Physics 22000

General Physics

Lecture 6 – Projectile Motion

Fall 2016 Semester

Prof. Matthew Jones

SUPPLEMENTAL INSTRUCTION

Free Study Sessions!

Rachel Hoagburg

Come to SI for more help in PHYS 220

Tuesday and Thursday	7:30-8:30PM Shreve C113

Office Hour

Tuesday 1:30-2:30 4th floor of Krach

For other SI-linked courses and schedules, visit purdue.edu/si or purdue.edu/boilerguide

Review of Chapter 3

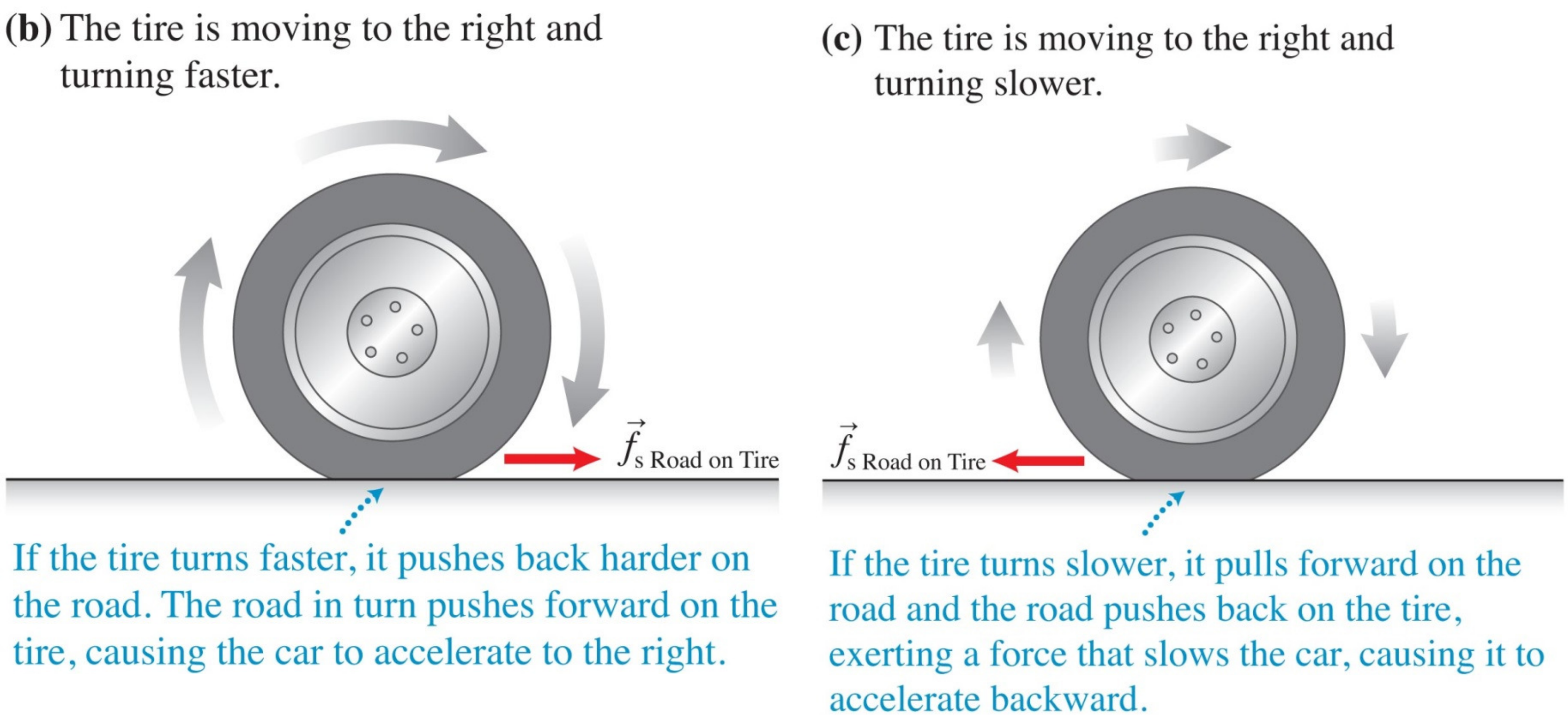
- So far we learned how to add forces as vectors to calculate the net force on an object
  - We used free-body diagrams
  - We added the components of each vector in the x- and y-directions
- We considered several types of forces
  - Force of the earth on an object (gravity)
  - Tension in strings
  - Static and kinetic friction
- So far we still only considered linear motion
  - Motion was constrained to a straight line

Using Newton's laws to explain how static friction helps a car start and stop

- Increasing or decreasing a car's speed involves static friction between the tire's region of contact and the pavement.
  - To move faster, turn the tire faster. The tire then pushes back harder on the pavement, and the pavement pulls forward more on the tire (Newton's third law).

Using Newton's laws to explain how static friction helps a car start and stop *(Cont'd)*

– To slow down, turn the tire more slowly. The tire pulls forward on the pavement, which in turn pushes back on the tire to accelerate the car backward.

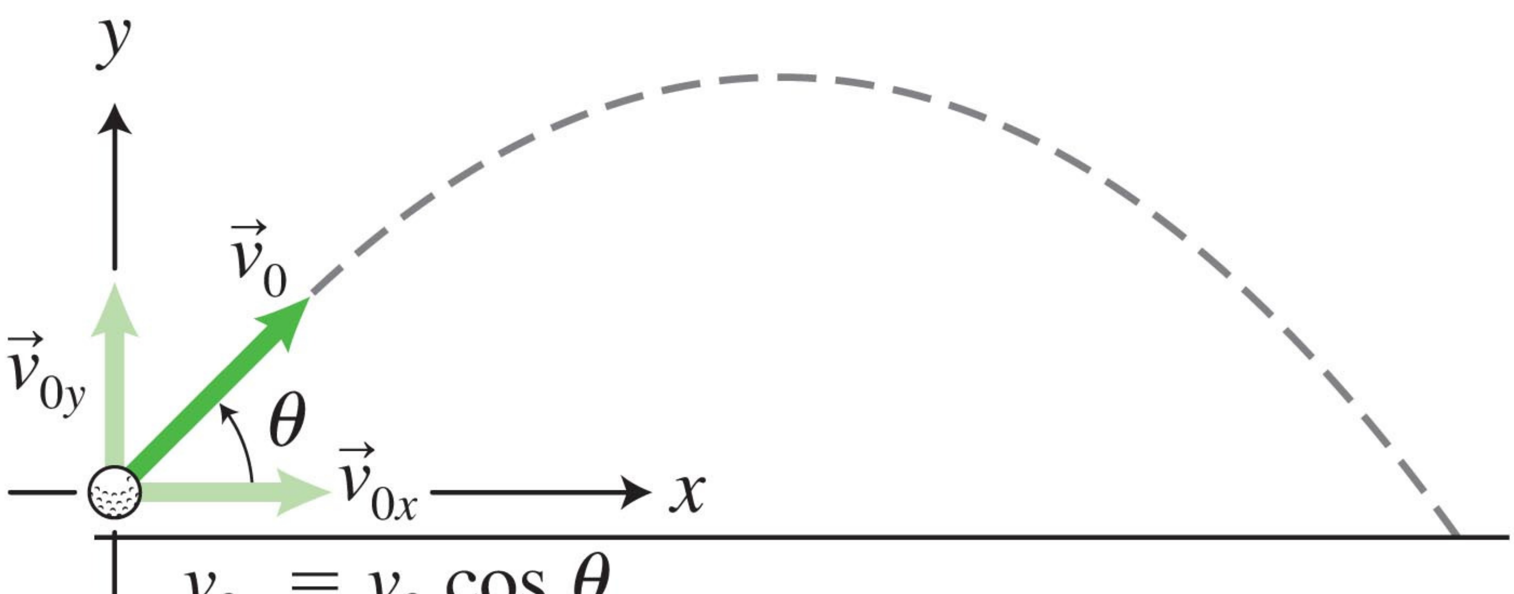


Tip

- Some people think a car's engine exerts a force on the car that starts the car's motion and helps it maintain a constant speed despite air resistance. In fact, the forces the engine exerts on other parts of the car are internal forces. Only external forces exerted by objects in the environment can affect the car's acceleration. The engine rotates the wheels, and the wheels push forward or backward on the ground. It is the ground (an external object) that pushes backward or forward on the wheels, causing the car to slow down or speed up. **The force responsible for this backward or forward push is the static friction force that the road exerts on the tires.**

Projectile Motion

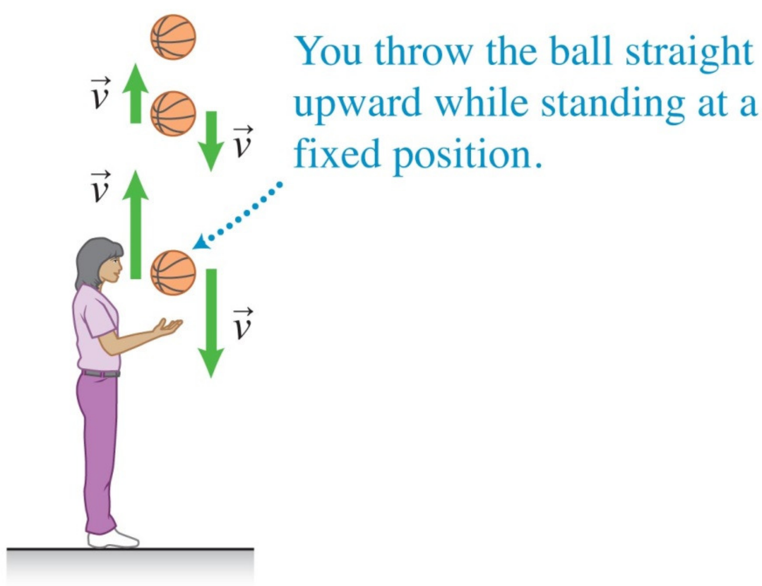
- Projectiles are objects launched at an angle with respect to some horizontal surface.

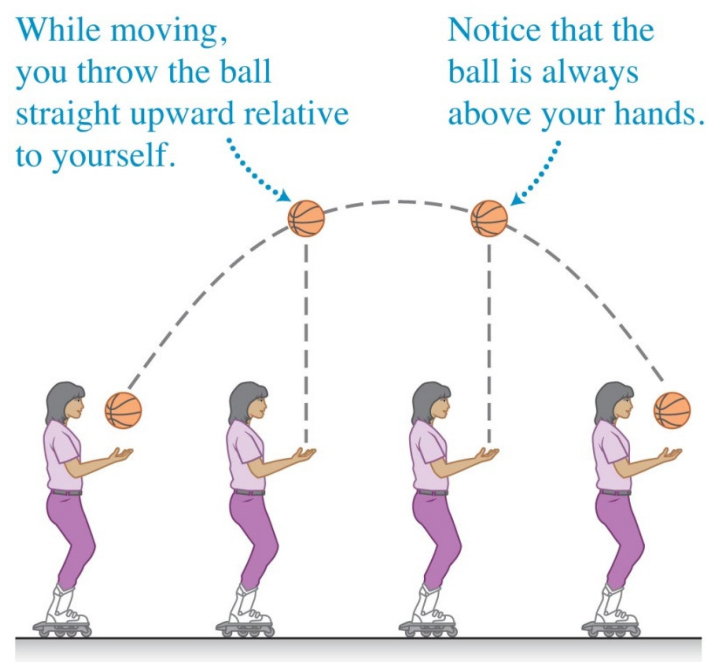


$$v_{0x} = v_0 \cos\theta$$
$$v_{0y} = v_0 \sin\theta$$

Creating Projectile Motion

- Throw the ball straight up while moving on rollerblades.
  - As long as you do not change your speed or direction while the ball is in flight, it will land back in your hands.





Qualitative analysis of projectile motion in the *y*-axis

- A ball thrown straight up in the air by a person moving horizontally on rollerblades will land back in the person's hand.
- Earth exerts a gravitational force on the ball, so its upward speed decreases until it stops at the highest point, and then its downward speed increases until it returns to your hands.
- With respect to you on the rollerblades, the ball simply moves up and down.

Qualitative analysis of projectile motion in the *x*-axis

- A ball thrown straight up in the air by a person moving horizontally on rollerblades will land back in the person's hand.
- The ball also moves horizontally.
- No object exerts a horizontal force on the ball. Thus, according to Newton's first law, the ball's horizontal velocity does not change once it is released and is the same as the person's horizontal component of velocity.

Qualitative analysis of projectile motion in the *x*- and *y*-axes

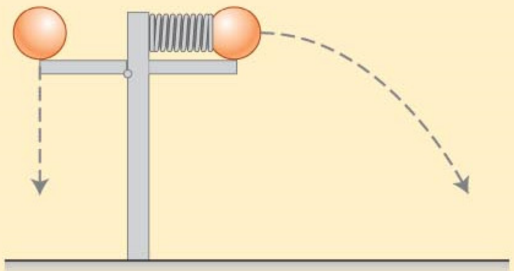
- A ball thrown straight up in the air by a person moving horizontally on rollerblades will land back in the person's hand.
- The ball continues moving horizontally as if it were not thrown upward.
- The ball moves up and down as if it does not move horizontally.
- It seems that the horizontal and vertical motions of the ball are independent of each other. **We need to test this pattern!**

Experiment to Test Independence of Motion in x- and y-directions

TESTING EXPERIMENT TABLE

3.6 Testing the independence of horizontal and vertical motions.

VIDEO 3.6

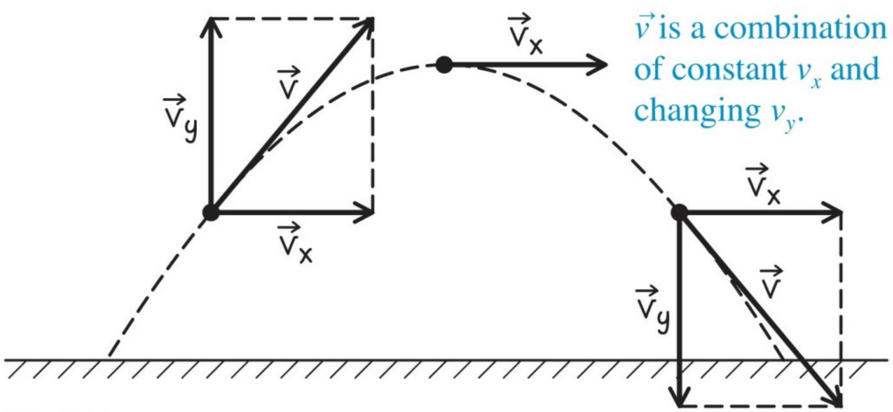
Testing experiment	Prediction	Outcome
One ball is shot horizontally when a compressed spring is released. Simultaneously, a second ball is dropped. Which ball hits the surface first? 	Both balls start with zero initial vertical speed; thus their vertical motions are identical. Since we think that the vertical motion is independent of the horizontal motion, we predict that they will land at the same time.	When we try the experiment, the balls do land at the same time.

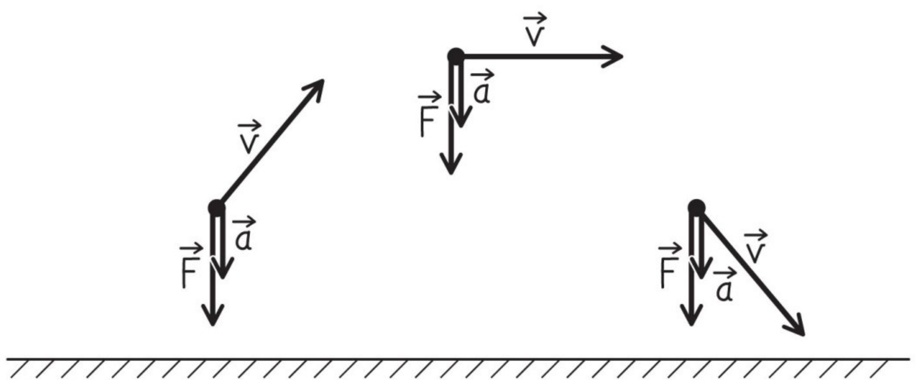
Conclusion

The outcome supports the idea of independent horizontal and vertical motions. We've failed to disprove that idea.

Conceptual Exercise: Throwing a Ball

- You throw a tennis ball as a projectile. Arrows represent the ball's instantaneous velocity and acceleration and the force or forces exerted on the ball by other objects when at the three positions shown in the diagram.



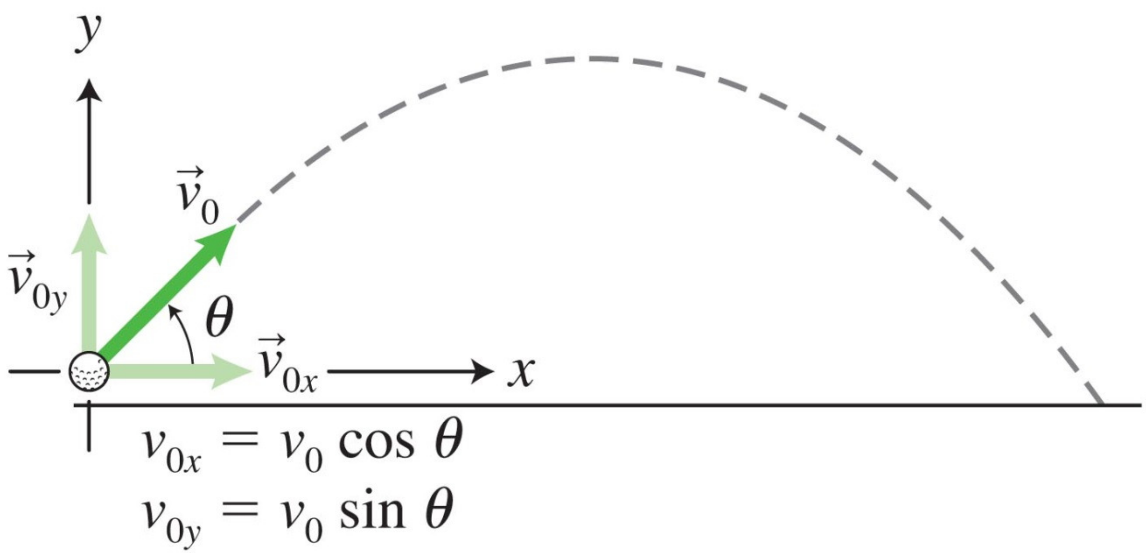


Quantitative Analysis of Projectile Motion: Acceleration

- The equations of motion for velocity and constant acceleration are used to analyze projectile motion quantitatively.
- The x-component (in the horizontal direction) of a projectile's acceleration is zero.
- The y-component (in the vertical direction) of a projectile's acceleration is $-g$.
  - The force is mg—the force of gravity that Earth exerts on the projectile.

Quantitative Analysis of Projectile Motion: Velocity

- For a projectile launched at speed v_0 at an angle θ with respect to the horizontal axis:



- Its initial x- and y- velocity components are

$$v_{0x} = v_0 \cos\theta$$
$$v_{0y} = v_0 \sin\theta$$

- The x-component of the velocity remains constant during the flight because the acceleration in the x-direction is zero.

Quantitative Analysis of Projectile Motion: Using Kinematic Equations

- Projectile motion in the x-direction:

$$(a_x = 0)$$

$$v_x = v_{0x} = v_0 \cos\theta$$

$$x = x_0 + v_{0x}t$$

$$= x_0 + (v_0 \cos\theta)t$$

- Projectile motion in the y-direction:

$$(a_y = -g)$$

$$v_y = v_{0y} + a_y t$$

$$= v_0 \sin\theta + (-g)t$$

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$$

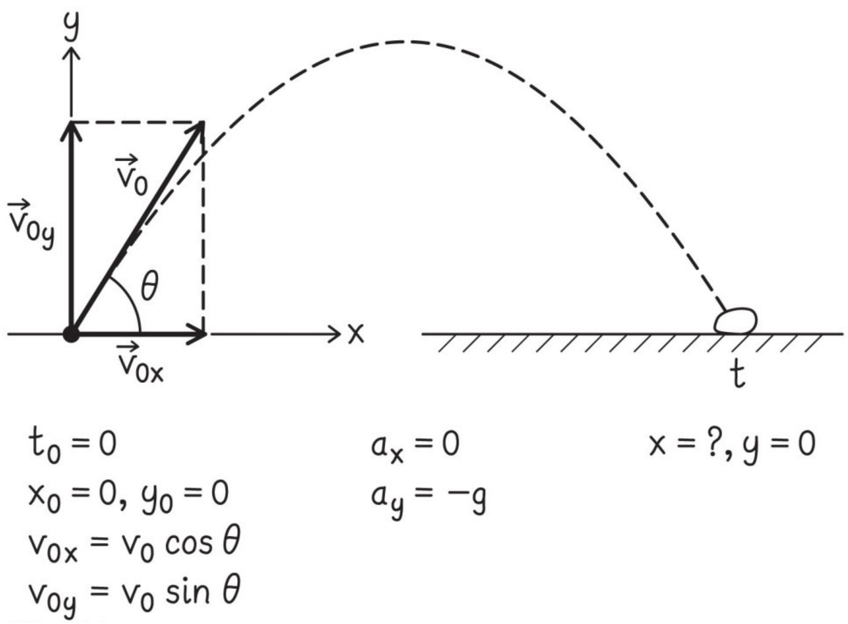
$$= y_0 + (v_0 \sin\theta)t - \frac{1}{2}gt^2$$

- The y-equations can be used to determine the time interval for the projectile's flight.
- The x-equations can be used to determine how far the projectile travels in the horizontal direction during that time interval.

Example: Best angle for farthest flight

- While rioting in down town West Lafayette, you want to throw a rock the farthest possible horizontal distance. You keep the initial speed of the rock constant and find that the horizontal distance it travels depends on the angle at which it leaves your hand.
- What is the angle at which you should throw the rock so that it travels the longest horizontal distance, assuming it is always thrown with the same initial speed?

Example: Best Angle for Farthest Flight



- Vertical position at time t:

$$y(t) = y_0 + (v_0 \sin\theta)t - \frac{1}{2}gt^2$$

- Initial height: $y_0 = 0$
- Final height: $y(t) = 0$

$$\left.\begin{array}{l} y_0 = 0 \\ y(t) = 0 \end{array}\right\} (v_0 \sin\theta)t - \frac{1}{2}gt^2 = 0$$

Example: Best Angle for Farthest Flight

$$(v_0 \sin\theta)t - \frac{1}{2}gt^2 = 0$$

- This has two solutions. One is just $t = 0$.
- The other solution will be when $t \neq 0$, so we can divide by t to obtain:

$$(v_0 \sin\theta) - \frac{1}{2}gt = 0$$

- Now solve for t:

$$t = \frac{2\, v_0 \sin\theta}{g}$$

Example: Best Angle for Farthest Flight

- Next, substitute t into equation for horizontal motion:

$$x = x_0 + (v_0\ \cos\theta)t$$

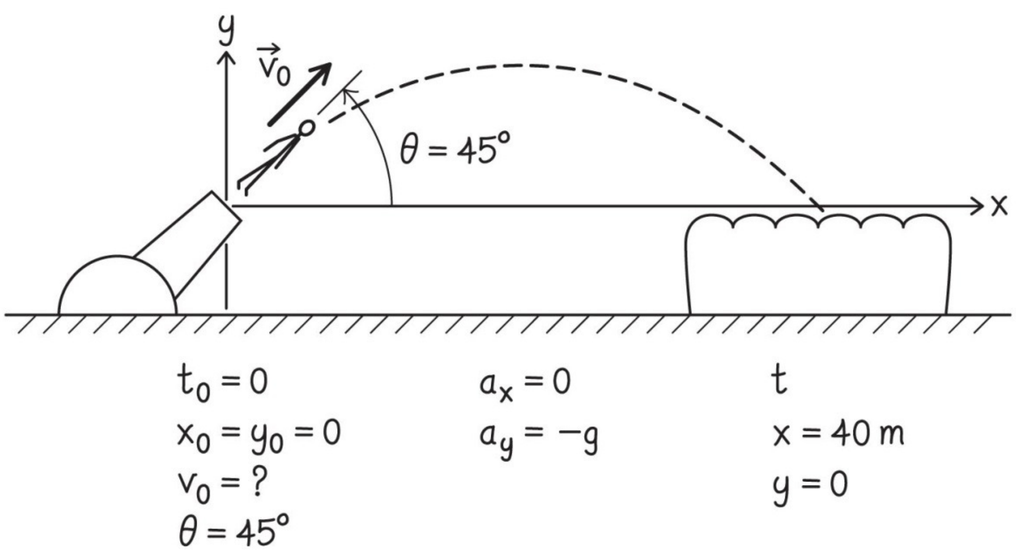
- Initial position: $x_0 = 0$
- Final position:
- Trigonometric identity (just google them):

$$2\cos\theta\sin\theta = \sin 2\theta$$

- This is maximal when $2\theta = 90°$ $(\sin 90° = 1)$
- ***Maximum distance is when*** $\boldsymbol{\theta = 45°}$

Example: Shot From a Cannon

- Super Dave Osborne is to be shot from a cannon that is oriented at $45\ °$ above the horizontal.
- What is the muzzle velocity needed for him to land in a net located $40\ m$ away at the same height as the muzzle of the cannon?



Example: Shot From a Cannon

- Remember, we already worked out that

$$x = (v_0 \ \cos\theta)\left(\frac{2\, v_0 \ \sin\theta}{g}\right)$$

- And we used the trigonometric identity:

$$2\cos\theta\sin\theta = \sin 2\theta$$

to obtain:

$$x = \frac{v_0^2 \sin 2\theta}{g}$$

- But when $\theta = 45°$, $\sin 2\theta = 1$, so

$$v_0 = \sqrt{x\ g} = \sqrt{(40\, m)(9.8\, m/s^2)}$$
$$= \mathbf{20}\, \boldsymbol{m/s} = \mathbf{71\ km/h}$$

Application: The Hippie Jump

- Examples:
  - https://youtu.be/R_C7nMs0bKA
  - https://youtu.be/mhyCCJ3EUcQ
- It is important to jump *straight up*
- If you don't, then Newton's third law says that you exert a force on the skateboard in the horizontal direction.
- This force causes you and the skateboard to accelerate in opposite horizontal directions
- The horizontal velocities are no longer equal and you will miss the skateboard when you land.