Purdue University Department of Physics

Physics 24100

Electricity & Optics

Lecture 10 – Chapter 25 sec. 1-3

Fall 2012 Semester

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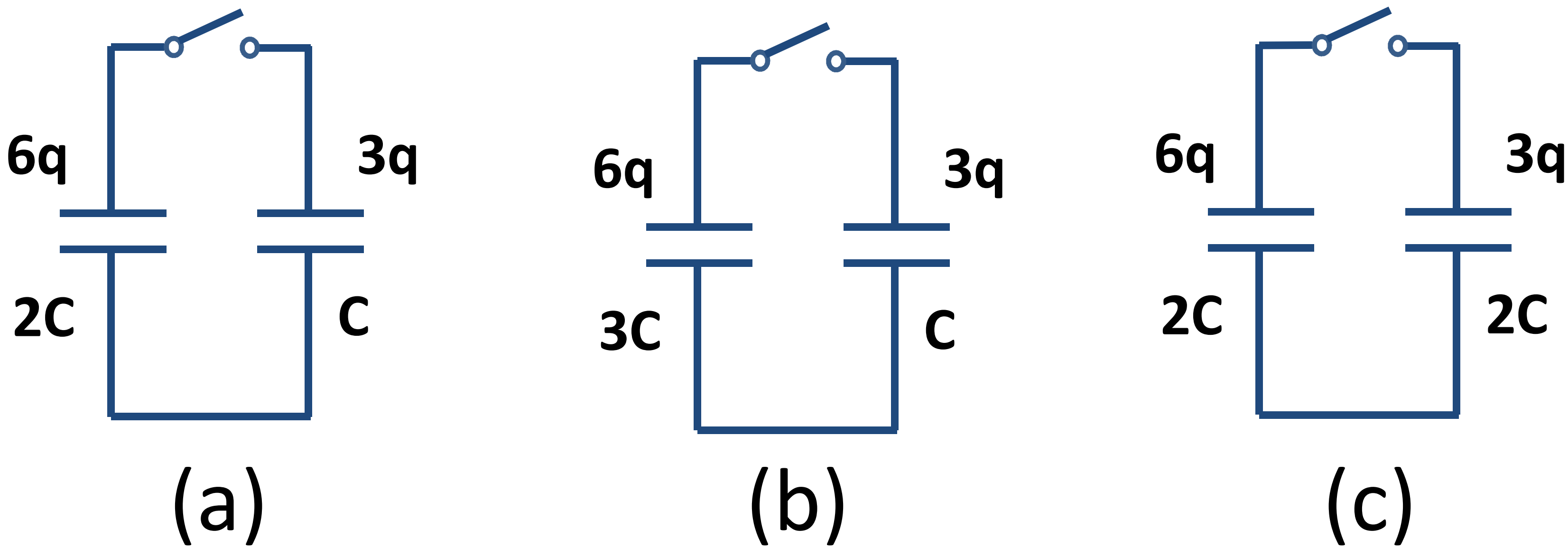
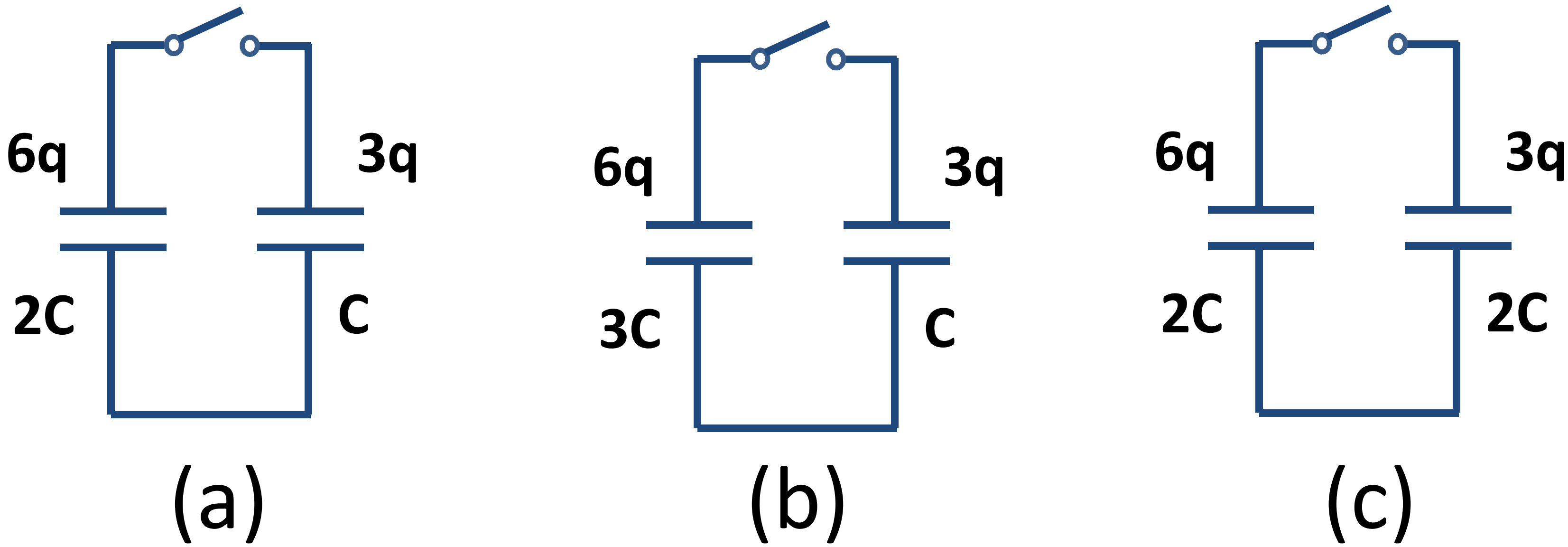
Tuesday's Question

- Three circuits, consisting of two capacitors and a switch, are initially charged as indicated.
- After the switches are closed, in which circuit will the charge on the **left** *increase*?



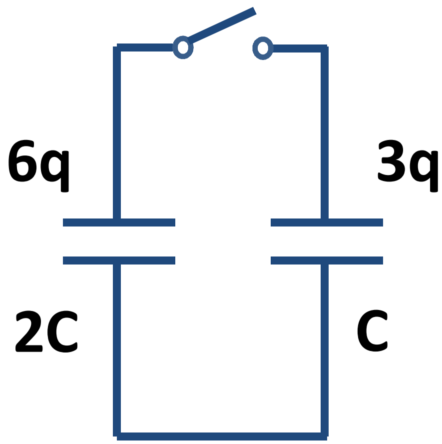
(d) None of them

Tuesday's Question



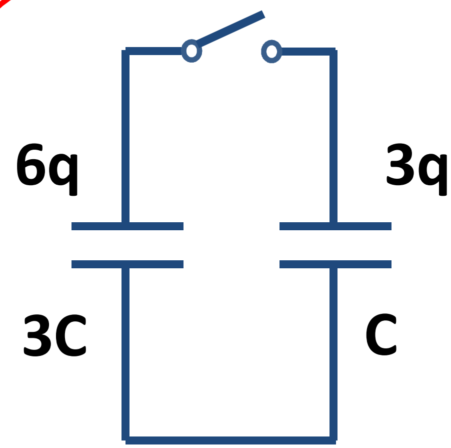
- Charge is conserved, $Q = 9q$
- Calculate equivalent capacitance, C_{equiv}
- Then calculate, $V = Q/C_{equiv}$
- Finally, calculate, $Q_{left} = C_{equiv}V$

Tuesday's Question



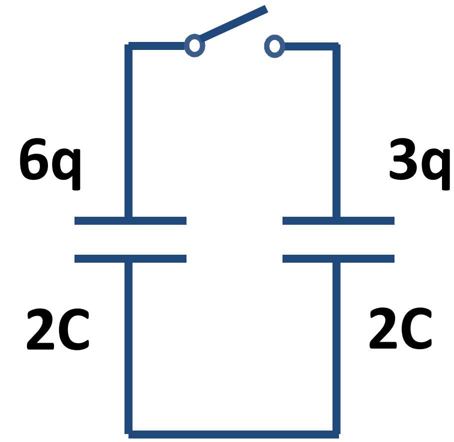
(a)

$$Q = 9q$$
$$C_{equiv} = 3C$$
$$V = \frac{Q}{C_{equiv}} = \frac{3q}{C}$$
$$Q_{left} = 2C \times \frac{3q}{C} = 6q$$



(b)

$$Q = 9q$$
$$C_{equiv} = 4C$$
$$V = \frac{Q}{C_{equiv}} = \frac{9q}{4C}$$
$$Q_{left} = 3C \times \frac{9q}{4C}$$
$$= \frac{27}{4}q > 6q$$



(c)

$$Q = 9q$$
$$C_{equiv} = 4C$$
$$V = \frac{Q}{C_{equiv}} = \frac{9q}{4C}$$
$$Q_{left} = 2C \times \frac{9q}{4C} = \frac{9}{2}q$$
$$< 6q$$

Mini-Review

- Lecture 1: $\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{Q_1 Q_2}{r^2}\hat{r}$
- Lecture 2: $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\hat{r}$ and $\vec{F} = q\vec{E}$
- Lecture 3: $\Delta\vec{E}(\vec{x}_p) = \frac{1}{4\pi\epsilon_0}\frac{\Delta Q(\vec{x}_s)}{r^2}\hat{r} \rightarrow \vec{E}(\vec{x}_p) = \frac{1}{4\pi\epsilon_0}\int \frac{\hat{r}}{r^2}dQ$
- Lecture 4: $\phi_{net} = \oint_S \hat{n}\cdot\vec{E}\,dA = \frac{Q_{inside}}{\epsilon_0}$
- Lecture 5: $\vec{E}$ near conductors and insulators
- Lecture 6: $\Delta V = -\int_a^b \vec{E}\cdot d\vec{\ell}$ and $\vec{E} = -\vec{\nabla}V$

$$V(r) = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}$$

Mini-Review

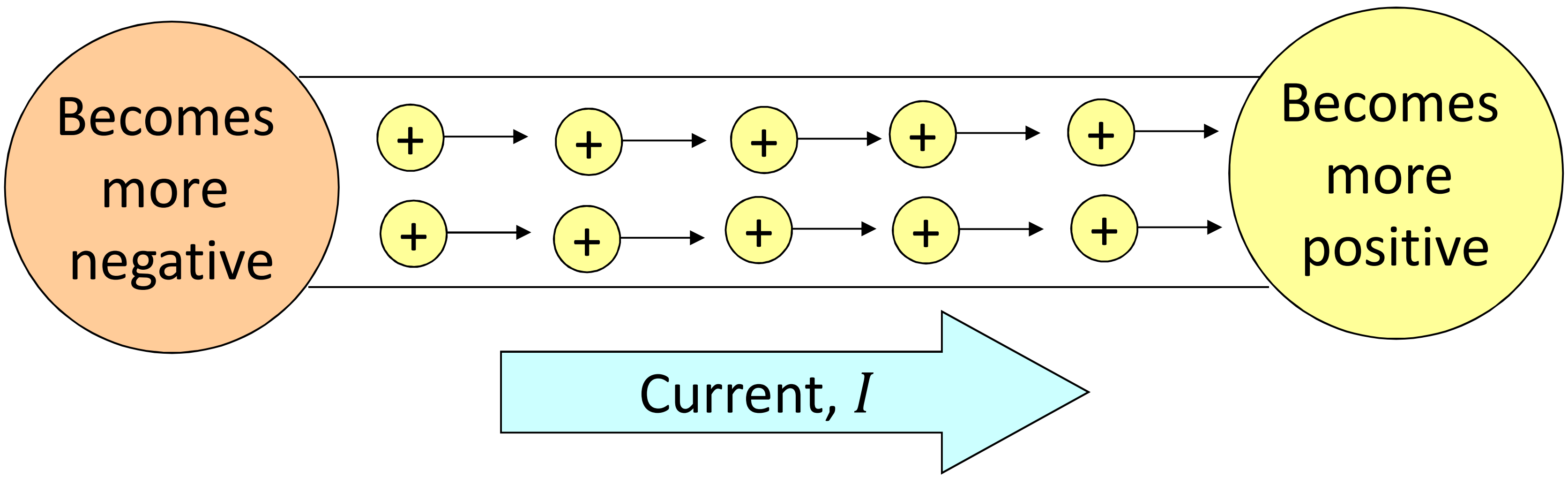
- Lecture 7: $V(\vec{x}) = \frac{1}{4\pi\epsilon_0}\sum_i \frac{Q_i}{r_i}$ ⟹ $V(\vec{x}) = \frac{1}{4\pi\epsilon_0}\int \frac{dQ}{r}$
- Lecture 8: $C = \frac{Q}{V}$ and $U = \frac{1}{2}CV^2$
- Lecture 9: $C_{\parallel} = C_1 + C_2$ and $C_{series} = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}$
- But wait! There's more…
  - No $\vec{E}$ field inside a conductor
  - Principle of superposition
  - Surface charge densities
  - $\vec{E}$ and V for example geometries
  - Work and energy
  - Dielectrics and $\epsilon = \kappa\,\epsilon_0$
  - Lots of examples and clicker questions…

Electrostatic Equilibrium

- No net motion of charge
- Insulators:
  - No free charges
- Conductors:
  - Charges are pushed by any electric field until their own electric field cancels the original one
  - The motion stops when charges accumulate at a surface
  - The net electric field in the conductor is zero
- What if charge is added to or removed from a surface as quickly as it accumulates?
  - The charge will continue to flow…
  - Not a state of electrostatic equilibrium

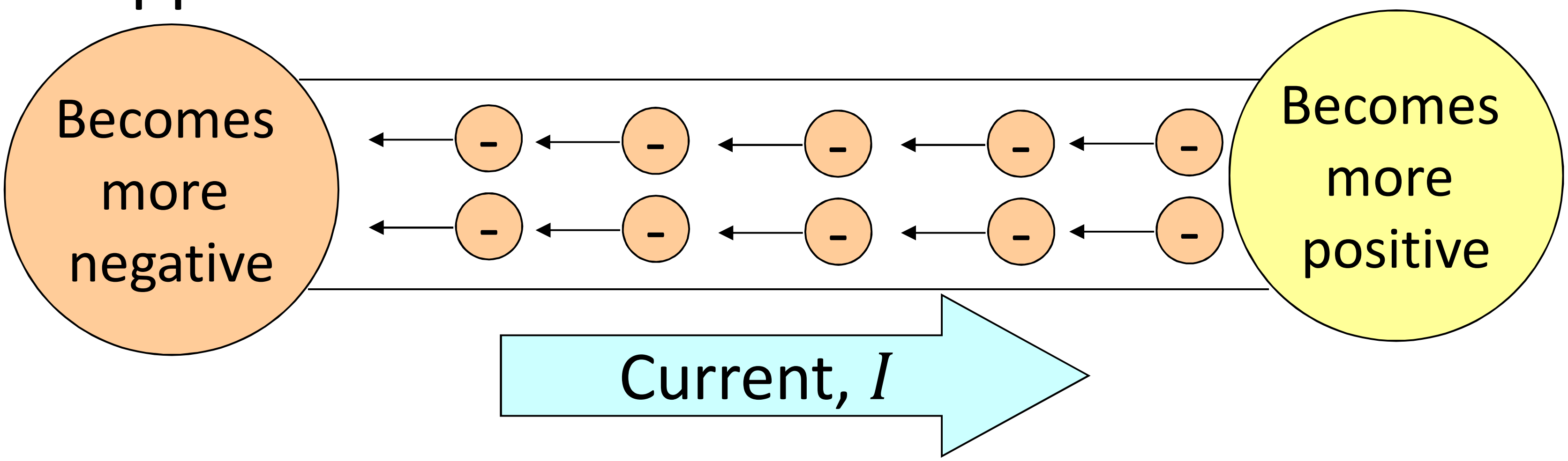
Electric Current

- When the motion of charge carriers are not restricted, they will flow.
- By convention, the direction of an electric current is in the direction that positive charge carriers move:



Electric Current

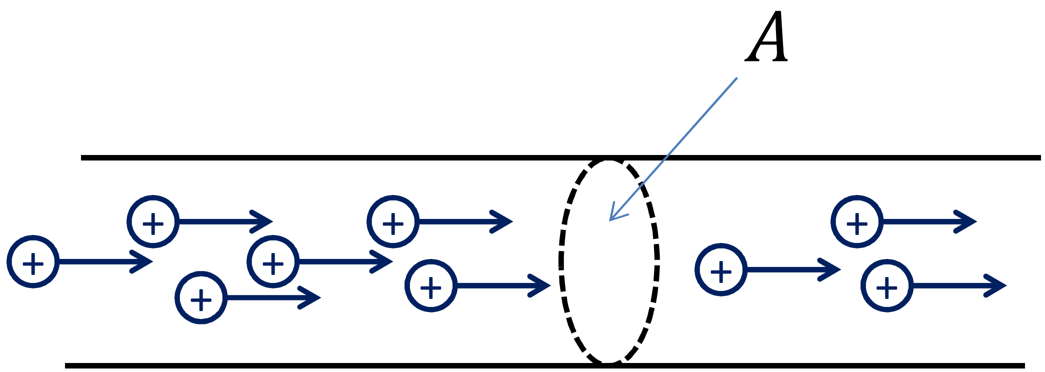
- If they are negatively charged, then the current is opposite their motion:



- In metals, the charge carriers are electrons
- In chemical solutions or ionized gasses, the charge carriers can be both positive and negative.

Electric Current

- Electric current is the net positive charge moving across a surface per unit time:

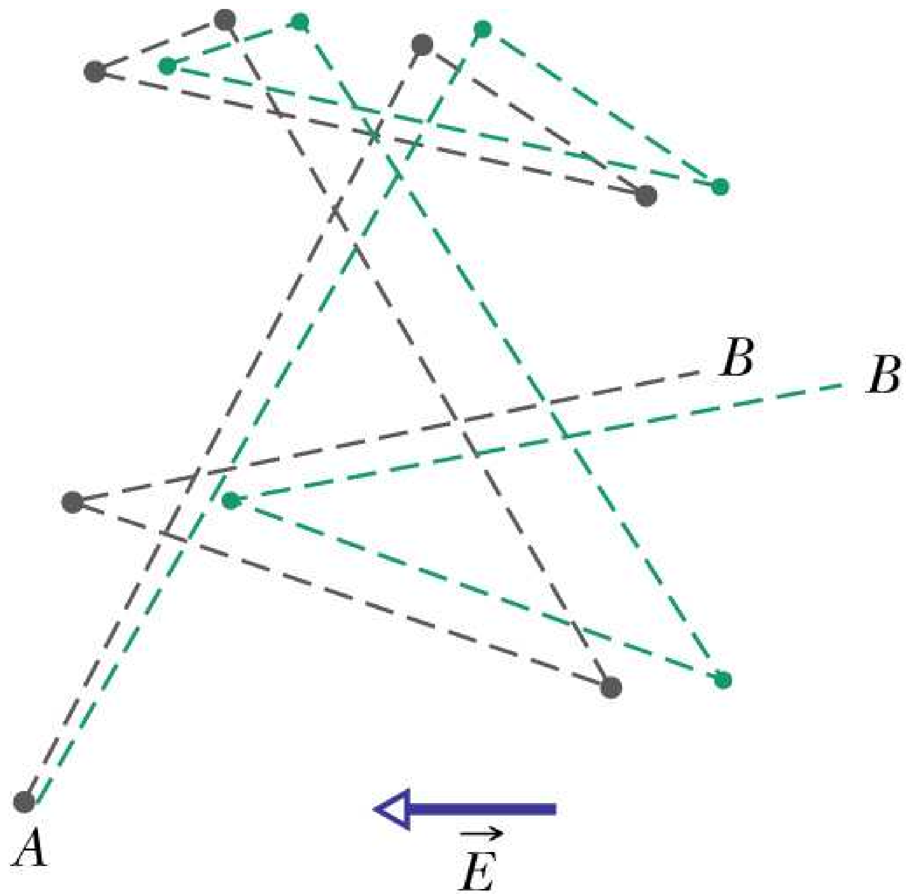


$$I = \frac{\Delta Q}{\Delta t}$$

Units: $Amperes = \frac{Coulombs}{second}$

Drift Velocity

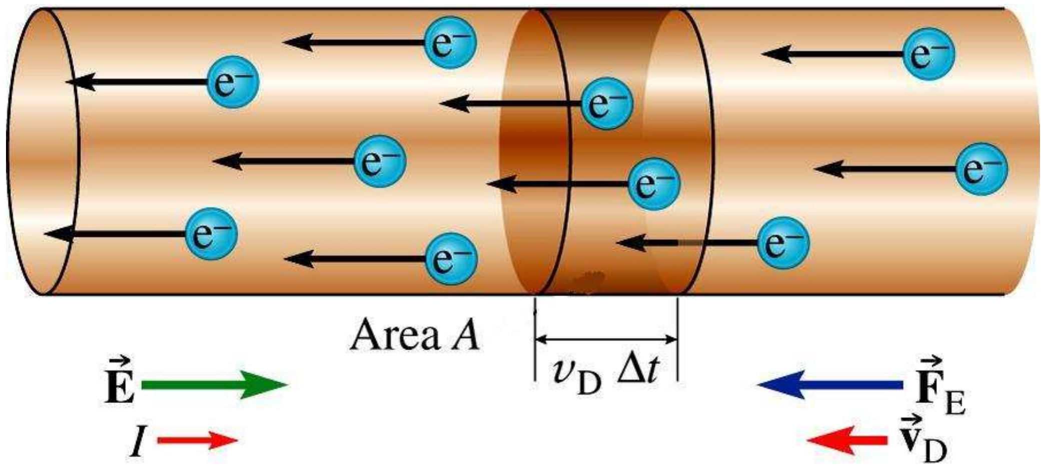
- Motion of individual charges is usually not uniform:



The average distance moved per unit time is the drift velocity:

$$v_D = \frac{\overline{\Delta x}}{\Delta t}$$

Drift speed, total charge and current

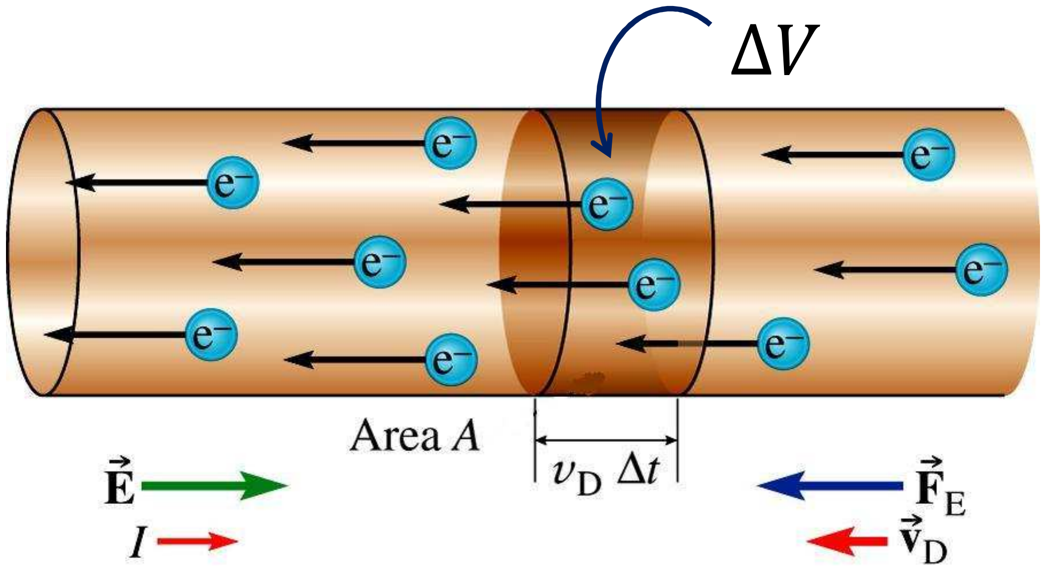


$$n = \frac{\text{\# of charge carriers}}{\text{unit volume}}$$

$$q = \text{charge of each carrier}$$

$$v_D = \text{drift velocity}$$

Drift speed, total charge and current



$$\Delta Q = n\, q\, \Delta V$$

$$\Delta V = A\, v_D \Delta t$$

$$I = \frac{\Delta Q}{\Delta t} = \frac{n\, q\, A\, v_D \Delta t}{\Delta t} = n\, q\, A\, v_D$$

Example

- What is the drift velocity in #12 AWG copper wire carrying 1 ampere of current?
  - What's the diameter of #12 AWG???
    - Google "wire gauge"… it's roughly 2 mm
  - How many charge carriers?
    - Assume one charge carrier per copper atom
    - How many copper atoms per unit volume?
    - How many copper atoms per unit mass?

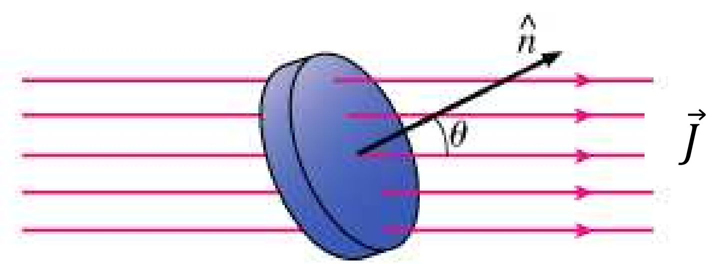
  Atomic mass: $\boldsymbol{m = 63.546\ g/mol}$

  Density of copper: $\boldsymbol{\rho = 8.94\ g/cm^3}$

Current Density

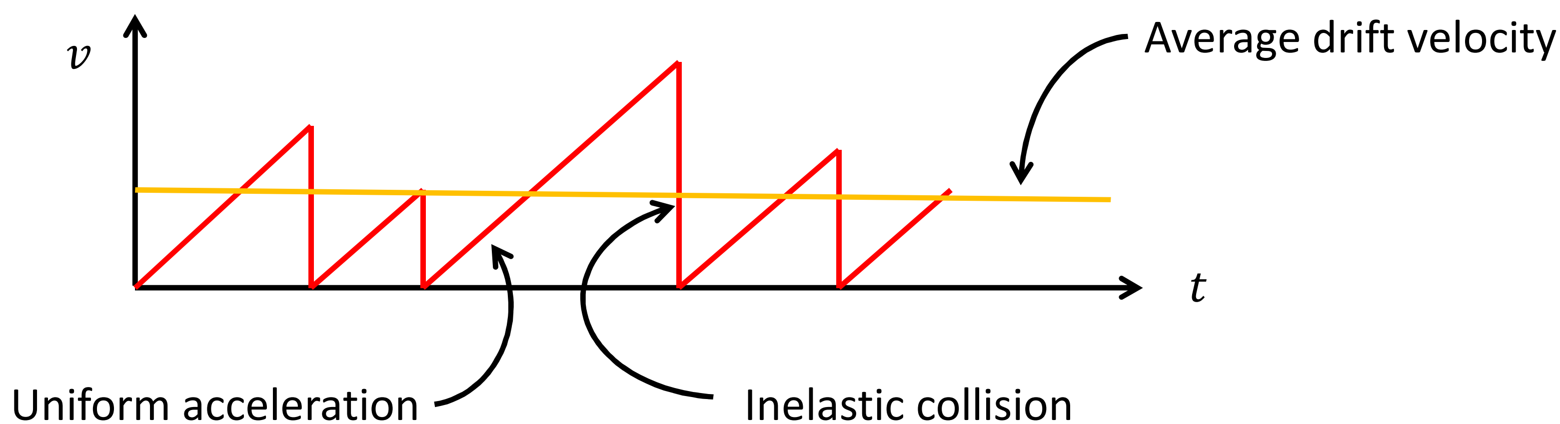
- The flow of charge might not be uniformly across a surface
  - The magnitude of the local current might change
  - The direction of the drift velocity could change
- Current: $\boldsymbol{I = n\, q\, v_D\, A}$
- Current density: $\boldsymbol{\vec{J} = n\, q\, \vec{v}_D}$
- They are related:

$$I = \int_S \vec{J} \cdot d\vec{A}$$



Resistance

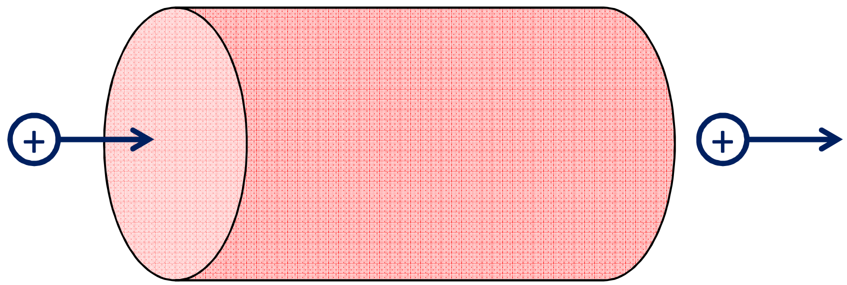
- Electrons in a metal do not accelerate indefinitely
  - They eventually hit an atom in the metal
  - The collision is inelastic and the electron loses all, or some of its energy
- Instantaneous vs average velocity:



- Resistance is a property of a material related to how rapidly charge carriers lose energy

Resistance

Potential energy:
$U = q\,V$



Potential energy:
$U' = q\,V'$

- The greater the current, the more energy is transferred to the material through inelastic collisions.
- Electric potential difference: $\Delta V \propto I$

$$\Delta V = R\,I \quad \Rightarrow \quad R = \frac{\Delta V}{I}$$

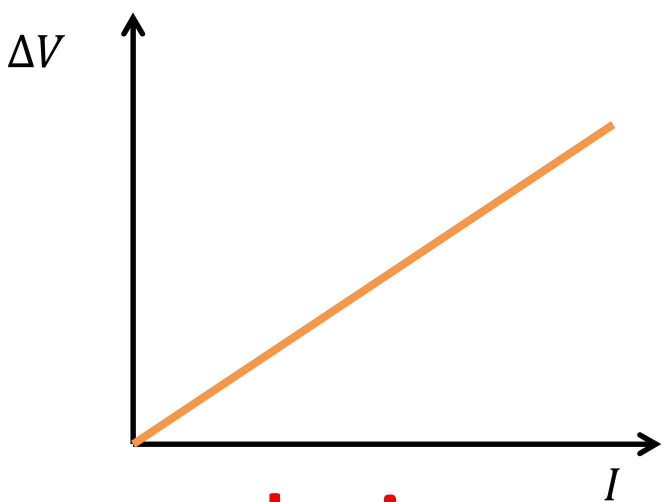
$$Ohms = \frac{Volts}{Ampere}$$

Ohm's Law

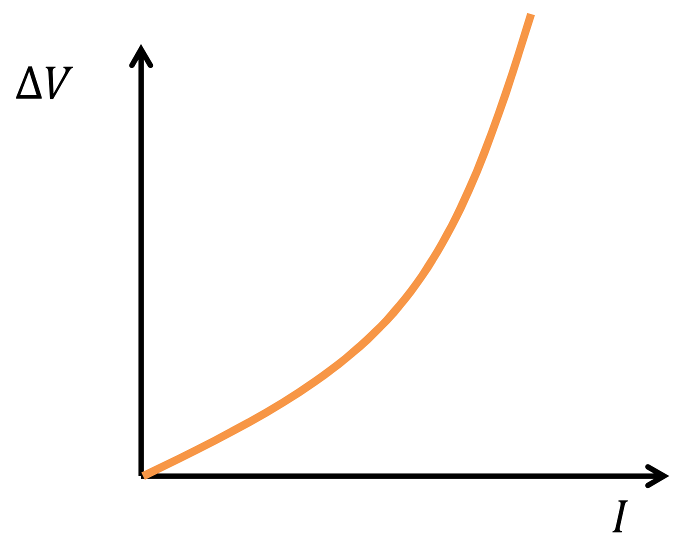
- Potential difference is proportional to current

$$\Delta V = I\, R$$

- This is usually a good approximation…



ohmic



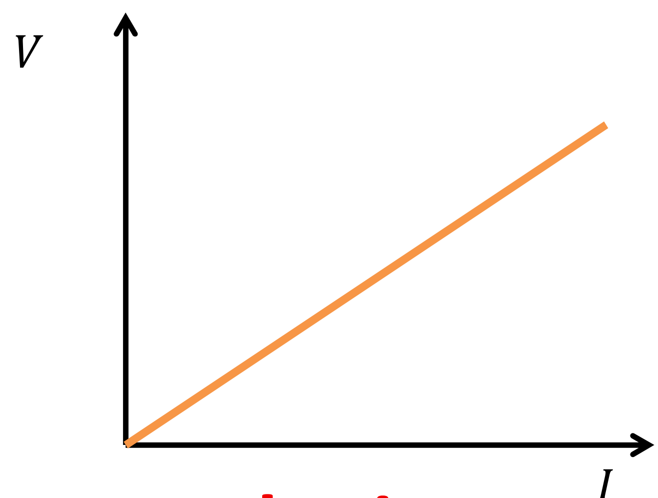
non-ohmic

Ohm's Law

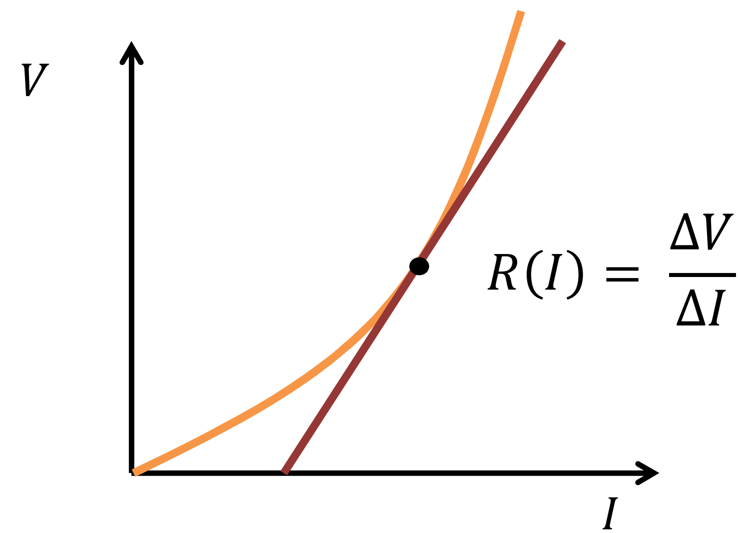
- Potential difference is proportional to current

$$\Delta V = I\,R$$

- This is usually a good approximation…

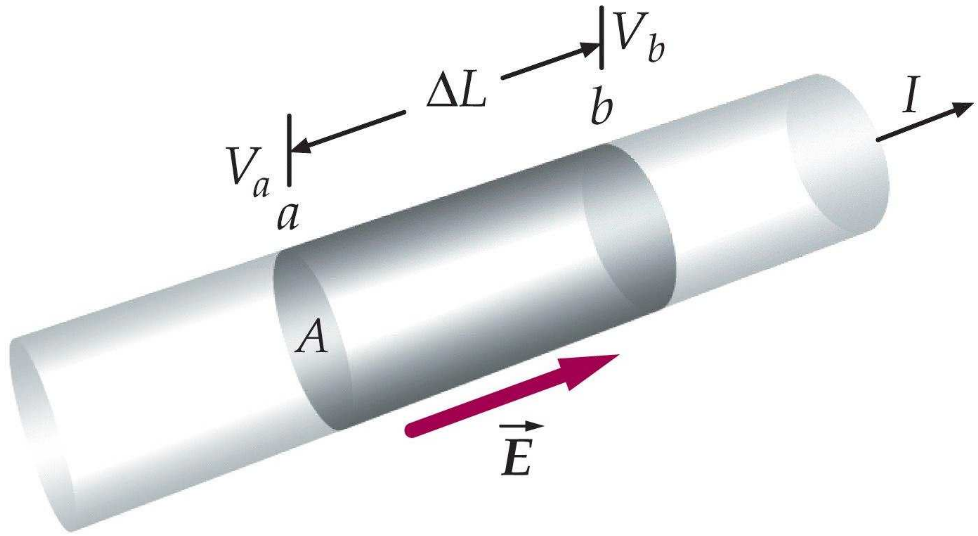


ohmic



non-ohmic

Resistance depends on geometry



- Resistance is proportional to ΔL
- Resistance is inversely proportional to A
- Resistivity, $\boldsymbol{\rho}$, is independent of geometry

$$R = \frac{\boldsymbol{\rho}\,\Delta L}{A}$$

Resistivity Depends on Temperature

- In general, resistivity increases with temperature

$$\Delta\rho \propto \Delta T$$

- The temperature coefficient, $\boldsymbol{\alpha}$, is defined as the fractional change in resistance:

$$\alpha = \frac{1}{\rho}\frac{\Delta\rho}{\Delta T}$$

- Resistivity and the temperature coefficient are usually given for a particular reference temperature (for example, $20\ ^\circ\mathrm{C}$)

Resistivities and Temperature Coefficients

Material	Resistivity, ρ $(\Omega \cdot m)$	Temp. coeff., $\alpha\ (K^{-1})$
Ag	1.6×10^{-8}	3.8×10^{-3}
Cu	1.7×10^{-8}	3.9×10^{-3}
W	5.5×10^{-8}	4.5×10^{-3}
Si	640	-7.5×10^{-2}
Si, n-type	8.7×10^{-4}	
Si, p-type	2.8×10^{-3}	
glass	10^{10}–10^{14}	

Temperature Dependence

$$\rho - \rho_0 = \rho_0 \alpha (T - T_0)$$
$$\rho(T) = \rho_0 (1 + \alpha (T - T_0))$$

- Also true for resistance: $R = \rho L / A$

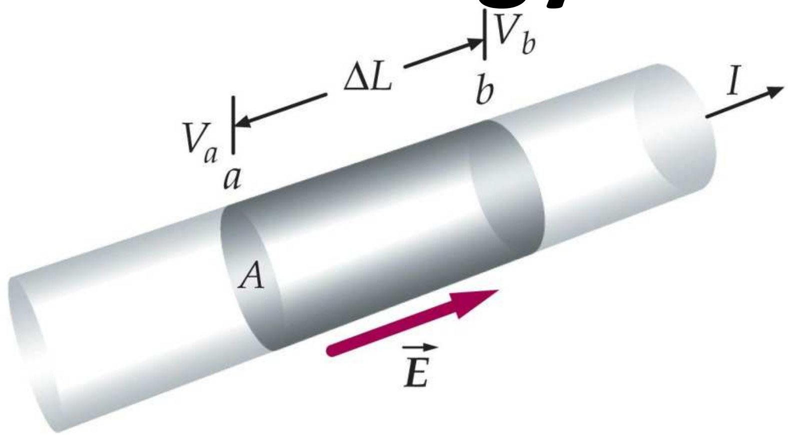
$$R - R_0 = R_0 \alpha (T - T_0)$$
$$R(T) = R_0 (1 + \alpha (T - T_0))$$

Example

- What is the resistance of a 10 cm long Tungsten wire with a diameter of 0.2 mm at 20 °C and at $3000\ \mathrm{K}$?

$$\rho_0 = 5.5 \times 10^{-8}\ \Omega \cdot m$$
$$\alpha = 4.5 \times 10^{-3}\ K^{-1}$$

Rate of Energy Loss



- Charges moving through a resistor lose energy

$$\Delta U = q\,\Delta V = q\,I\,R$$

- Total energy lost per unit time:

$$P = n\,q\,A\,v_D\ \times I\,R$$
$$P = I^2 R$$

- Electric potential energy is converted into heat.

Clicker Question

- The resistance across the human body is approximately $2\ k\Omega$
- If it takes only $50\ mA$ of current to kill a human, what voltage could be lethal?

(a) 0.1 Volts

(b) 1 Volt

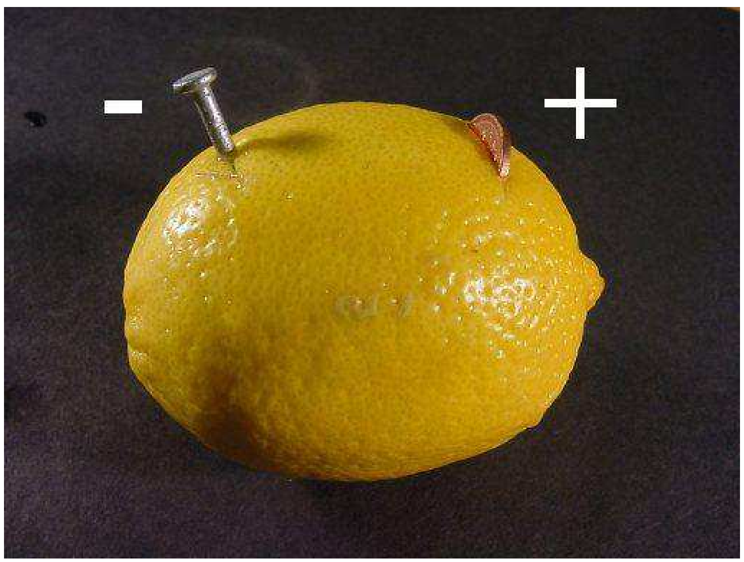
(c) 10 Volts

(d) 100 Volts

(e) 1000 Volts

Electric Current

- A chemical battery is a *source* of electric potential
  - The chemical reaction creates a potential difference across the poles:



The chemical reaction maintains a constant potential difference.

Positive charges at the + end have a greater electric potential than positive charges at the – end.

There must be an electric field between the poles.

If free charges were present, they would be accelerated by the electric field – the field does work on the charges.

Their potential energy decreases as they move towards the – pole.