PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 22000

General Physics

Lecture 9 – Impulse and Linear Momentum

Fall 2016 Semester

Prof. Matthew Jones

First Midterm Exam

Tuesday, October 4th, 8:00-9:30 pm

Location: PHYS 112 and WTHR 200.

Covering material in chapters 1-6

(but probably not too much from chapter 6)

Multiple choice, probably about 25 questions, 15 will be conceptual, 10 will require simple computations.

A formula sheet will be provided.

You can bring one page of your own notes.

SUPPLEMENTAL INSTRUCTION

Free Study Sessions!

Rachel Hoagburg

Come to SI for more help in PHYS 220

Tuesday and Thursday	7:30-8:30PM Shreve C113

Office Hour

Tuesday 1:30-2:30 4th floor of Krach

For other SI-linked courses and schedules, visit purdue.edu/si or purdue.edu/boilerguide

Newton’s Laws

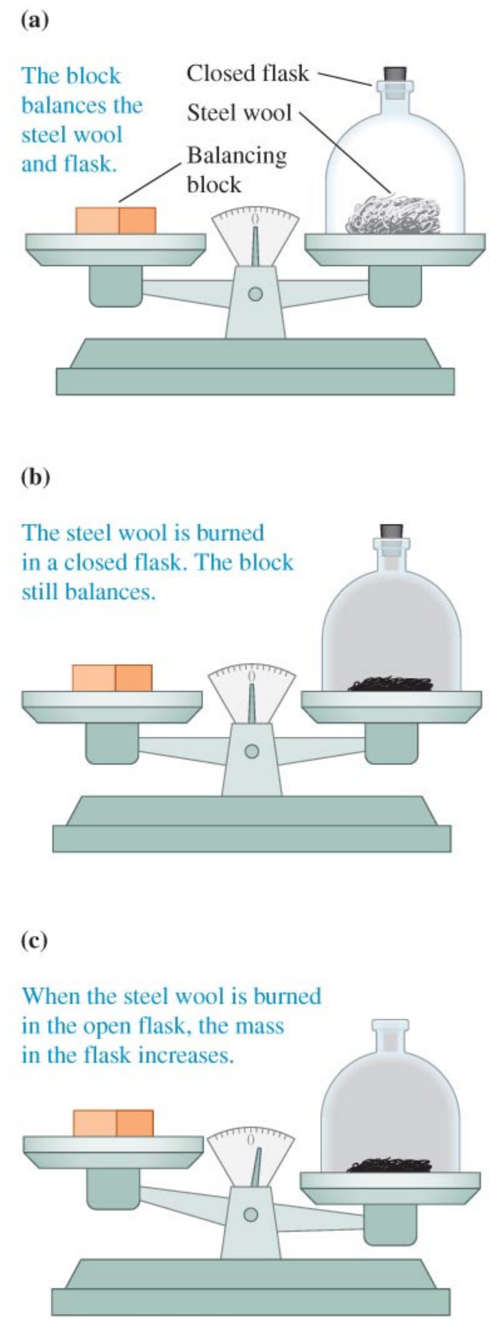
- To use Newton’s Laws effectively, we need to know the force in order to calculate the acceleration and determine the resulting motion.
- In many cases we might not know exactly what the force is at any instant in time.
- We can still deduce what the resulting motion will be using two new concepts: ***impulse*** and ***linear momentum***.

First: Accounting for Mass

- The mass of a log in a campfire decreases as the log burns. What happens to the "lost" mass from the log?
- If we choose only the log as the system, the mass of the system decreases as it burns.
- Air is needed for burning. What happens to the mass if we choose the surrounding air and the log as the system?

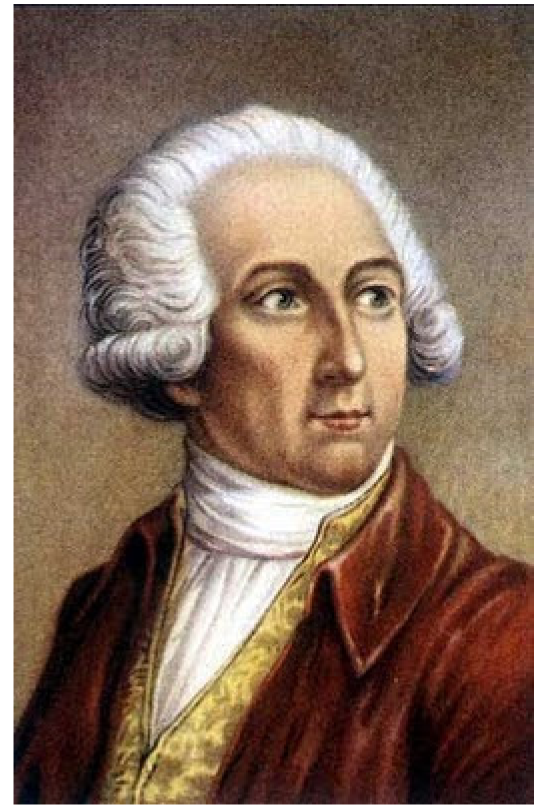
Accounting for Mass

- Is the mass of a system always a constant value?
- Burn some steel wool in a closed container…
  - The total mass doesn't change.
- Burn it in an open container, allowing fresh air to enter…
  - More steel wool burns and the resulting stinky mess is more massive.



Law of Constancy of Mass

- Lavoisier defined an isolated system as a group of objects that interact with each other but not with external objects.
- When a system of objects is isolated (a closed container), its mass equals the sum of the masses of components and remains constant in time.
- When the system is not isolated, any change in mass is equal to the amount of mass leaving or entering the system.



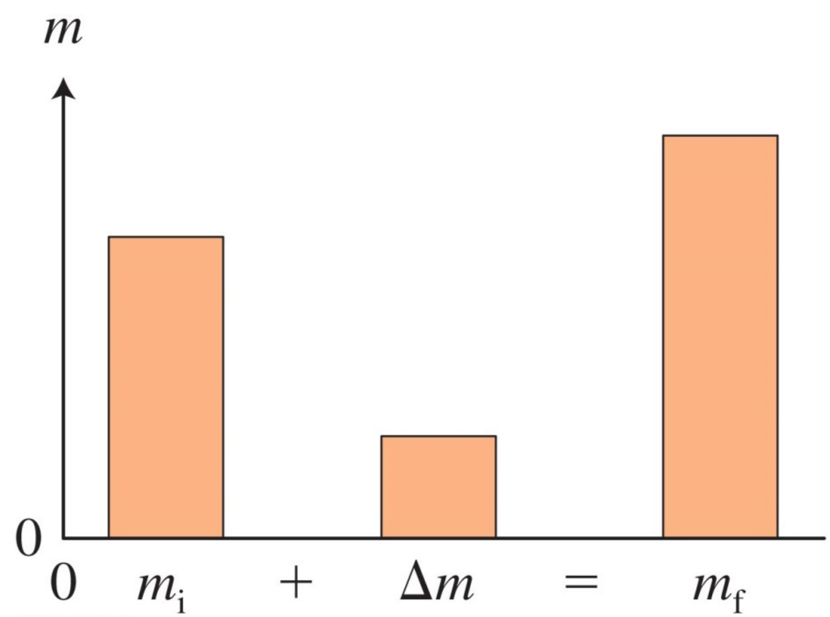
Lavoisier – Born 1743. Died 1794 by the guillotine during the second phase of the French Revolution. Contemporary of Benjamin Franklin (1706-1790), George Washington (1732-1799) and Mozart (1756-1791). Note the powdered wig…

Accounting for Changing Mass

$$\begin{pmatrix} \textit{initial} \text{ mass of} \\ \text{system at earlier} \\ \text{clock reading} \end{pmatrix} + \begin{pmatrix} \textit{new} \text{ mass entering or} \\ \text{leaving system between} \\ \text{the two clock readings} \end{pmatrix} = \begin{pmatrix} \textit{final} \text{ mass of} \\ \text{system at later} \\ \text{clock reading} \end{pmatrix}$$

- The mass is constant if there is no flow of mass in or out of the system.
- The mass changes in a predictable way if there is some flow of mass between the system and the environment.

Mass Bar Chart



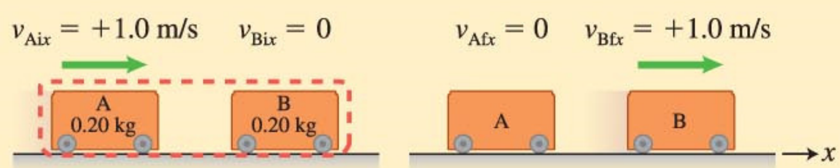
- The left bar represents the initial mass of the system, the central bar represents the mass added or taken away, and the right bar represents the mass of the system in the final situation.
- The height of the left bar plus the height of the central bar equals the height of the right bar.

Observing Collisions of Two Carts

OBSERVATIONAL EXPERIMENT TABLE

5.1 **Collisions in a system of two carts (all velocities are with respect to the track).**

VIDEO 5.1

Observational experiment	Analysis
Experiment 1. Cart A (0.20 kg) moving right at 1.0 m/s collides with cart B (0.20 kg), which is stationary. Cart A stops and cart B moves right at 1.0 m/s.	 The direction of motion is indicated with a plus and a minus sign.

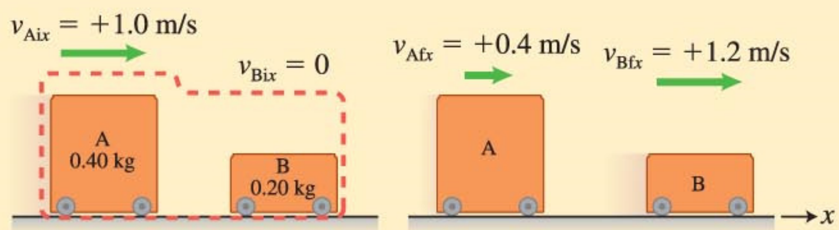
- *Speed:* The sum of the speeds of the system objects is the same before and after the collision: 1.0 m/s + 0 m/s = 0 m/s + 1.0 m/s.
- *Mass • speed:* The sum of the products of mass and speed is the same before and after the collision: 0.20 kg(1.0 m/s) + 0.20 kg(0 m/s) = 0.20 kg(0 m/s) + 0.20 kg(1.0 m/s).
- *Mass • velocity:* The sum of the products of mass and the *x*-component of velocity is the same before and after the collision: 0.20 kg(+1.0 m/s) + 0.20 kg(0) = 0.20 kg(0) + 0.20 kg(+1.0 m/s).

Observing Collisions of Two Carts

OBSERVATIONAL EXPERIMENT TABLE

5.1 **Collisions in a system of two carts (all velocities are with respect to the track).** ***(Continued)***

VIDEO 5.1

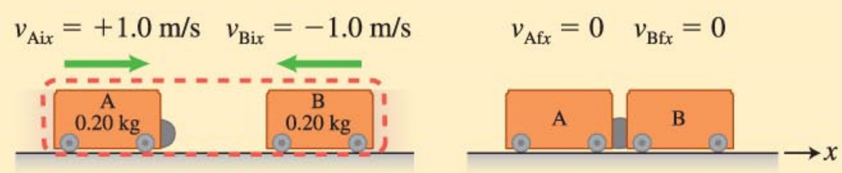
Observational experiment	Analysis
Experiment 2. Cart A (0.40 kg) moving right at 1.0 m/s collides with cart B (0.20 kg), which is stationary. After the collision, both carts move right, cart B at 1.2 m/s, and cart A at 0.4 m/s.	 ■ *Speed:* The sum of the speeds of the system objects is not the same before and after the collision: $1.0\text{ m/s} + 0\text{ m/s} \neq 0.4\text{ m/s} + 1.2\text{ m/s}$. ■ *Mass • speed:* The sum of the products of mass and speed is the same before and after the collision: $0.40\text{ kg}(1.0\text{ m/s}) + 0.20\text{ kg}(0\text{ m/s}) = 0.40\text{ kg}(0.4\text{ m/s}) + 0.20\text{ kg}(1.2\text{ m/s})$. ■ *Mass • velocity:* The sum of the products of mass and the *x*-component of velocity is the same before and after the collision: $0.40\text{ kg}(+1.0\text{ m/s}) + 0.20\text{ kg}(0) = 0.40\text{ kg}(+0.4\text{ m/s}) + 0.20\text{ kg}(+1.2\text{ m/s})$.

Observing Collisions of Two Carts

OBSERVATIONAL EXPERIMENT TABLE

5.1 Collisions in a system of two carts (all velocities are with respect to the track). *(Continued)*

VIDEO 5.1

Observational experiment	Analysis
Experiment 3. Cart A (0.20 kg) with a piece of clay attached to the front moves right at 1.0 m/s. Cart B (0.20 kg) moves left at 1.0 m/s. The carts collide, stick together, and stop.	 ■ *Speed:* The sum of the speeds of the system objects is not the same before and after the collision: $1.0\ \text{m/s} + 1.0\ \text{m/s} \neq 0\ \text{m/s} + 0\ \text{m/s}$. ■ *Mass · speed:* The sum of the products of mass and speed is not the same before and after the collision: $0.20\ \text{kg}(1.0\ \text{m/s}) + 0.20\ \text{kg}(1.0\ \text{m/s}) \neq 0.20\ \text{kg}(0\ \text{m/s}) + 0.20\ \text{kg}(0\ \text{m/s})$. ■ *Mass · velocity:* The sum of the products of mass and the *x*-component of velocity is the same before and after the collision: $0.20\ \text{kg}(+1.0\ \text{m/s}) + 0.20\ \text{kg}(-1.0\ \text{m/s}) = 0.20\ \text{kg}(0\ \text{m/s}) + 0.20\ \text{kg}(0\ \text{m/s})$.

Patterns

One quantity remains the same before and after the collision in each experiment—the sum of the products of the mass and *x*-velocity component of the system objects.

Observing Collisions of Two Carts

- One quantity remains the same before and after the collision in each experiment: the sum of the products of the mass and *x*-velocity component of the system objects.
- Hypothesis to test: The sum of mass times velocity is the quantity characterizing motion that is constant in an isolated system.

Testing the Hypothesis

TESTING EXPERIMENT TABLE

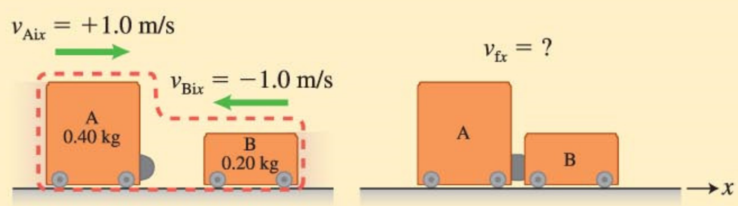
5.2 **Testing the idea that $\Sigma m\vec{v}$ in an isolated system remains constant (all velocities are with respect to the track).**

VIDEO 5.2

Testing experiment

Cart A (0.40 kg) has a piece of modeling clay attached to its front and is moving right at 1.0 m/s. Cart B (0.20 kg) is moving left at 1.0 m/s. The carts collide and stick together. Predict the velocity of the carts after the collision.

Prediction



The system consists of the two carts. The direction of velocity is noted with a plus or minus sign of the velocity component:

$$(0.40\text{ kg})(+1.0\text{ m/s}) + (0.20\text{ kg})(-1.0\text{ m/s}) = (0.40\text{ kg} + 0.20\text{ kg})v_{fx}$$

or

$$v_{fx} = (+0.20\text{ kg}\cdot\text{m/s})/(0.60\text{ kg}) = +0.33\text{ m/s}$$

After the collision, the two carts should move right at a speed of about 0.33 m/s.

Outcome

After the collision, the carts move together toward the right at close to the predicted speed.

Conclusion

Our prediction matched the outcome. This result gives us increased confidence that this new quantity $m\vec{v}$ might be the quantity whose sum is constant in an isolated system.

Linear Momentum

Linear Momentum The linear momentum $\vec{p}$ of a single object is the product of its mass m and velocity $\vec{v}$:

$$\vec{p} = m\vec{v} \tag{5.1}$$

Linear momentum is a vector quantity that points in the same direction as the object's velocity $\vec{v}$ (**Figure 5.3**). The SI unit of linear momentum is (kg · m/s). The total linear momentum of a system containing multiple objects is the vector sum of the momenta (plural of momentum) of the individual objects.

$$\vec{p}_{\text{net}} = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_n\vec{v}_n = \Sigma m\vec{v}$$

Important Points About Linear Momentum

- Linear momentum is a vector quantity; it is important to consider the direction in which the colliding objects are moving before and after the collision.
- Momentum depends on the velocity of the object, and the velocity depends on the choice of the reference frame. Different observers will measure different momenta for the same object.
- To establish that momentum is a conserved quantity, we need to ensure that the momentum of a system changes in a predictable way for systems that are not isolated.

Momentum of an Isolated System is Constant

Momentum constancy of an isolated system The momentum of an isolated system is constant. For an isolated two-object system:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f} \tag{5.2}$$

Because momentum is a vector quantity and Eq. (5.2) is a vector equation, we will work with its x- and y-component forms:

$$m_1 v_{1i\,x} + m_2 v_{2i\,x} = m_1 v_{1f\,x} + m_2 v_{2f\,x} \tag{5.3x}$$

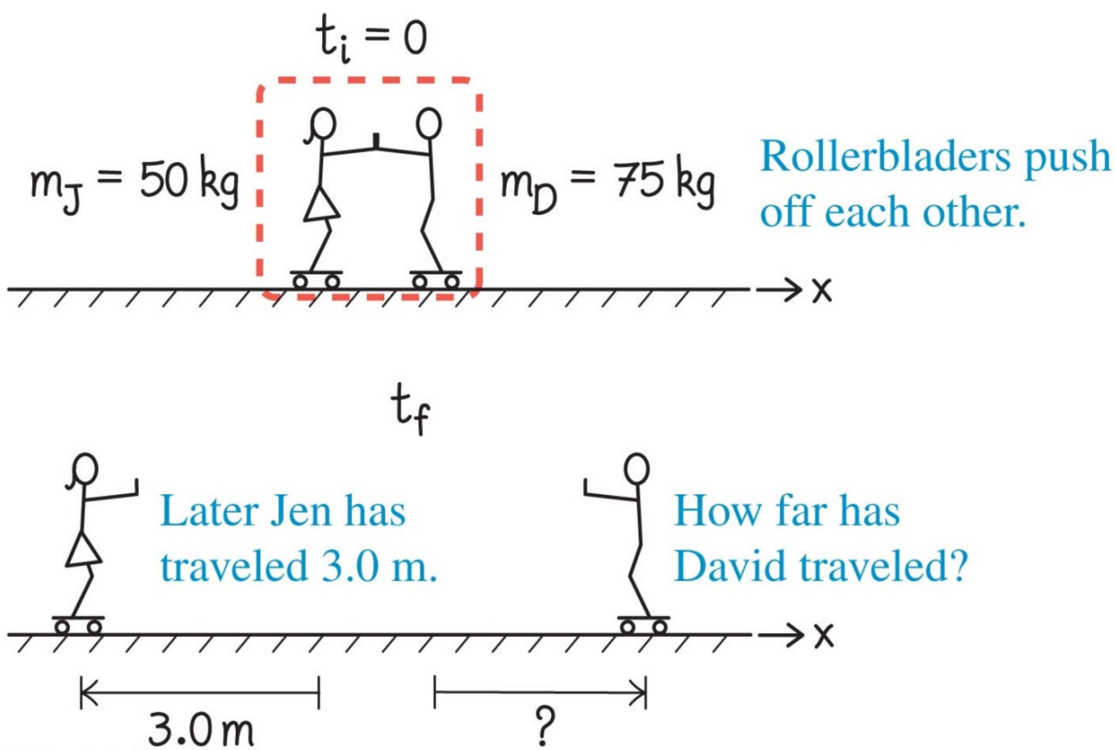
$$m_1 v_{1i\,y} + m_2 v_{2i\,y} = m_1 v_{1f\,y} + m_2 v_{2f\,y} \tag{5.3y}$$

- To describe a system with more than two objects, we simply include a term on each side of the equation for each object in the system.

Example: Two Rollerbladers

- Jen (50 kg) and David (75 kg), both on rollerblades, push off each other abruptly. Each person coasts backward at approximately constant speed. During a certain time interval, Jen travels 3.0 m.

- How far does David travel during that same time interval?



Example: Two Rollerbladers

- What is the initial momentum?
- What is Jen's final velocity?

$$v_{J,x} = \frac{x_J}{t_f} = \frac{-3.0\ m}{t_f}$$

- What is Jen's final momentum?

$$p_{J,x} = m_J v_{J,x}$$

- What is David's final momentum?

$$p_{D,x} = m_D v_{D,x} = m_D \frac{x_D}{t_f} = -p_{J,x} = -m_J \frac{x_J}{t_f}$$

Example: Two Rollerbladers

- David's final momentum:

$$m_D \frac{x_D}{\cancel{t_f}} = -m_J \frac{x_J}{\cancel{t_f}}$$

- David's final distance:

$$x_D = -\frac{m_J}{m_D} x_J = -\frac{50\ kg}{75\ kg}(-3.0\ m)$$
$$= 2.0\ m$$

Importance of Linear Momentum

- In the last example, we were able to determine the velocity by using the principle of momentum constancy.
  - **We did not need any information about the forces involved.**
  - This is a very powerful result, because in all likelihood the forces exerted were not constant.
- The kinematics equations we have used assumed constant acceleration of the system (and thus constant forces).

Impulse due to a Force Exerted on a Single Object

- We need a way to account for change in momentum when the net external force on a system is not zero
- A relationship can be derived from Newton's laws and kinematics:

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i} = \frac{\sum \vec{F}}{m}$$

$$m\vec{v}_f - m\vec{v}_i = \vec{p}_f - \vec{p}_i = \underbrace{\sum \vec{F}(t_f - t_i)}_{\text{Impulse}}$$

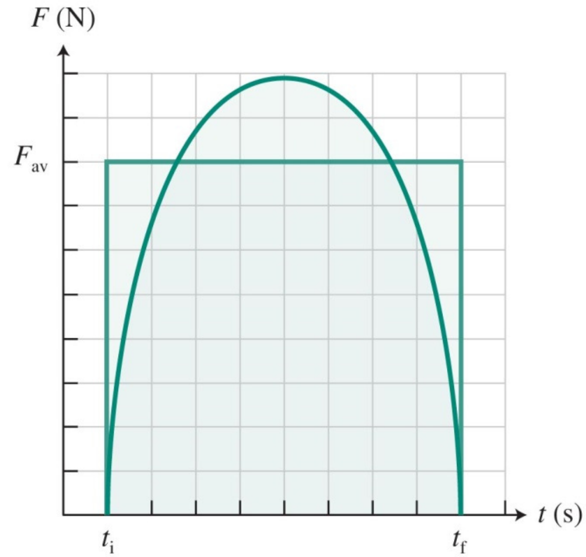
Two Important Points about Impulse

- This equation is Newton's second law written in a different form—one that involves the physical quantity momentum:

$$m\vec{v}_f - m\vec{v}_i = \vec{p}_f - \vec{p}_i = \sum \vec{F}(t_f - t_i)$$

- Both force and time interval affect momentum: a small force exerted over a long time interval can change the momentum of an object by the same amount as a large force exerted over a short time interval.

Impulse: The product of the external force exerted on an object and the time interval



Impulse The impulse $\vec{J}$ of a force is the product of the average force $\vec{F}_{av}$ exerted on an object during a time interval $(t_f - t_i)$ and that time interval:

$$\vec{J} = \vec{F}_{av}(t_f - t_i) \tag{5.5}$$

Impulse is a vector quantity that points in the direction of the force. The impulse has a plus or minus sign depending on the orientation of the force relative to a coordinate axis. The SI unit for impulse is $N \cdot s = (kg \cdot m/s^2) \cdot s = kg \cdot m/s$, the same unit as momentum.

Impulse-momentum equation for a single object

Impulse-momentum equation for a single object If several external objects exert forces on a single-object system during a time interval $(t_f - t_i)$, the sum of their impulses $\Sigma\vec{J}$ causes a change in momentum of the system object:

$$\vec{p}_f - \vec{p}_i = \Sigma\vec{J} = \Sigma\vec{F}_{\text{on System}}(t_f - t_i) \qquad (5.6)$$

The x- and y-scalar component forms of the impulse-momentum equation are

$$p_{fx} - p_{ix} = \Sigma F_{\text{on System}\,x}(t_f - t_i) \qquad (5.7x)$$

$$p_{fy} - p_{iy} = \Sigma F_{\text{on System}\,y}(t_f - t_i) \qquad (5.7y)$$

- If the magnitude of the force changes during the time interval considered, then we just use the average force.

Example: An abrupt stop in a car

- A 60-kg person is traveling in a car that is moving at 16 m/s with respect to the ground when the car hits a barrier. The person is not wearing a seat belt, but is stopped by an air bag in a time interval of 0.20 s.
- Determine the average force that the air bag exerts on the person while stopping him.

Abrupt stop in a car

- What is the initial momentum?

$$p_i = m\,v_i$$

- What is the final momentum?

$$p_f = 0$$

- What is the change in momentum?

$$p_f - p_i = -m\,v_i = J = \bar{F}\left(t_f - t_i\right)$$

- What is the average force?

$$\bar{F} = \frac{-m\,v_i}{t_f - t_i} = \frac{-(60\;kg)(16\,m/s)}{0.2\;s} = -4800\;N$$

(The force is opposite the direction of the initial velocity.)

Using Newton’s laws to understand the constancy of momentum

- Newton's third law provides a connection between our analyses of two colliding carts.
- Interacting objects at each instant exert equal-magnitude but oppositely directed forces on each other:

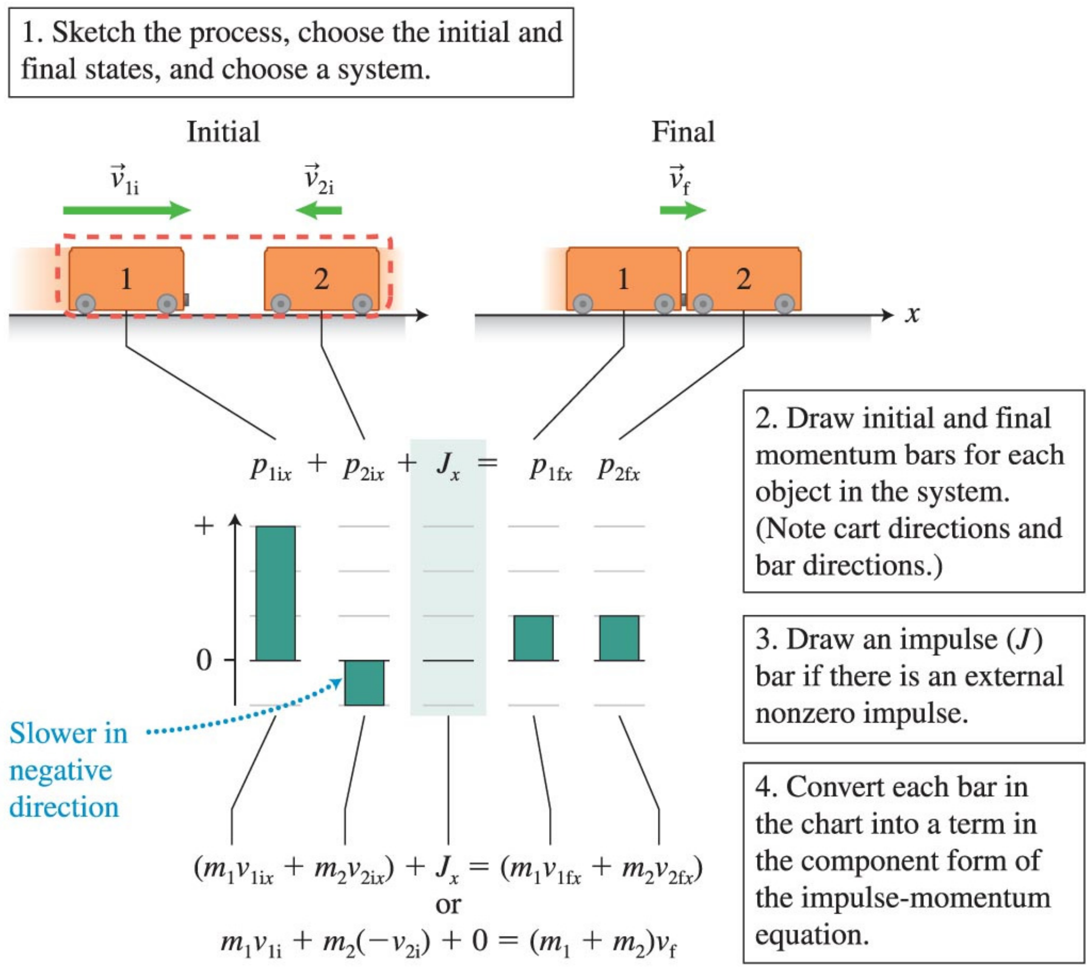
$$\underbrace{m_1\vec{v}_{1\mathrm{i}} + m_2\vec{v}_{2\mathrm{i}}}_{\text{Initial momentum}} = \underbrace{m_1\vec{v}_{1\mathrm{f}} + m_2\vec{v}_{2\mathrm{f}}}_{\text{Final momentum}}$$

- With an additional, external impulse:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} + J = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Impulse-momentum bar charts

REASONING SKILL Constructing a qualitative impulse-momentum bar chart.

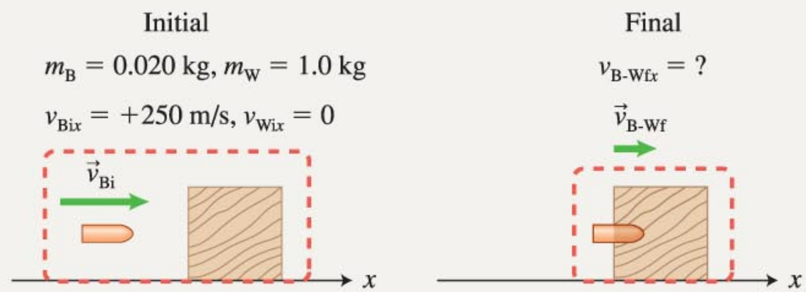


Example: Measuring the speed of a bullet

Sketch and translate

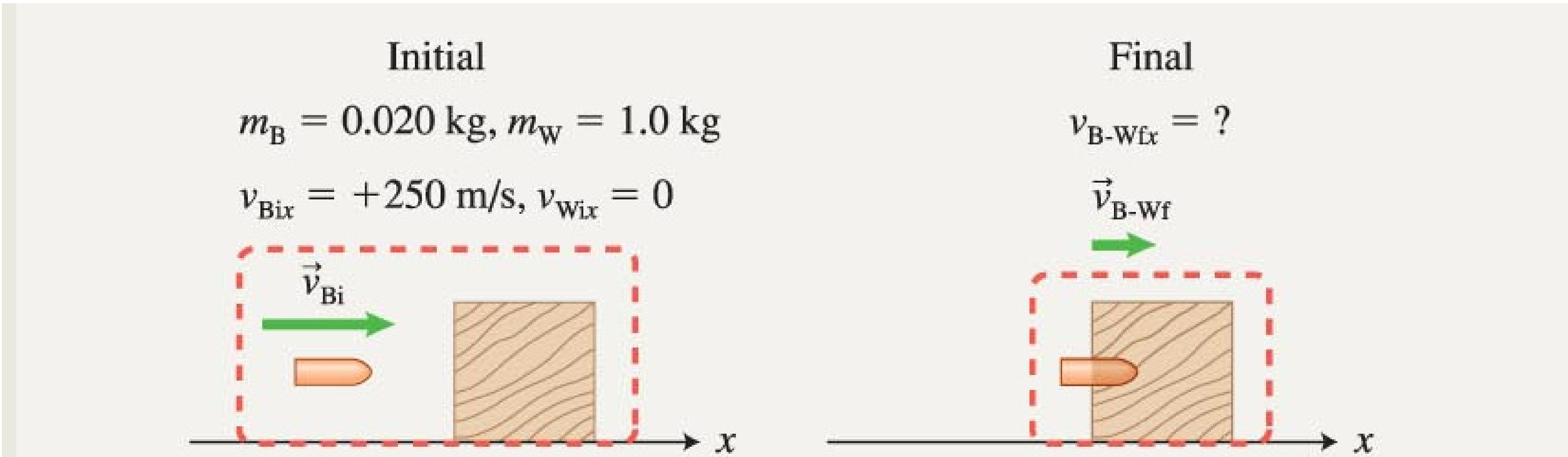
- Sketch the initial and final states and include appropriate coordinate axes. Label the sketches with the known information. Decide on the object of reference.
- Choose a system based on the quantity you are interested in; for example, a multi-object isolated system to determine the velocity of an object, or a single-object nonisolated system to determine an impulse or force.

The left side of the sketch below shows the bullet traveling in the positive x-direction with respect to the ground; it then joins the wood. All motion is along the x-axis; the object of reference is Earth. The system includes the bullet and wood; it is an isolated system since the vertical forces balance. The initial state is immediately before the collision; the final state is immediately after.



$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} + \cancelto{0}{J} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Example: Measuring the speed of a bullet



$$m_B \vec{v}_{Bi} + m_W \cancelto{0}{\vec{v}_{Wi}} = (m_B + m_W)\vec{v}_{B-Wf}$$

$$\vec{v}_{B-Wf} = \frac{m_B \vec{v}_{Bi}}{m_B + m_W} \text{ or } \vec{v}_{Bi} = \frac{m_B + m_W}{m_B} \vec{v}_{B-Wf}$$

Determining the Speed of a Bullet

- It might be easier to measure the distance it takes for an object to stop during a collision.
- The stopping distance can be measured after a collision, such as how far a car's front end crumples or the depth of a hole left by a meteorite.
- Impulse-momentum tells us information about the stopping time; we must use kinematics to relate this to distance.

Determining the stopping time interval from the stopping distance

- Assume that the acceleration of the object while stopping is constant.
- The average velocity is the average of the initial and final velocities $v_{avg} = \frac{1}{2}(v_f + v_i)$
- The stopping displacement and the stopping time interval are related by

$$x_f - x_i = v_{avg}(t_f - t_i) = \frac{1}{2}(v_{fx} + v_{ix})(t_f - t_i)$$

- Solve for the stopping time:

$$t_f - t_i = \frac{2(x_f - x_i)}{v_{fx} + v_{ix}}$$

Example: Stopping the fall of a movie stunt diver

- The record for the highest movie stunt fall without a parachute is 71 m, held by 80-kg Super-Dave Osborne. His fall was stopped by a large air cushion, into which he sank about 4.0 m. His speed was approximately 36 m/s when he reached the top of the air cushion.
- Estimate the average force that the cushion exerted on this stunt diver's body while stopping him.

Example

- Impulse-momentum relation:

$$p_f - p_i = -m\, v_i = J = \bar{F}\left(t_f - t_i\right)$$

$$\bar{F} = \frac{p_f - p_i}{t_f - t_i}$$

- We know the initial momentum: $p_i = -mv_i$
- We don't know the time interval (yet)…

$$t_f - t_i = \frac{2\left(x_f - x_i\right)}{v_{fx} + v_{ix}}$$

- We know the initial velocity, and the final velocity is zero.

Example

$$\bar{F} = \frac{p_f - p_i}{t_f - t_i} = -\frac{p_i v_i}{2\left(x_f - x_i\right)} = -\frac{m\, v_i^2}{2\left(x_f - x_i\right)}$$

$$= -\frac{(80\ kg)(36\, m/s)^2}{2(-4\ m)}$$

$$= 12{,}960\ N$$

(ouch)

Jet Propulsion

- Cars change velocity because of an interaction with the road; a ship's propellers push water backward.
- A rocket in empty space has nothing to push against.
  - If the rocket and fuel are at rest before the rocket fires its engines, the momentum is zero. Because there are no external impulses, after the rocket fires its engines, the momentum should still be zero.
  - Burning fuel is ejected backward at high velocity, so the rocket must have nonzero forward velocity.

Thrust

- Thrust is the force exerted by the fuel on a rocket during jet propulsion.
- Typical rocket thrusts measure approximately 10^6 N, and exhaust speeds are more than 10 times the speed of sound.
- Thrust provides the impulse necessary to change a rocket's momentum.
  - The same principle is at work when you blow up a balloon, but then open the valve and release it, and when you stand on a skateboard with a heavy ball and throw the ball away from you.

Assumptions for Jet Propulsion

- In reality, a rocket burns its fuel gradually rather than in one short burst; thus its mass is not a constant number but changes gradually.
- To solve jet propulsion problems without calculus, we need to assume the fuel burns in a short enough burst to ignore the change in mass when the thrust is applied.

Final Example: Meteorite impact

- Arizona's Meteor Crater was produced 50,000 years ago by the impact of a 3×10^8–kg meteorite traveling at 1.3×10^4 m/s. The crater is approximately 200 m deep.

- Estimate (1) the change in Earth's velocity as a result of the impact and (2) the average force exerted by the meteorite on Earth during the collision.

Example

- Change in the earth's velocity:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

- Assume that the earth is initially at rest
  - this defines the reference frame we will use

$$m_M v_M = (m_M + m_E)v_{E+M}$$

$$v_{E+M} = \frac{m_M v_M}{m_M + m_E}$$

$$= \frac{(3 \times 10^8\ kg)(1.3 \times 10^4\ m/s)}{(3 \times 10^8\ kg) + (6 \times 10^{24}\ kg)} = 6.5 \times 10^{-13}\ m/s$$

- The average force can be determined the same way as in the previous example.