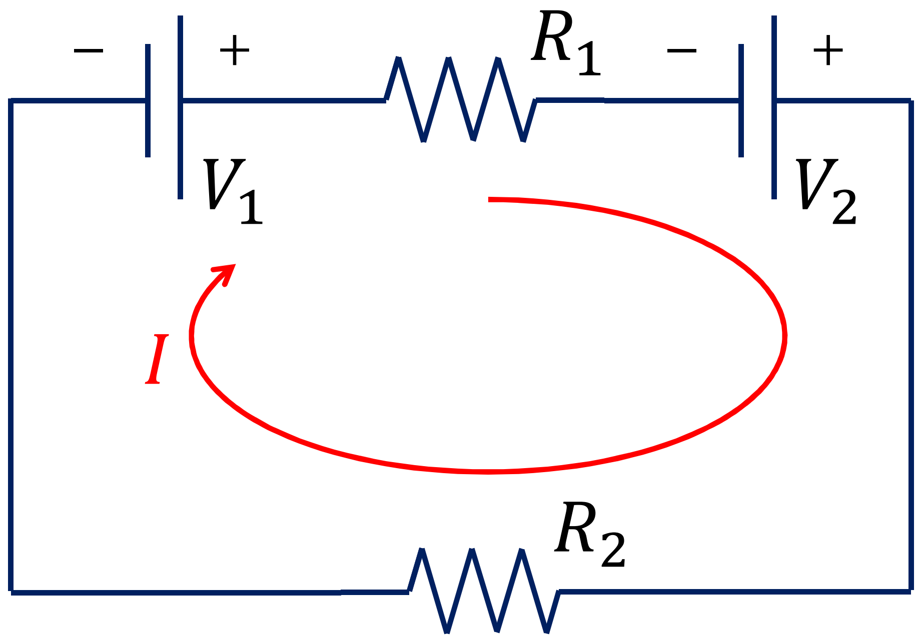
- Step 2: Apply Kirchhoff's Loop Rule…




$$V_1$$

# Circuit Analysis

- Step 3: Solve for $I$…

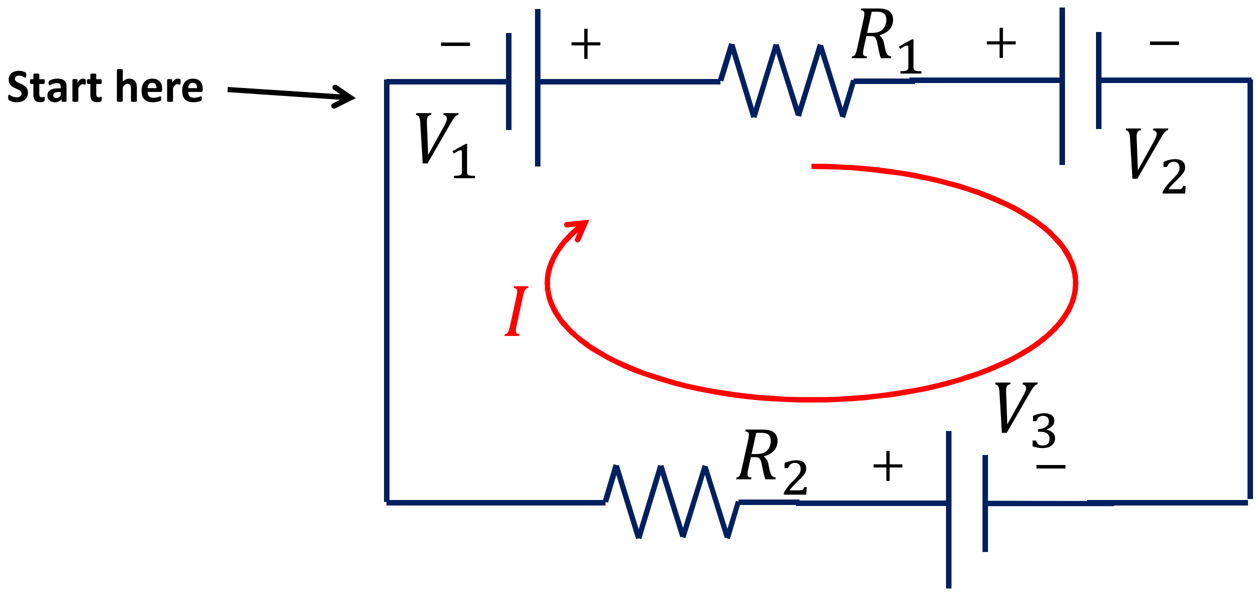



$$I = \frac{V_1 + V_2}{R_1 + R_2}$$

What if $I$ is negative? Then it means the current flows in the opposite direction.

# Question

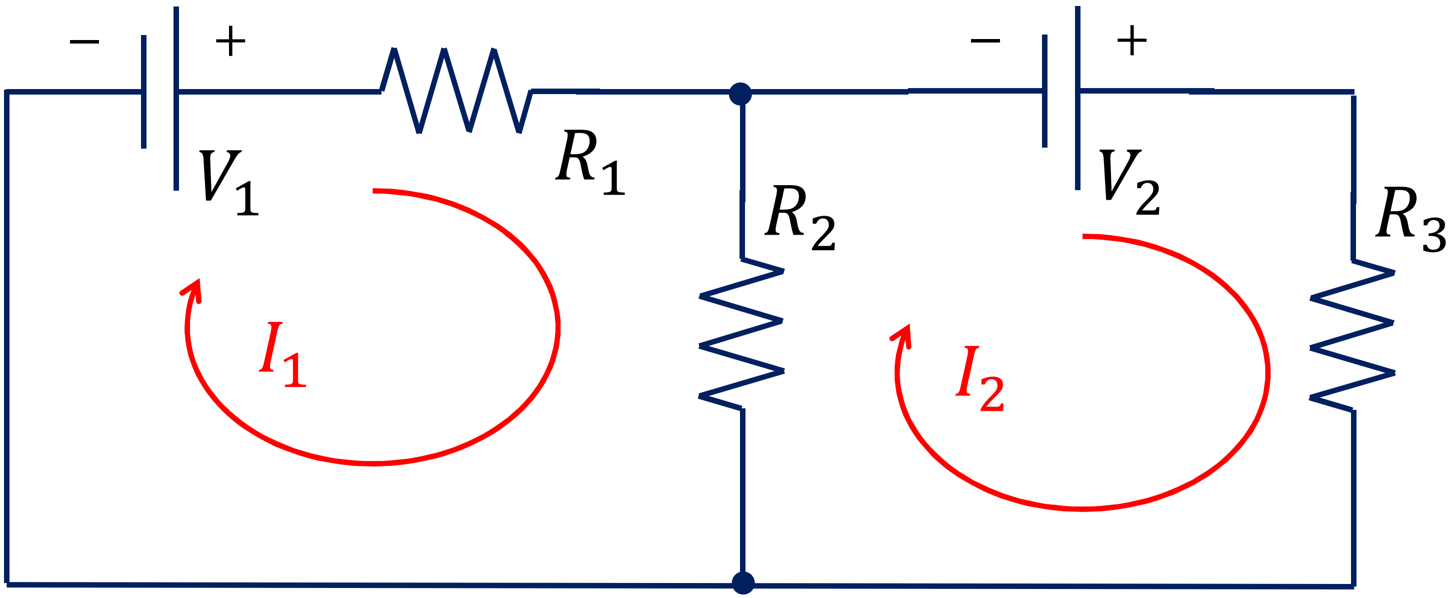
- Which formula shows the correct application of Kirchhoff's Loop rule to the following circuit:




(a) $V_1 - I\,R_1 + V_2 + V_3 - I\,R_2 = 0$
(b) $-V_1 - I\,R_1 - V_2 + V_3 - I\,R_2 = 0$
(c) $V_1 - I\,R_1 - V_2 + V_3 - I\,R_2 = 0$
(d) $V_1 + I\,R_1 + V_2 + V_3 + I\,R_2 = 0$

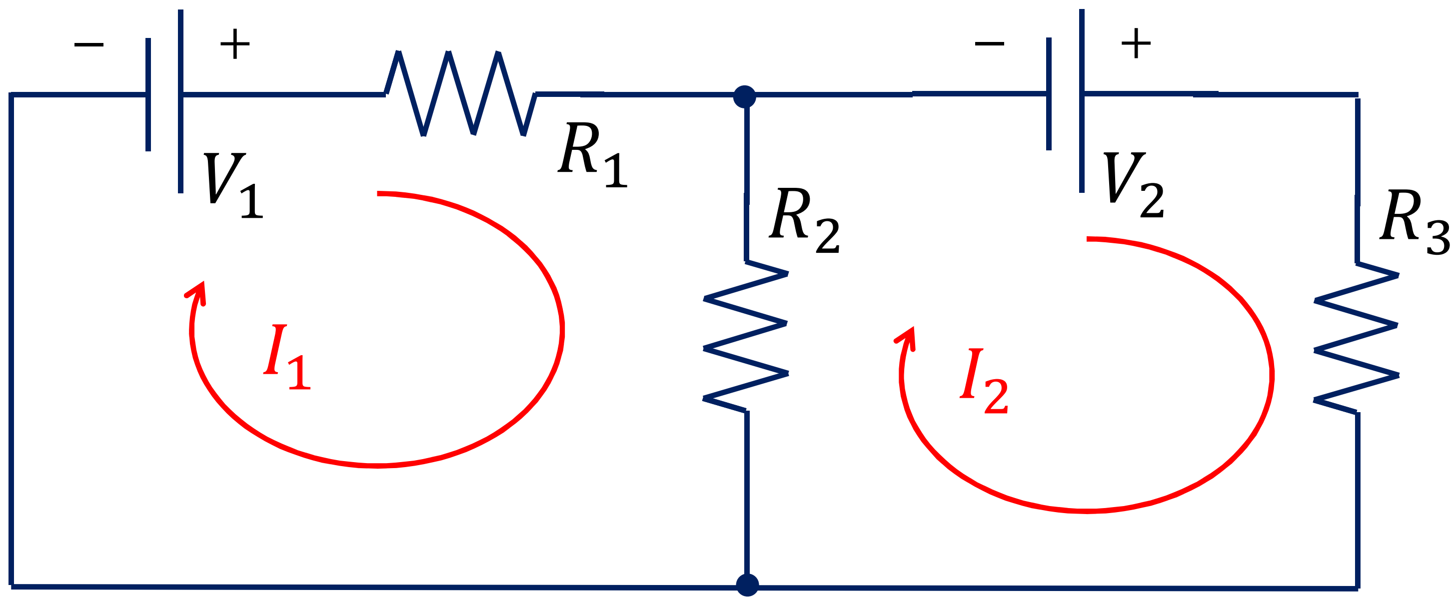
# Two Loops

- Step 1: Assign currents to each loop



# Two Loops

- Step 2: Apply Kirchhoff's Loop rule




$$V_1 - I_1R_1 - (I_1 - I_2)R_2 = 0$$
$$V_2 - I_2R_3 - (I_2 - I_1)R_2 = 0$$

# Two Loops

$$V_1 - I_1 R_1 - (I_1 - I_2) R_2 = 0$$
$$V_2 - I_2 R_3 - (I_2 - I_1) R_2 = 0$$

This is a system of linear equations… write them as a matrix equation:

$$\begin{pmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

This should *always* be a symmetric matrix.

# Kramer's Rule

- This is the "formula" that gives you the solution to a system of linear equations:

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

$$x = \frac{\begin{vmatrix} c_1 & b_1 \\ c_2 & b_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1}$$

$$y = \frac{\begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}}{\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}} = \frac{a_1 c_2 - a_2 c_1}{a_1 b_2 - a_2 b_1}$$

# Two Loops

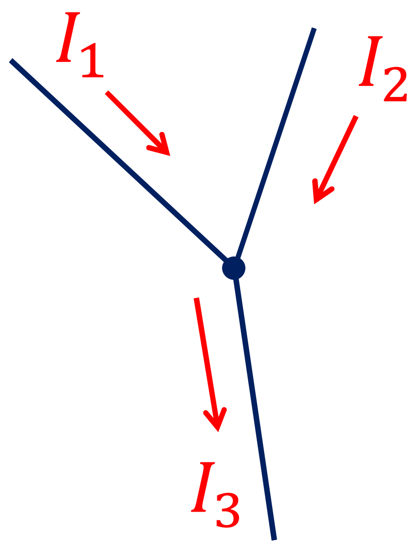
$$\begin{pmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

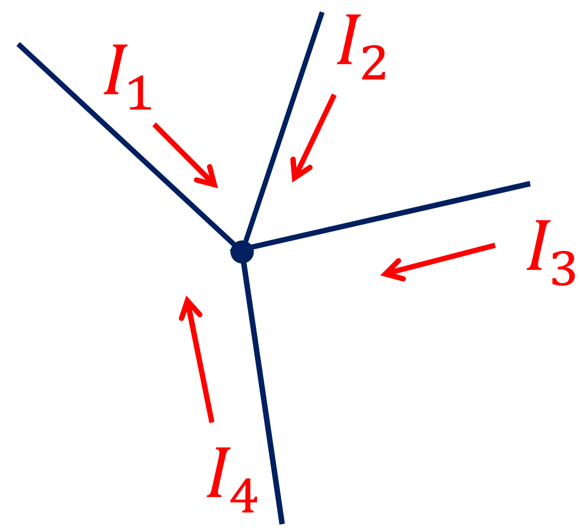
$$I_1 = \frac{\begin{vmatrix} V_1 & -R_2 \\ V_2 & R_2 + R_3 \end{vmatrix}}{\begin{vmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 \end{vmatrix}} = \frac{V_1(R_2 + R_3) + V_2 R_2}{(R_1 + R_2)(R_2 + R_3) - (R_2)^2}$$

$$I_2 = \frac{\begin{vmatrix} R_1 + R_2 & V_1 \\ -R_2 & V_2 \end{vmatrix}}{\begin{vmatrix} R_1 + R_2 & -R_2 \\ -R_2 & R_2 + R_3 \end{vmatrix}} = \frac{V_2(R_1 + R_2) + V_1 R_2}{(R_1 + R_2)(R_2 + R_3) - (R_2)^2}$$

# Kirchhoff's Node Rule

- The sum of the currents entering a node must equal the sum of the currents leaving.



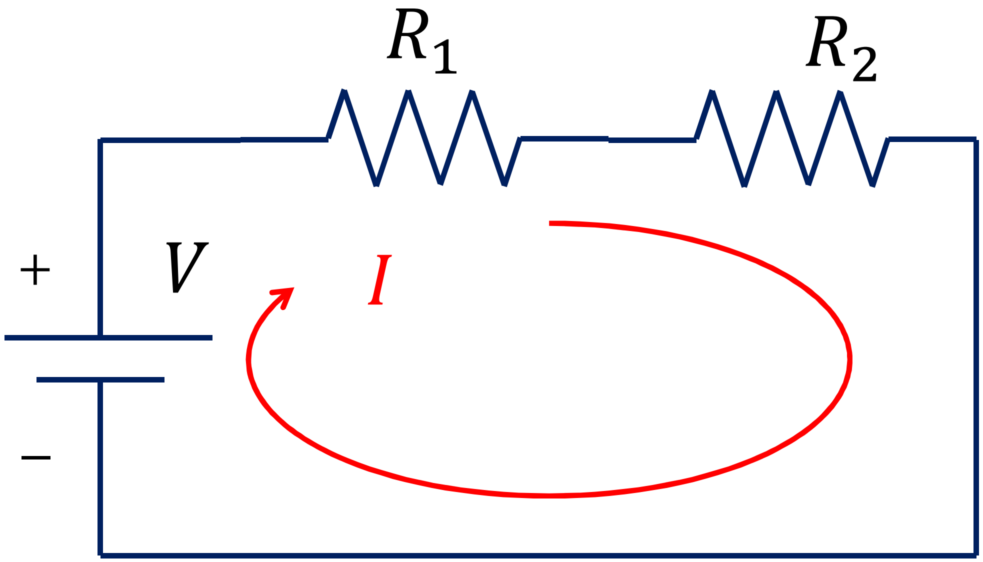

$$I_1 + I_2 = I_3$$

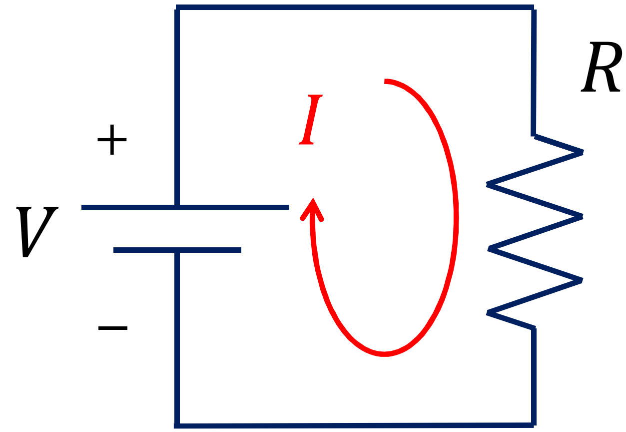
$$I_1 + I_2 + I_3 + I_4 = 0$$

(at least one of these must be negative)

# Equivalent Series Resistors

- What resistance $R$ would produce the same current as the series combination of $R_1$ and $R_2$?



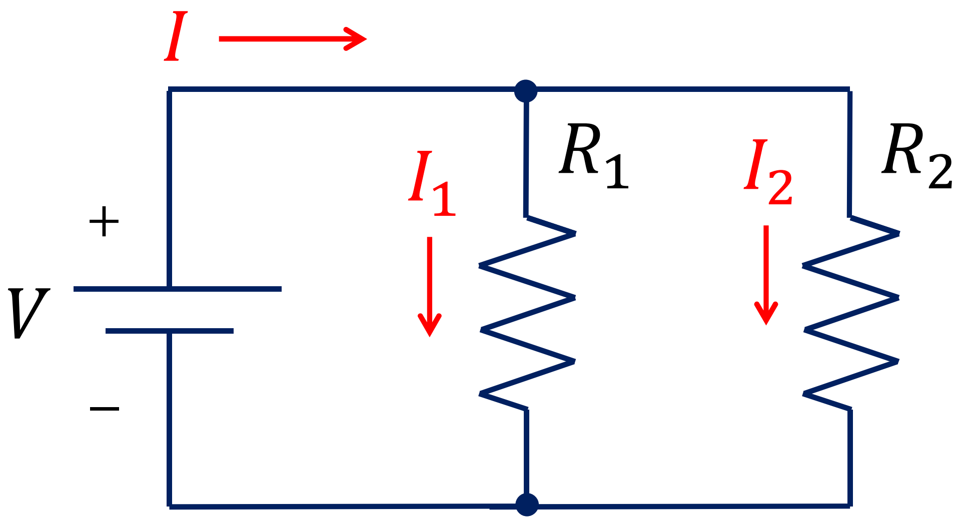

$$V - I\,R_1 - I\,R_2 = 0$$

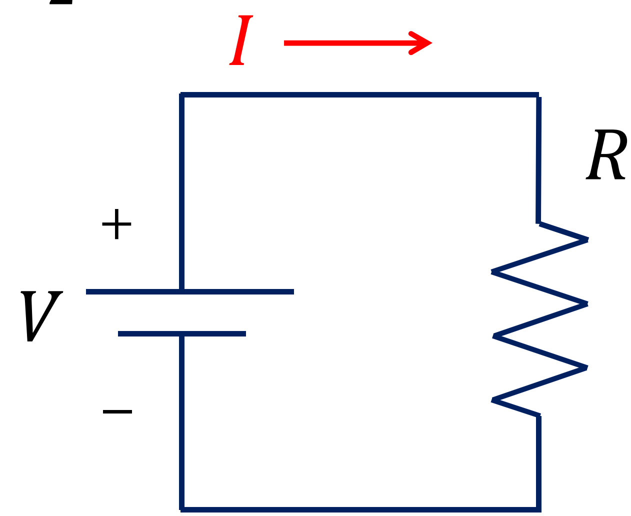
$$I = \frac{V}{R_1 + R_2}$$

$$\boldsymbol{R = R_1 + R_2}$$

# Equivalent Parallel Resistance

- What resistance $R$ would produce the same current as the parallel combination of $R_1$ and $R_2$?




The voltage across each resistor is equal.

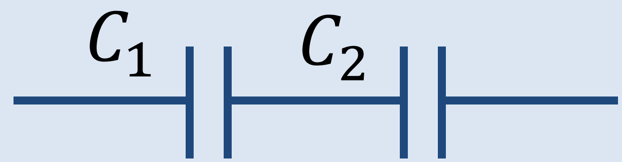
$$V = I_1 R_1$$
$$V = I_2 R_2$$
$$I = I_1 + I_2 = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$$

$$I = \frac{V}{R}$$

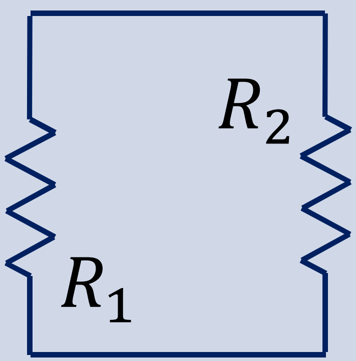
$$\boldsymbol{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}}$$

# Series and Parallel

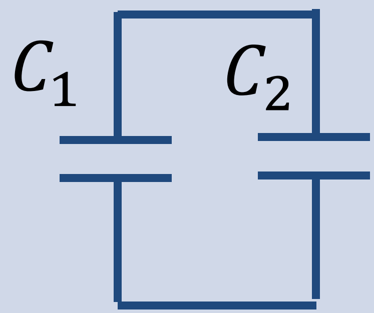

$$R = R_1 + R_2$$




$$C = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$$

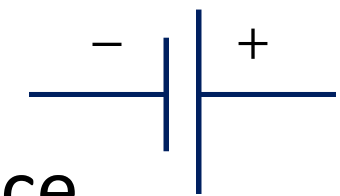



$$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

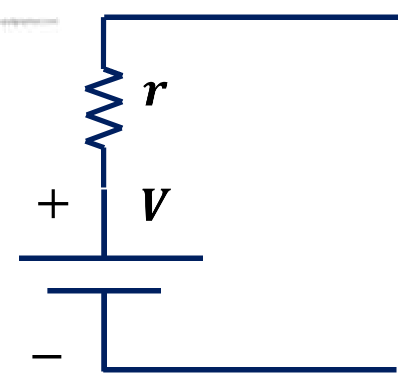



$$C = C_1 + C_2$$

# Internal Resistance

- The symbol 

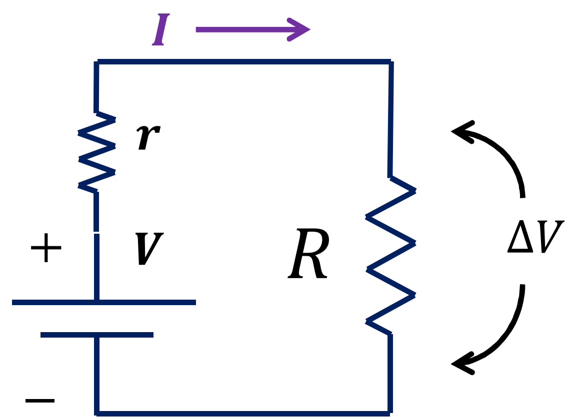
represents an *ideal* voltage source.
  - The voltage is the same no matter how much current flows through it.
- A real voltage source is equivalent to an ideal voltage source in series with a resistor.
  - The resistor is called the “internal resistance”
  - It is usually small.



# Internal Resistance

- The voltage across a load depends on the current.

$$I = \frac{V}{r + R}$$

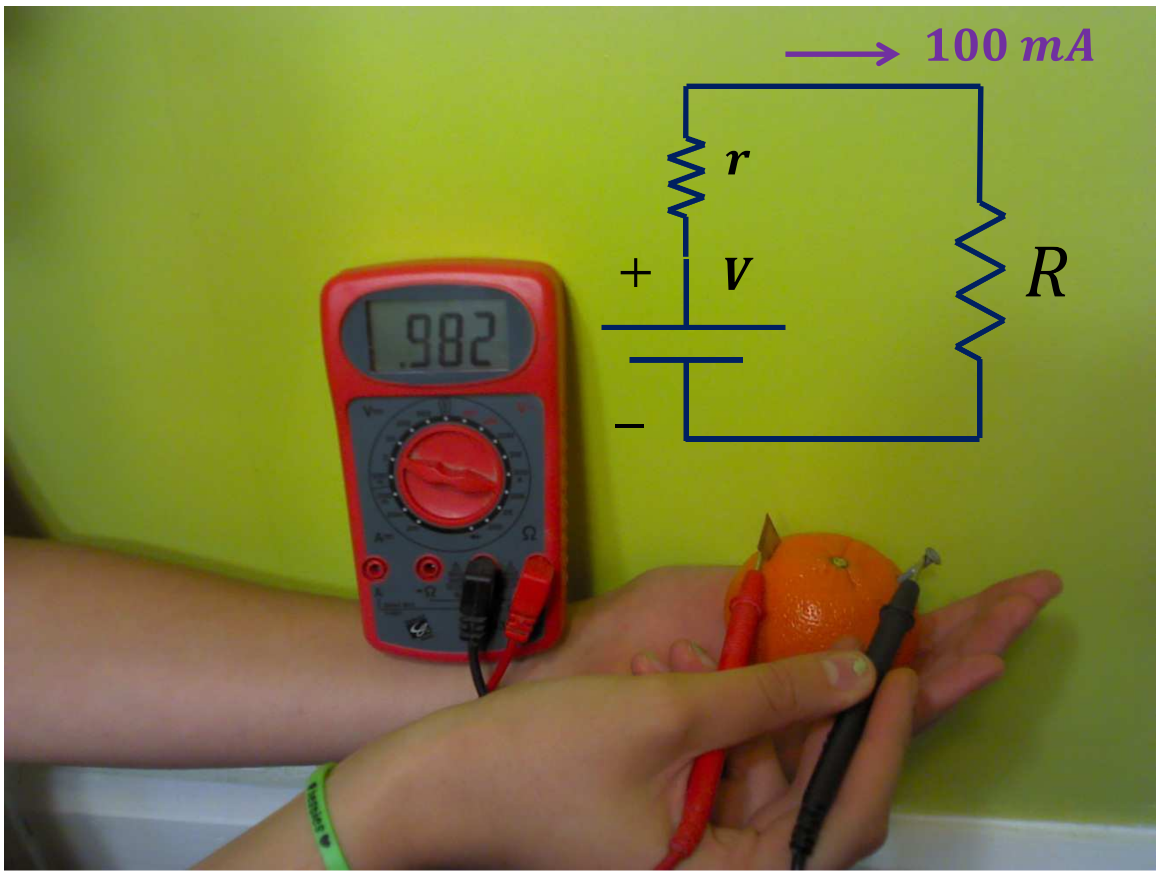



$$V - Ir - \Delta V = 0 \quad \text{so} \quad \Delta V = V - Ir$$

# Question

When there is no load, the orange provides 0.982 Volts.

What is the internal resistance of the orange if the voltage drops to 0.082 volts when the current is 100 mA?




(a) 9 Ω

(b) 90 Ω

(c) 0.9 Ω

(d) 10 Ω

(e) not enough information