Purdue University Department of Physics

Physics 22000

General Physics

Lecture 8 – Planetary Motion

Fall 2016 Semester

Prof. Matthew Jones

First Midterm Exam

Tuesday, October 4th, 8:00-9:30 pm

Location: PHYS 112 and WTHR 200.

Covering material in chapters 1-6

(but probably not too much from chapter 6)

Multiple choice, probably about 25 questions, 15 will be conceptual, 10 will require simple computations.

A formula sheet will be provided.

You can bring one page of your own notes.

SUPPLEMENTAL INSTRUCTION

Free Study Sessions!

Rachel Hoagburg

Come to SI for more help in **PHYS 220**

Tuesday and Thursday	7:30-8:30PM Shreve C113

Office Hour

Tuesday 1:30-2:30 4th floor of Krach

For other SI-linked courses and schedules, visit purdue.edu/si or purdue.edu/boilerguide

Uniform Circular Motion

- If an object moves in a circle of radius r with constant speed v then it must be accelerated.

$$a = \frac{v^2}{r}$$

- This acceleration changes the direction of the velocity.
- The direction of this acceleration is always towards the center of the circle.
- If the object has mass m, then the force that is responsible for this acceleration is

$$F_r = \frac{mv^2}{r}$$

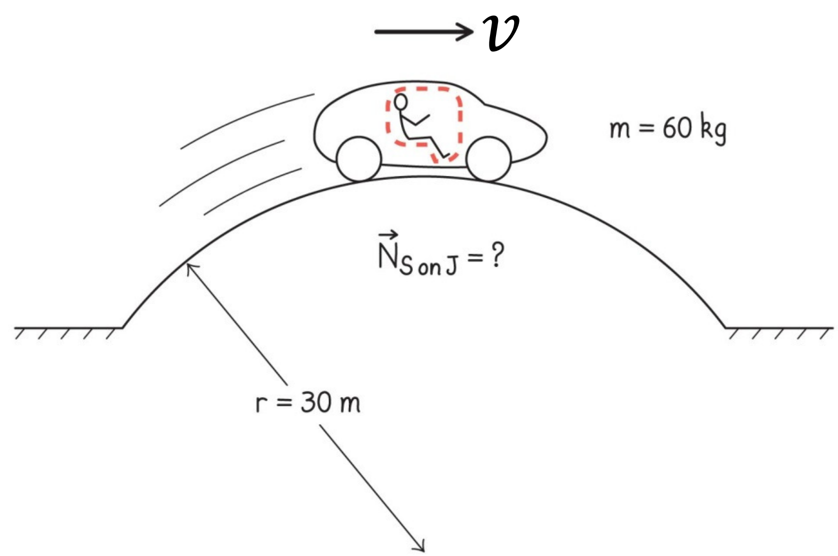
and points towards the center of the circle.

Tip for circular motion

- There is no special force that causes the radial acceleration of an object moving at constant speed along a circular path.
- This acceleration is caused by all of the forces exerted on the system object by other objects.
- Add the radial components of these regular forces.
- This sum is what causes the radial acceleration of the system object.

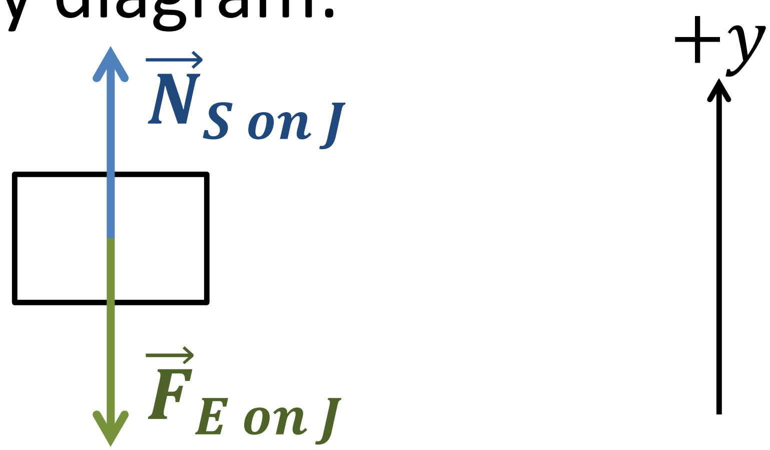
Example

- A certain road, on the outskirts of Lafayette, has a bump in it that is approximately circular with a radius of curvature of $r = 30\ m$.
- What speed does Jacqueline have to be driving in order to feel weightless?
- "Weightless" means that the normal force vanishes.



Example

- Draw the free-body diagram:



- Net force:

$$\sum F_y = N_{S\ on\ J,y} + F_{E\ on\ J,y} = N_{S\ on\ J,y} - mg = -\frac{mv^2}{r}$$

- When $N_{S\ on\ J,y} = 0$, $v^2 = gr$

$$v = \sqrt{mgr} = \sqrt{(9.8\ m/s^2)(30\ m)}$$
$$= 17.1\ m/s = 38.4\ \text{mph}$$

Planetary Motion

- Newton was among the first to hypothesize that the Moon moves in a circular orbit around Earth because Earth pulls on it, continuously changing the direction of the Moon's velocity.
- He wondered if the force exerted by Earth on the Moon was the same type of force that Earth exerted on falling objects, such as an apple falling from a tree.
- Newton compared the acceleration of the Moon if it could be modeled as a point particle near Earth's surface to the acceleration of the moon observed in its orbit…

Motion of the Moon

$$a = v^2/r$$

- The orbit of the moon is approximately circular with $r = 3.8 \times 10^8\ m = 60\ R_E$.
- The period of rotation is $T = 27.3$ days.
- The speed of the moon around the earth is

$$v = \frac{2\pi r}{T}$$

- The acceleration is therefore

$$a = \frac{4\pi^2 r}{T^2}$$

Motion of the Moon

$$a_{r \text{ at } R=60R_E} = \frac{4\pi^2 R}{T^2} = \frac{4\pi^2(3.8 \times 10^8 \text{ m})}{[(27.3 \text{ days})(86{,}400 \text{ s/day})]^2}$$

$$= \frac{4\pi^2(3.8 \times 10^8 \text{ m})}{(2.36 \times 10^6 \text{ s})^2} = 2.69 \times 10^{-3} \text{ m/s}^2$$

$$\frac{a_{r \text{ at } R=60R_E}}{a_{r \text{ at } R=1R_E}} = \frac{F_{\text{E on Moon at } R=60R_E}/m_{\text{Moon}}}{F_{\text{E on Moon at } R=1R_E}/m_{\text{Moon}}} = \frac{F_{\text{E on Moon at } R=60R_E}}{F_{\text{E on Moon at } R=1R_E}}$$

$$\frac{F_{\text{E on Moon at } R=60R_E}}{F_{\text{E on Moon at } R=1R_E}} = \frac{a_{r \text{ at } R=60R_E}}{a_{r \text{ at } R=1R_E}} = \frac{2.69 \times 10^{-3} \text{ m/s}^2}{9.8 \text{ m/s}^2} = \frac{1}{3600} = \frac{1}{60^2}$$

$$\Rightarrow F_{\text{E on Moon at } R=60R_E} = \frac{1}{60^2} F_{\text{E on Moon at } R=1R_E}$$

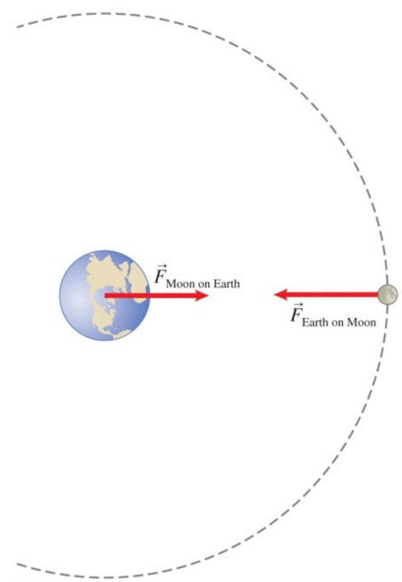
$$F_{\text{E on Moon at } r} \propto \frac{1}{r^2}$$

Motion of the Moon

- You might wonder why if Earth pulls on the Moon, the Moon does not come closer to Earth in the same way that an apple falls from a tree.
- The difference in these two cases is the speed of the objects. The apple is at rest with respect to Earth before it leaves the tree, and the Moon is moving tangentially.
- If Earth stopped pulling on the Moon, it would fly away along a straight line

Dependence of Gravitational Force on Mass

- Newton deduced $F_{E\ on\ M} \propto m_{Moon}$ by recognizing that acceleration would equal g only if the gravitational force was directly proportional to the system object's mass.
- Newton deduced $F_{E\ on\ M} \propto m_{Earth}$ by applying the third law of motion.



The Law of Universal Gravitation

- Newton deduced

$$F_{E\ on\ M} = F_{M\ on\ E} \propto \frac{m_{Earth} m_{Moon}}{r^2}$$

but did not know the proportionality constant

- In fact, at that time the mass of the Moon and Earth were unknown.
- Later scientists determined the constant of proportionality:

$$F_{g\ 1\ on\ 2} = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11}\ N \cdot m^2/kg^2$$

- *G* is the universal gravitational constant, indicating that this law works anywhere in the universe for any two masses.

The Universal Gravitational Constant

- *G* is very small.

$$G = 6.67 \times 10^{-11}\, N \cdot m^2/(kg^2)$$
$$= 6.67 \times 10^{-11}\, m^3/s^2\, /kg$$

- For two objects of mass 1 kg that are separated by 1 m, the gravitational force they exert on each other equals 6.67 x 10^{-11} N.
- The gravitational force between everyday objects is small enough to ignore in most calculations.

Newton’s Law of Universal Gravitation

Newton’s law of universal gravitation The magnitude of the attractive gravitational force that an object with mass m_1 exerts on an object with mass m_2 separated by a center-to-center distance r is:

$$F_{g\,1\,\mathrm{on}\,2} = G\frac{m_1 m_2}{r^2} \tag{4.14}$$

where $G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$ is known as the gravitational constant.

Making sense of the gravitational force that everyday objects exert on Earth

- It might seem counterintuitive that objects exert a gravitational force on Earth; Earth does not seem to react every time someone drops something.
- Acceleration is force divided by mass, and the mass of Earth is very large, so Earth's acceleration is very small.
- For example, the acceleration caused by the gravitational force a tennis ball exerts on Earth is 9.2×10^{-26} m/s^2.

Free-fall Acceleration

- We can now understand why free-fall acceleration on Earth equals 9.8 m/s^2.

$$a_{\text{object}} = \frac{F_{\text{Earth on object}}}{m_{\text{object}}} = \frac{(GM_{\text{Earth}} m_{\text{object}})/R_{\text{Earth}}^2}{m_{\text{object}}} = \frac{GM_{\text{Earth}}}{R_{\text{Earth}}^2}$$

Inserting Earth's mass $M_{\text{Earth}} = 5.97 \times 10^{24}$ kg and its radius $R_{\text{Earth}} = 6.37 \times 10^{6}$ m, we have

$$\begin{aligned} a_{\text{object}} &= \frac{GM_{\text{Earth}}}{R_{\text{Earth}}^2} \\ &= \frac{(6.67 \times 10^{-11}\,\text{N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24}\,\text{kg})}{(6.37 \times 10^{6}\,\text{m})^2} = 9.8\,\text{N/kg} = 9.8\,\text{m/s}^2 \end{aligned}$$

- This agrees with what we measure, providing a consistency check that serves the purpose of a testing experiment.

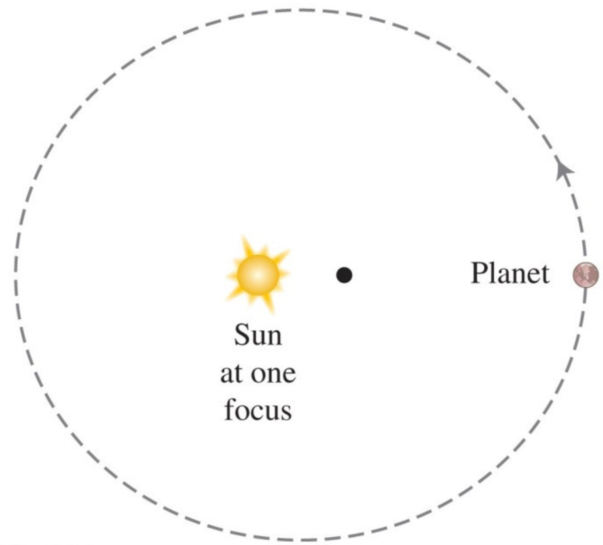
Kepler's Laws and the Law of Universal Gravitation

- About 50 years before Newton's work, Johannes Kepler (1571–1630) crafted his three laws of planetary motion.
- Newton used the law of universal gravitation to explain Kepler's laws.
- This work contributed to scientists' confidence in the law of universal gravitation.

A contemporary of William Shakespeare, 1564-1616. You can tell by his awesome ruff.

Kepler's First Law of Planetary Motion

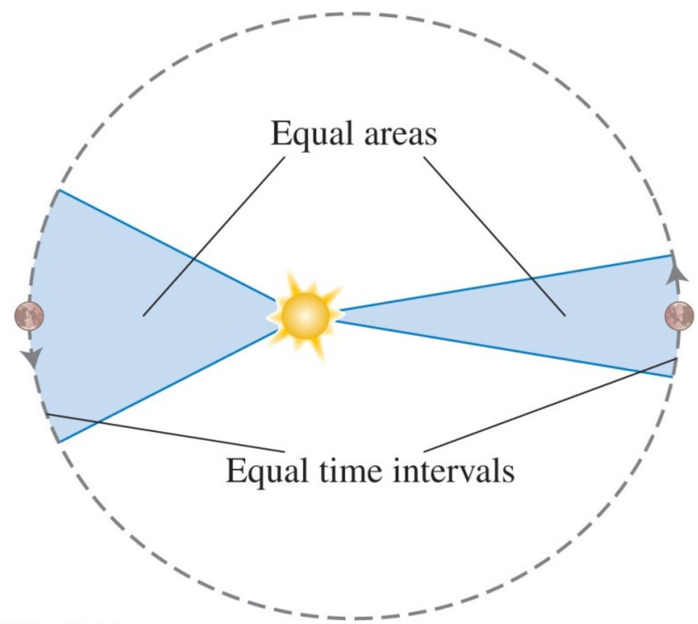
- The orbits of all planets are ellipses, with the Sun located at one of the ellipse's foci.



- The shape of the planetary orbits is close to circular for most planets. In those cases, the foci of the ellipse are very close to the center of the circular orbit that most closely approximates the ellipse.

Kepler's Second Law of Planetary Motion

- When a planet travels in an orbit, an imaginary line connecting the planet and the Sun continually sweeps out the same area during the same time interval, independent of the planet's distance from the Sun.



Kepler's Third Law of Planetary Motion

- The square of the period T of the planet's motion (the time interval to complete one orbit) divided by the cube of the semi-major axis of the orbit (which is half the maximum diameter of the elliptical orbit and the radius r of a circle) equals the same constant for all the known planets:

$$\frac{T^2}{r^3} = K$$

Gravitational and Inertial Mass

- We see that mass enters the equations of motion in two ways:

$$\sum F_x = \boldsymbol{m_i} a_x \quad \text{(inertial mass)}$$

$$F_{E\ on\ m} = G\frac{M\boldsymbol{m_g}}{r^2} \quad \text{(gravitational mass)}$$

- Electrostatic forces also act at a distance:

$$F_{Q\ on\ m} = k\frac{Qq}{r^2} \quad (Q \text{ and } \boldsymbol{q} \text{ are electric charges})$$

- Why on earth is it that $\boldsymbol{m_i} \equiv \boldsymbol{m_g}$???
- Einstein's theory of General Relativity provides the answer.

Limitations of the Law of Universal Gravitation

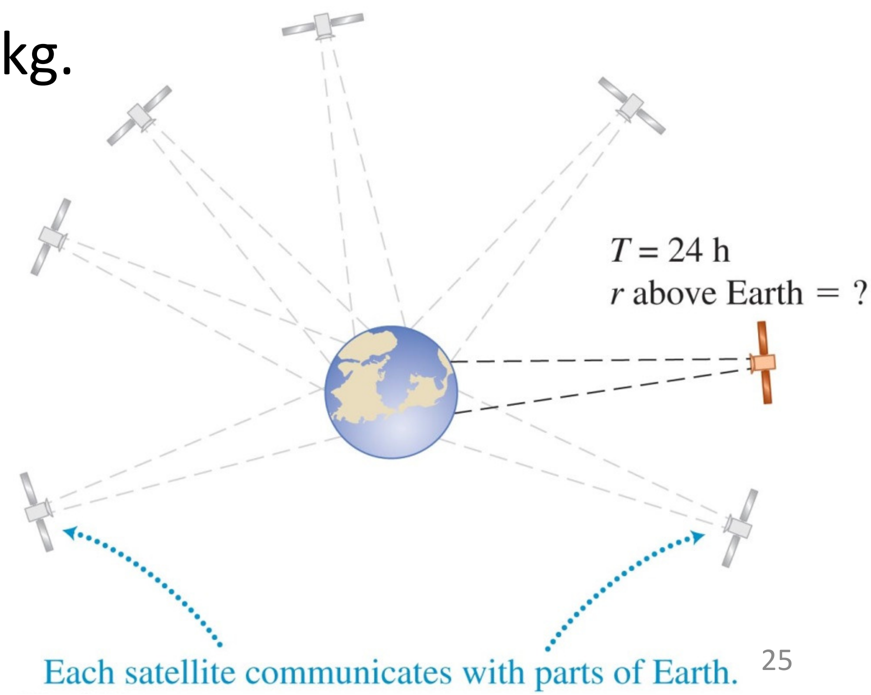
- Even with careful use of calculus, the Newton’s Law of Universal Gravitation cannot account for the details of some motions.
- When astronomers made careful observations of the orbit of Mercury, they noticed that its orbit exhibited some patterns that the law of universal gravitation could not explain.
- It was not until the early 20th century, when Einstein constructed a more advanced theory of gravity, that scientists could predict all of the details of the motion of Mercury.

Geostationary Satellites

- Geostationary satellites stay at the same location in the sky. This is why a satellite dish always points in the same direction.
- These satellites must be placed at a specific altitude that allows the satellite to travel once around Earth in exactly 24 hours while remaining above the equator.
- An array of such satellites can provide communications to all parts of Earth.

Example 4.8: Geostationary satellite

- You are in charge of launching a geostationary satellite into orbit.
- At which altitude above the equator must the satellite orbit be to provide continuous communication to a stationary dish antenna on Earth?
- The mass of Earth is 5.98×10^{24} kg.



Geostationary Satellite

- The *only* force acting on the satellite is gravity

$$F_r = ma_r = G\frac{M_{Earth}m}{r^2} = m\frac{v^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$G\frac{M_{Earth}}{r^2} = \frac{4\pi^2 r}{T^2}$$

$$r^3 = G\frac{M_{Earth}T^2}{4\pi^2}$$

$$r = \sqrt[3]{G\frac{M_{Earth}T^2}{4\pi^2}}$$

$$= \sqrt[3]{\frac{(6.67\times 10^{-11}\ m^3/s^2\ /kg)(5.98\times 10^{24} kg)(86{,}400\ s)^2}{4\pi^2}}$$

$$= 4.23\times 10^7\ m = 42{,}300\ km$$

$$R_E = 6{,}400\ km$$

Altitude of the orbit is $h = r - R_E = 42{,}300\ km\ - 6{,}400\ km = 35{,}900\ km$

Are Astronauts Weightless in the ISS?

- The International Space Station orbits approximately 0.50×10^6 m above Earth's surface, or 6.78×10^6 m from Earth's center.
- Compare the force that Earth exerts on an astronaut in the station to the force that Earth exerts on the same astronaut when he is on Earth's surface.

Astronauts are NOT weightless in the ISS

- Earth exerts a gravitational force on them!
  - This force causes the astronaut and space station to fall toward Earth at the same rate while they fly forward, staying on the same circular path.
- The astronaut is in free fall (as is the station).
  - If the astronaut stood on a scale in the space station, the scale would read zero even though the gravitational force is nonzero.
- Weight is a way of referring to the gravitational force, not the reading of a scale