Purdue University Department of Physics

Physics 22000

General Physics

Lecture 5 – Applying Newton's Laws

Fall 2016 Semester

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Review of Lectures 1, 2 and 3

- Algebraic description of linear motion with constant acceleration:

$$\boldsymbol{x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2}$$

$$\boldsymbol{v_x(t) = v_{0x} + a_x t}$$

- Newton's Laws:
    1. In an inertial reference frame, the motion of an object remains unchanged when there is no net force acting on it.
    2. Acceleration is proportional to the net force and inversely proportional to the mass of an object.
    3. Forces come in pairs, but act on different objects.

Review of Lectures 1,2 and 3

- We can relate velocity, distance and acceleration at any point in time:

$$\mathbf{2\, a_x(x - x_0) = v_x^2 - v_{0x}^2}$$

- Average acceleration:

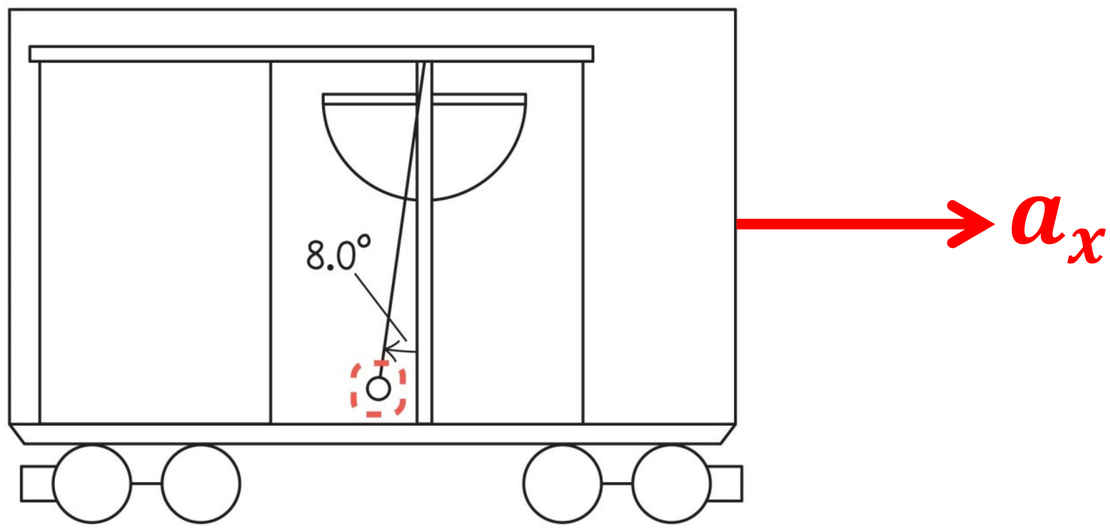
$$\mathbf{a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)}}$$

- The force acting on an object of mass $\boldsymbol{m}$ that will result in this acceleration is $\boldsymbol{F_x = m a_x}$.

Review of Lecture 4

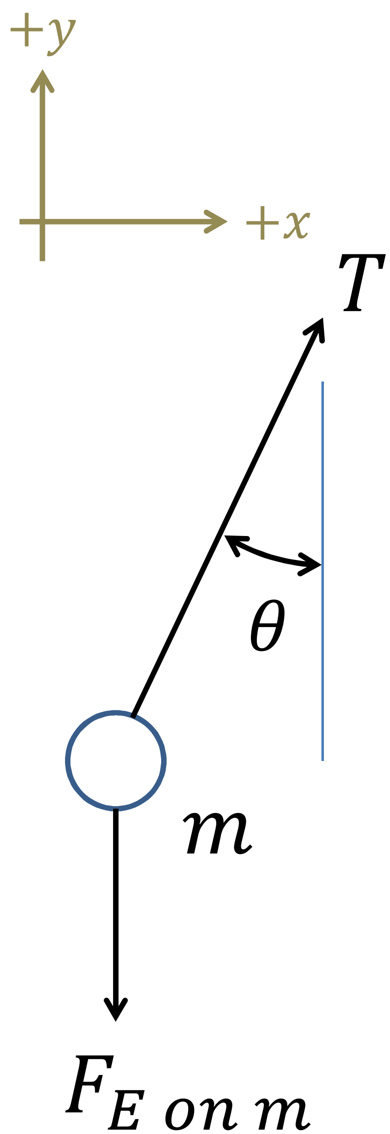
- Forces and other vectors in two dimensions
  - How to add vectors
- Free body diagrams
  - Draw all the forces acting on an object
  - Calculate the net force
  - Use $F_x = m\, a_x$ or in general, $\vec{F} = m\, \vec{a}$
- Special examples:
  - Incline plane (easier to choose x-axis to be in the direction of motion – not the horizontal direction)
  - Tension in strings

Example (just to get started)



- A train car moves with constant acceleration a_x causing a pendulum inside it to swing to an angle $\theta = 8°$ with respect to the vertical. What is the acceleration, a_x?

Free Body Diagram



- Forces in y-direction:

$$F_{E\ on\ m,y} + T\ \cos\theta = 0$$

$$T = \frac{-F_{E\ on\ m,y}}{\cos\theta}$$

$$= \frac{m\ g}{\cos\theta}$$

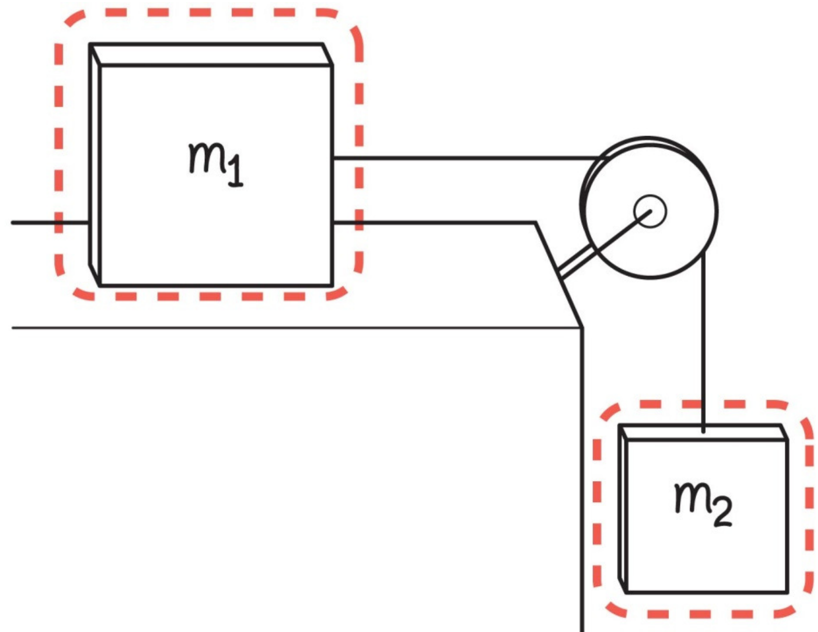
- Forces in x-direction:

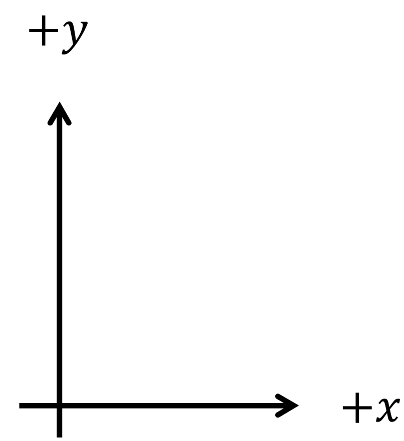
$$T\ \sin\theta = m\ a_x$$

- Acceleration:

$$a_x = \frac{T\ \sin\theta}{m} = g\ \tan\theta$$

Pulleys and Strings





- In Lecture 4 we worked it out:

$$a_1 = g\left(\frac{m_2}{m_1 + m_2}\right) = -a_2$$

- This assumed that the horizontal surface was frictionless. How do we treat friction?

Friction

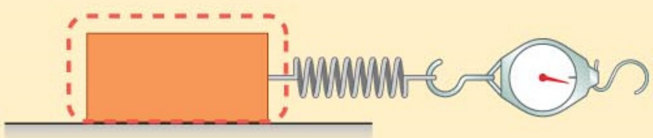
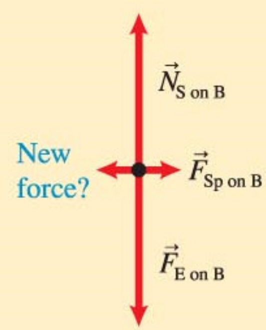
- There are two kinds of friction:
  - Static friction: the maximum horizontal force that can be applied before an object starts to move.
  - Kinetic friction: the horizontal force that acts in the direction opposite the motion of an object.
- How can we observe these forces?
  - Perform an experiment using spring scales to measure horizontal and vertical forces.

Static Friction

OBSERVATIONAL EXPERIMENT TABLE

3.1 Pulling a block with a spring scale.

VIDEO 3.1

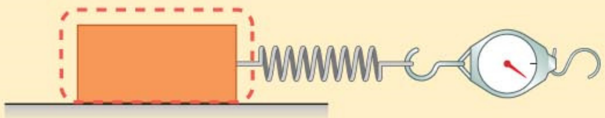
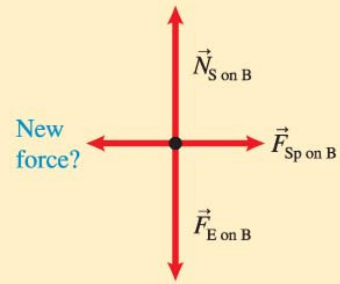
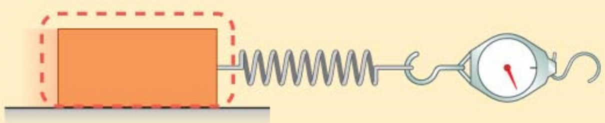
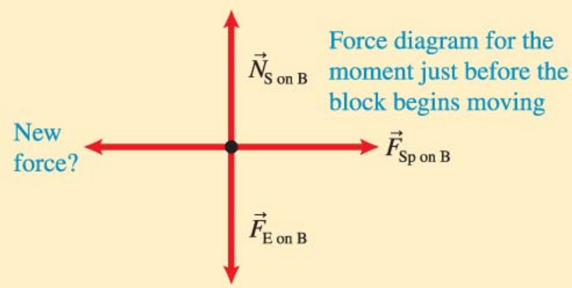
Observational experiment	Analysis
A block is at rest on the horizontal surface of a desk. 	
A spring scale pulls lightly on the block; the block does not move. 	

Static Friction

OBSERVATIONAL EXPERIMENT TABLE

3.1 Pulling a block with a spring scale. *(Continued)*

VIDEO 3.1

Observational experiment	Analysis
The spring scale pulls harder on the block; the block still does not move. 	
The spring scale pulls even harder on the block; the block finally starts moving. 	

Patterns

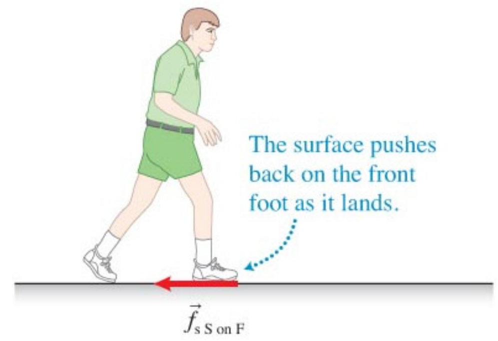
- In each of these experiments, the surface exerted a normal force on the block that balanced the downward gravitational force exerted by Earth on the block.
- As the spring scale exerted an increasing force on the block to the right, the block remained stationary (zero acceleration). The surface must have exerted an additional force—an increasing force on the block toward the left.
- Eventually, the spring scale exerted a strong enough force on the block that the block started sliding. Thus, the resistive force must have a maximum value.

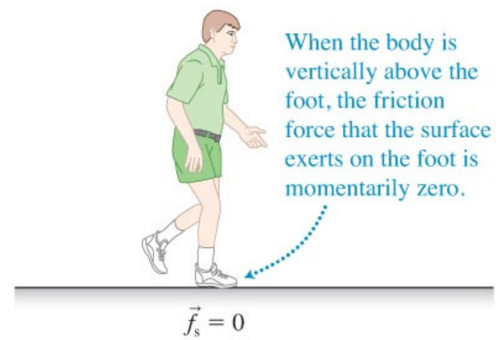
Static friction

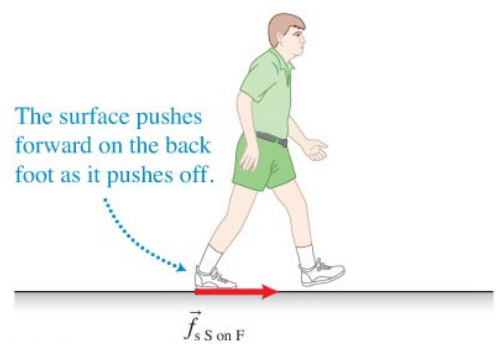
- Static friction force is parallel to the surfaces of two objects that are not moving in relation to each other and opposes the tendency of one object to move across the other.
- Static friction force changes magnitude to prevent motion, up to a maximum value called the maximum static friction force.
- When the external force exceeds this static friction force, the object starts moving.

Static friction can help us move

- Sometimes friction is a necessary phenomenon for movement—for example, when walking on a flat horizontal sidewalk.
- Static friction prevents your shoe from sliding at the start and end of a step.

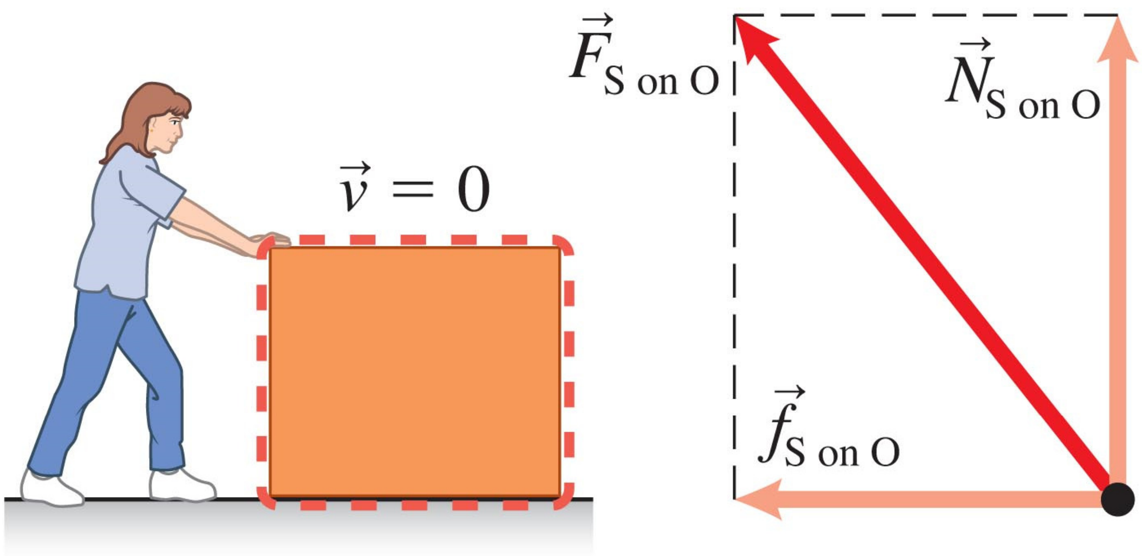






Static Friction and the Normal Force

- A surface really exerts one force, but we can analyze each component separately:



The force of the surface on the block is considered as two forces: the normal force $\vec{N}$ and the friction force $\vec{f}$.

Magnitude of Maximum Static Friction

What affects the magnitude of the maximum static friction between an object and a surface?

- How smooth is the surface? Compare a block sliding on (1) glass, (2) wood, and (3) a rubber mat.

$$\vec{f}_R > \vec{f}_W > \vec{f}_G$$

- How much surface area is in contact?
  – Maximum static friction is independent of surface area
- What is the mass of the object?

$$\vec{f}_{3kg} = 3\,\vec{f}_{1kg}$$
$$\vec{f}_{2kg} = 2\,\vec{f}_{1kg}$$

  – Maximum static friction is proportional to the normal force

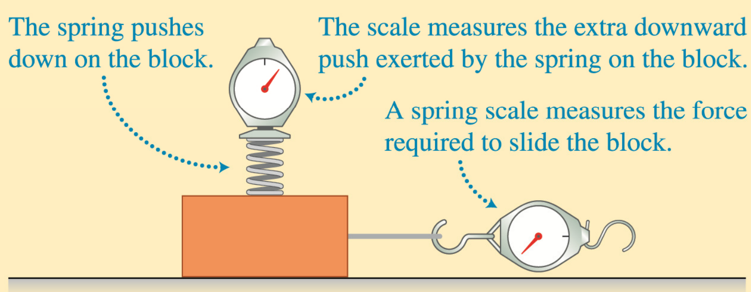
Maximum Static Frictional Force

TESTING EXPERIMENT TABLE

3.3 Does the maximum static friction force depend on mass?

VIDEO 3.3

Testing experiment	Prediction	Outcome
We use a spring attached to a spring scale to pull a 1-kg block. The mass of the block does not change, but we push down on the block with a spring that exerts a series of downward forces on it. For each of these downward forces, we use the pulling string and spring scale to determine the maximum static friction force the surface exerts on the block.	If the friction force is proportional to the mass of the block, the friction force should remain constant during the experiment.	The friction force changes—the harder we press on the block, the higher the maximum static friction force.



Conclusion

The outcome of the experiment did not match the prediction; the hypothesis requires a revision.

Measuring the Static Friction Force

- The hypothesis that the maximal static friction force depends on mass did not turn out to be correct.
- We must test the other proposed relationships.

Table 3.4 Maximum static friction force when a block presses harder against a surface.

Mass of the block	Extra downward force exerted on the 1-kg block	Normal force exerted by the surface on the block	Maximum static friction force	Ratio of maximum static friction force to normal force
1.0 kg	0.0 N	9.8 N	3.0 N	0.31
1.0 kg	5.0 N	14.8 N	4.5 N	0.30
1.0 kg	10.0 N	19.8 N	6.1 N	0.31
1.0 kg	20.0 N	29.8 N	9.1 N	0.31

- The data in the table indicate there is a constant ratio between the maximum static friction force and the normal force.

Relationship between normal force and friction force

- It makes sense that the static friction force should be proportional to the normal force.

$$\frac{f_{\text{s S on O max}}}{N_{\text{S on O}}} \approx 0.31$$

- The normal force and friction force are two perpendicular components of the same force—the force a surface exerts on an object!
  - If the force exerted by the surface on an object increases, the normal and friction forces increase proportionally.

Relationship between normal force and friction force

- The ratio of the maximum friction force to the normal force is constant in all trials.
- The proportionality constant is different for different surfaces; the proportionality depends on the types of contacting surfaces.
- The proportionality constant is greater for two rough surfaces contacting each other and smaller for smoother surfaces.

$$\mu_s = \frac{f_{s\,max}}{N}$$

- This ratio is the coefficient of static friction.

Coefficient of Static Friction

$$\mu_s = \frac{f_{s\,max}}{N}$$

- The coefficient of static friction is a measure of the relative difficulty of sliding two surfaces across each other.
- The easier it is to slide one surface on the other, the smaller the coefficient is.
- This coefficient is unit-less and typically has a value between 0 and 1.

Force of Static Friction

Static friction force When two objects are in contact and we try to pull one across the other, they exert a static friction force on each other. This force is parallel to the contacting surfaces of the two objects and opposes the tendency of one object to move across the other. The static friction force changes magnitude to prevent motion—up to a maximum value. This maximum static friction force depends on the roughness of the two surfaces (on the coefficient of static friction μ_s between the surfaces) and on the magnitude of the normal force N exerted by one surface on the other. The magnitude of the static friction force is always less than or equal to the product of these two quantities:

$$0 \leq f_s \leq \mu_s N \tag{3.5}$$

Assumptions for our static friction model

- Our equation $\mu_s = f_{s\,max}/N$ is reasonable only in situations in which the following conditions hold:
  - Relatively light objects are resting on relatively firm surfaces.
  - The objects never cause the surfaces to deform significantly (for example, they do not involve a car tire sinking into mud).

Kinetic friction

- *Kinetic* indicates that the surfaces in contact are moving relative to each other.
- A similar relationship exists as between the friction force and the normal force, but with two important differences:
  - Under the same conditions, the magnitude of the kinetic friction force is always lower than the maximum static friction force.
  - The resistive force exerted by the surface on the moving object has a constant value.

Force of Kinetic Friction

Kinetic friction force When an object slides along a surface, the surfaces exert kinetic friction forces on each other. These forces are exerted parallel to the contacting surfaces and oppose the motion of one surface relative to the other surface. The kinetic friction force depends on the surfaces themselves (on the coefficient of kinetic friction μ_k) and on the magnitude of the normal force N exerted by one surface on the other:

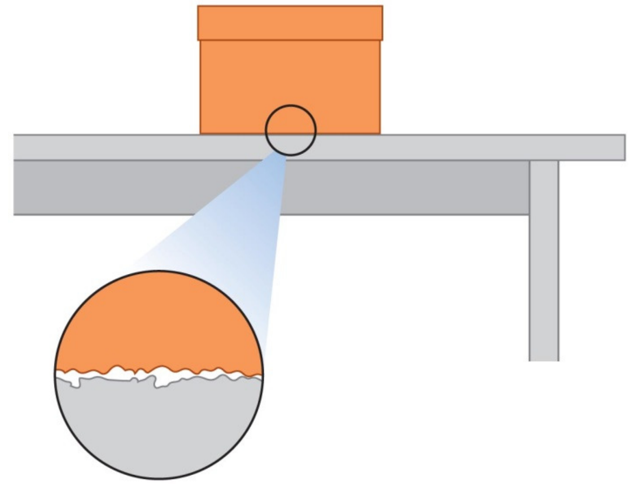
$$f_k = \mu_k N \qquad (3.6)$$

Assumptions for our kinetic friction model

- Our equation $f_k = \mu_k N$ is reasonable only in situations in which the following conditions hold:
  - It cannot be used for rolling objects.
  - It makes the same assumption about the rigidity of the surfaces as the model for static friction.
  - The objects cannot be moving at high speed.
- This equation does not have general applicability, but it is useful for rigid surfaces and objects moving at everyday speeds.

What causes friction?

- Even the slickest surfaces have bumps that can hook onto the bumps on another surface.



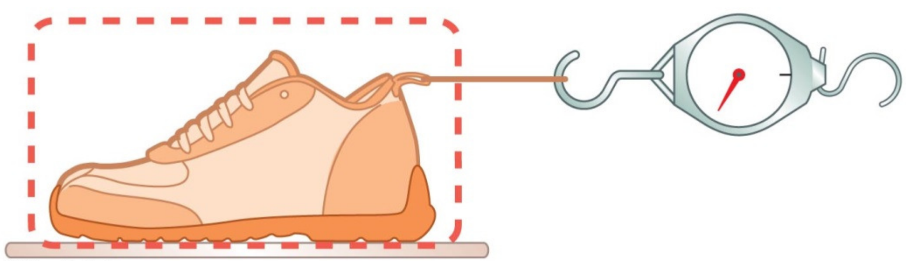
Rough edges on the contacting surfaces cause friction.

- Smoother surfaces should have reduced friction, consistent with our previous findings.
- If two surfaces are too smooth, friction increases again (for example, two pieces of plastic wrap).
  - This is due to attraction between particles at the surface that are too close together without typical surface bumpiness.

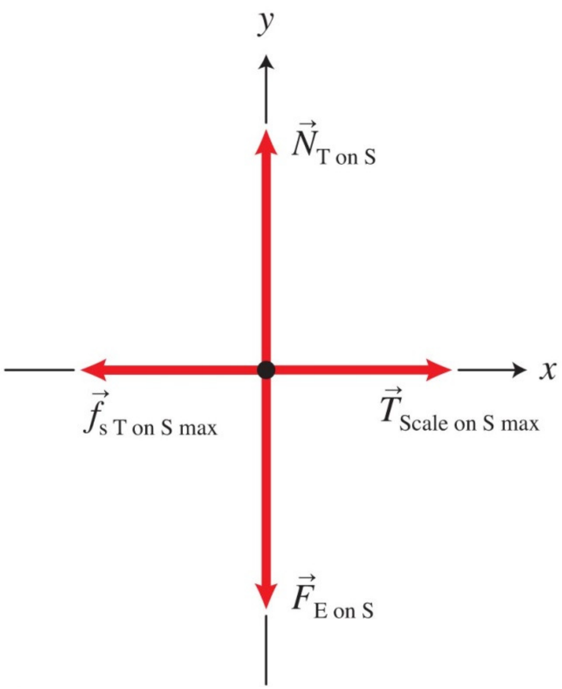
Determining friction experimentally: Experiment 1

- The shoe is pulled horizontally with a spring scale until the shoe begins to slide.

(a)



(b)



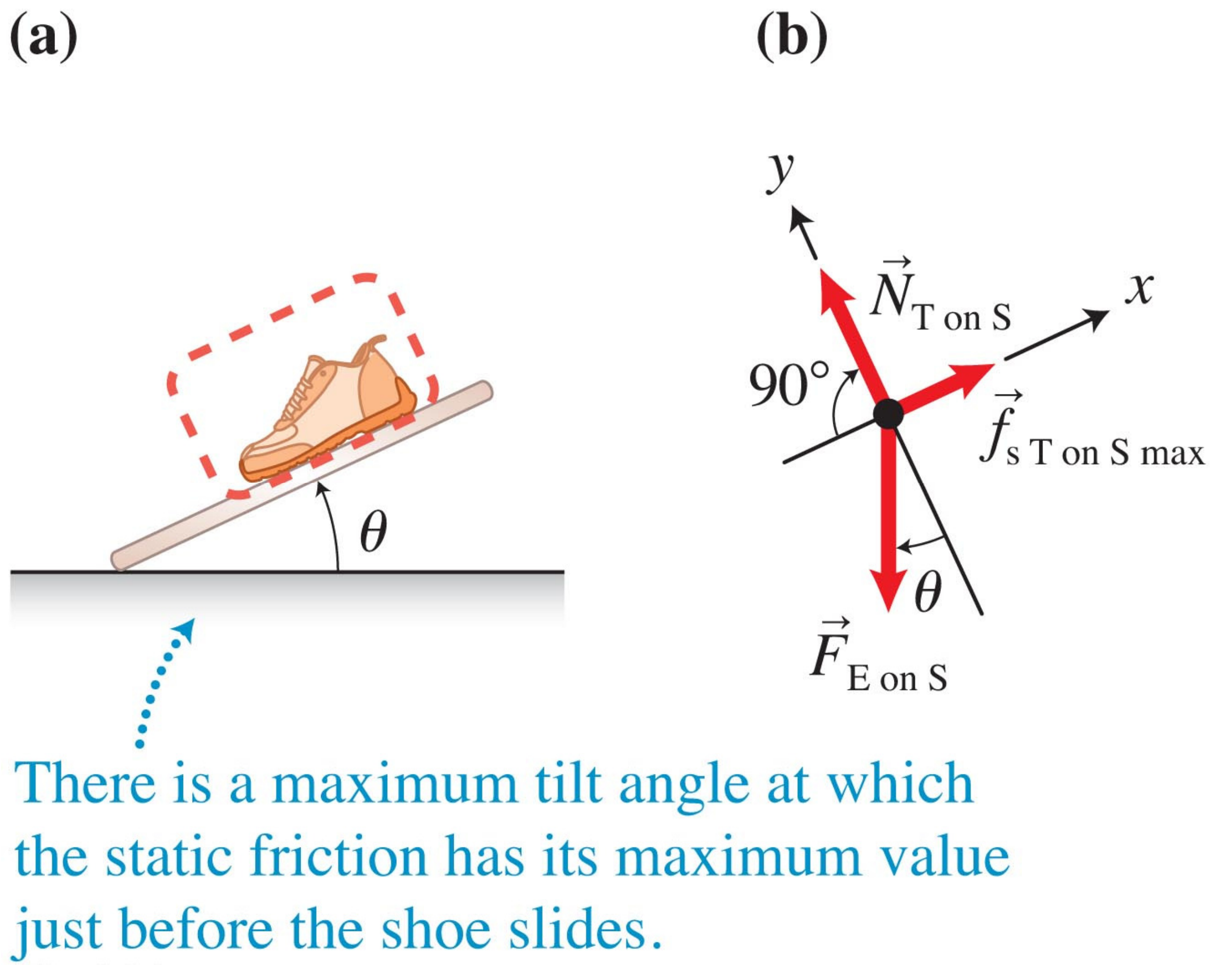
Determining friction experimentally: Experiment 1 *(Cont'd)*

- Just before the shoe starts to slide, its acceleration is zero, and the scale indicates the maximum force of static friction that the tile exerts on the shoe.

$$0 = T_{\text{Scale on S max}} - f_{\text{s T on S max}}$$

Determining friction experimentally: Experiment 2

- Place the shoe on the tile and tilt the tile until the shoe starts to slide.



Determining friction experimentally: Experiment 2 *(Cont'd)*

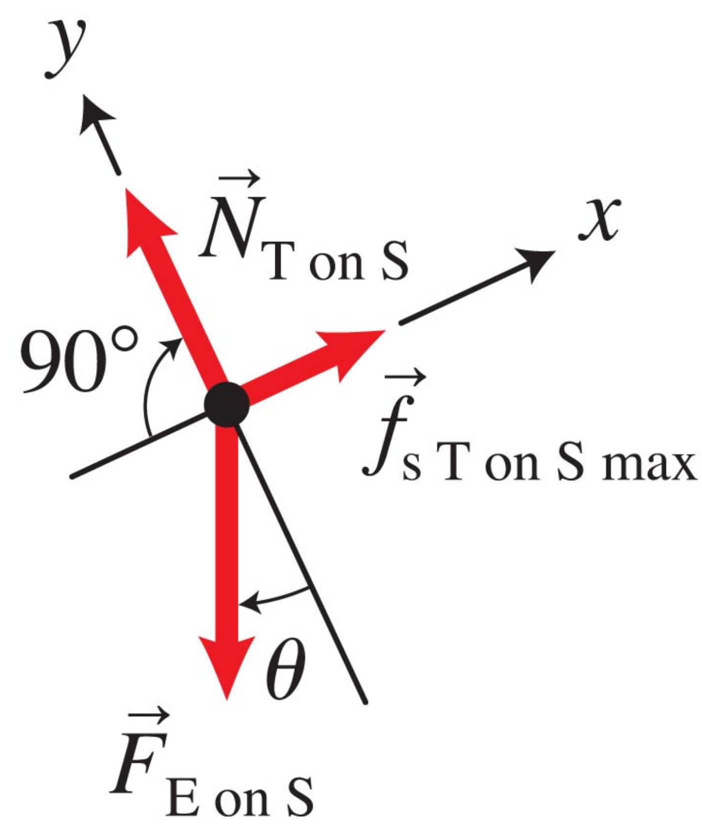
- Just before the shoe starts to slide, its acceleration is zero, and the static friction force is at its maximum.

$$y\text{-component equation:}\quad m_S \cdot 0 = N_{T\text{ on }S} \sin 90^\circ - m_S g \cos\theta + f_{s\,T\text{ on }S\,\max} \sin 0^\circ$$

$$x\text{-component equation:}\quad m_S \cdot 0 = N_{T\text{ on }S} \cos 90^\circ - m_S g \sin\theta + f_{s\,T\text{ on }S\,\max} \cos 0^\circ$$

Tip

- The magnitude of the normal force that a surface exerts on an object does not necessarily equal the magnitude of the gravitational force that Earth exerts on the object, especially when the object is on an inclined surface!



Example 3.6: Using skid marks for evidence

- If a driver slams on the brakes, the tires can lock, causing the car to skid.
- Police officers use the length of skid marks to estimate the initial speed of the vehicle.
- A car involved in a minor accident left 18.0-m skid marks on a horizontal road. After inspecting the car and road surface, the police officer decided the coefficient of kinetic friction was 0.80. The speed limit was 15.6 m/s on that street. Was the car speeding?

Example

- Relation between a, v_0 and x:

$$a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)}$$

- Newton's second law:

$$F_x = m\, a_x = \mu_k N = \mu_k\, m\, g$$

- Initial position, $x_0 = 0$; Final velocity, $v_x = 0$.

$$v_x^2 = 2\, a_x\, x = 2\, \mu_k\, g\, x$$

$$v_x = \sqrt{2\mu_k g\, x} = \sqrt{2(0.8)(9.8\, m/s^2)(18\, m)}$$

$$= 16.8\, m/s > 15.6\, m/s$$

(the car was speeding)

Other types of friction

- Rolling friction is caused by the surfaces of rolling objects indenting slightly as they turn.
  - This friction is decreased in tires that have been inflated to a higher pressure.
- In Chapter 11, we will learn about another type of friction: the friction that air or water exerts on a solid object moving through the air or water.
  - This is called a drag force.