PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 22000

General Physics

Lecture 4 – Applying Newton's Laws

Fall 2016 Semester

Prof. Matthew Jones

Review of Lectures 1, 2 and 3

- Algebraic description of linear motion with constant acceleration:

$$\boldsymbol{x(t) = x_0 + v_{0x}t + \frac{1}{2}a_x t^2}$$
$$\boldsymbol{v_x(t) = v_{0x} + a_x t}$$

- Newton's Laws:
    1. In an inertial reference frame, the motion of an object remains unchanged when there is no net force acting on it.
    2. Acceleration is proportional to the net force and inversely proportional to the mass of an object.
    3. Forces come in pairs, but act on different objects.

Review of Lectures 1,2 and 3

- We can relate velocity, distance and acceleration at any point in time:

$$\boldsymbol{2\ a_x(x - x_0) = v_x^2 - v_{0x}^2}$$

- Average acceleration:

$$\boldsymbol{a_x = \frac{v_x^2 - v_{0x}^2}{2(x - x_0)}}$$

- The force acting on an object of mass $\boldsymbol{m}$ that will result in this acceleration is $\boldsymbol{F_x = ma_x}$.

Example

- You are approaching a red light. You slow down from 50 km/h to rest over a distance of 100 m.
- If your mass is 80 kg, what force do you feel?
  - You are the system object
  - The seat and the seat belt apply the force

$$F = \frac{(80\,kg)\left(0 - \left[\frac{(50\,km/h)(1000\,m/km)}{3600\,s/h}\right]^2\right)}{2(100\,m - 0)}$$

$$= 77\,N$$

Motion and Forces in More than One Dimension

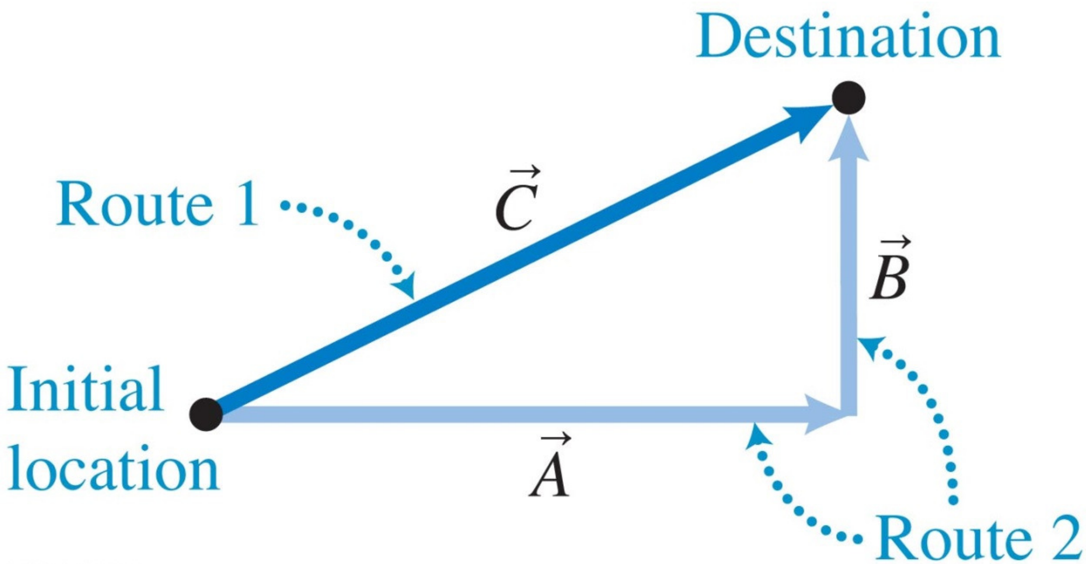
- So far we have mainly considered motion in only one dimension.
- We picked a coordinate axis (with an origin and a direction) to define the observer's reference frame.
- Newton's second law for the *components* along the x-axis:

$$a_x = \frac{\sum F_x}{m}$$

- *We can analyze motion and forces in more than one dimension by considering each component along different coordinate axes separately.*

Displacement in Two Dimensions

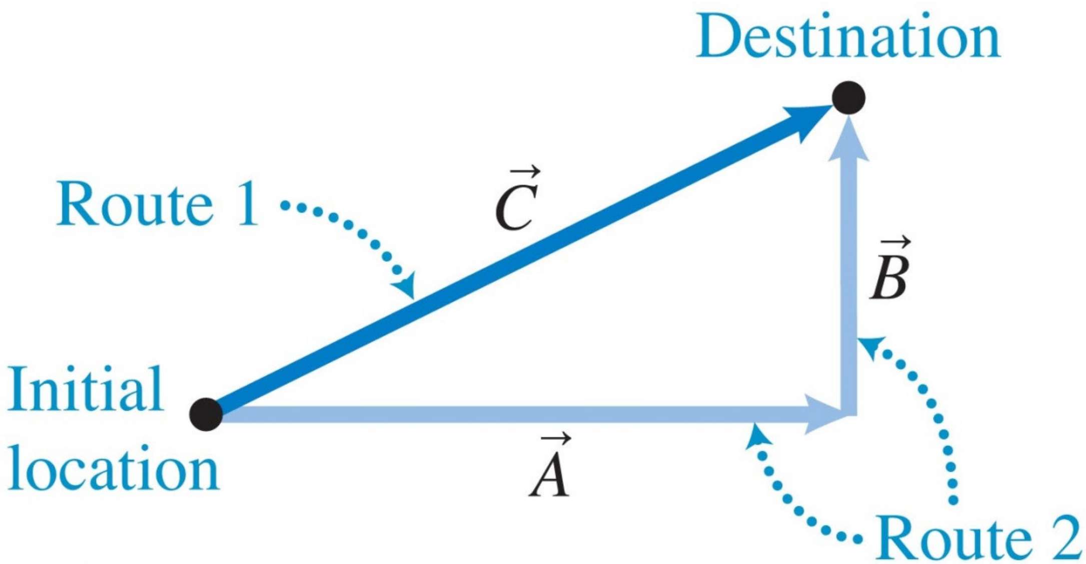
- Displacement starts from an object's initial position and ends at its final position.



- Route 1 is a direct path represented by a displacement vector.
  - Its tail represents your initial location and its head represents your final destination.

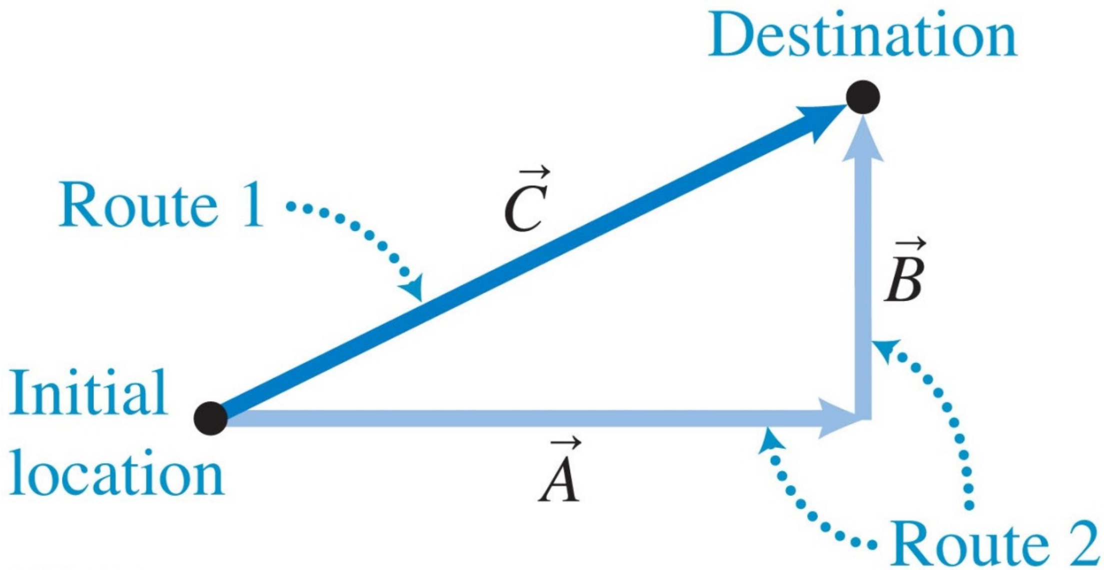
Displacement in Two Dimensions

- Displacement starts from an object's initial position and ends at its final position.



- Route 2 goes along two roads.
  - You first travel along A, then along B.
- **You end at the same final destination with either route.**

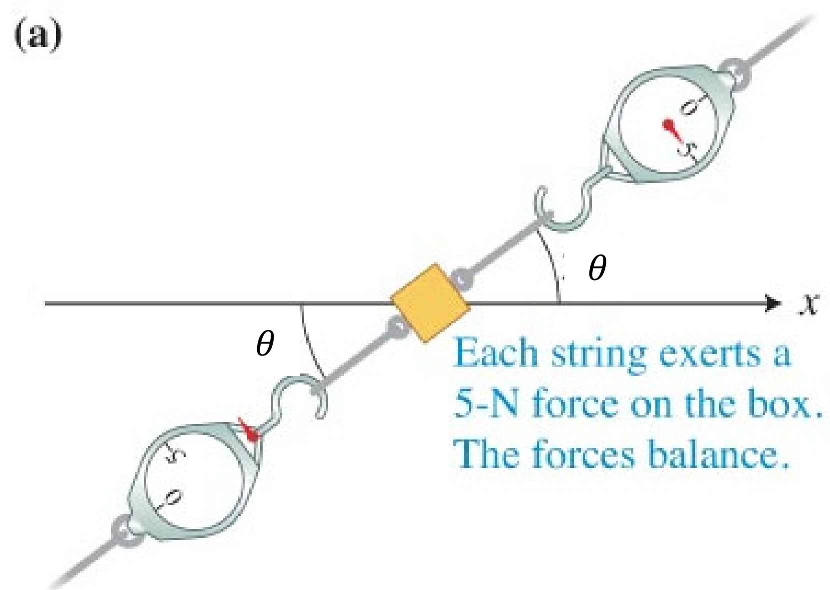
Displacement in Two Dimensions



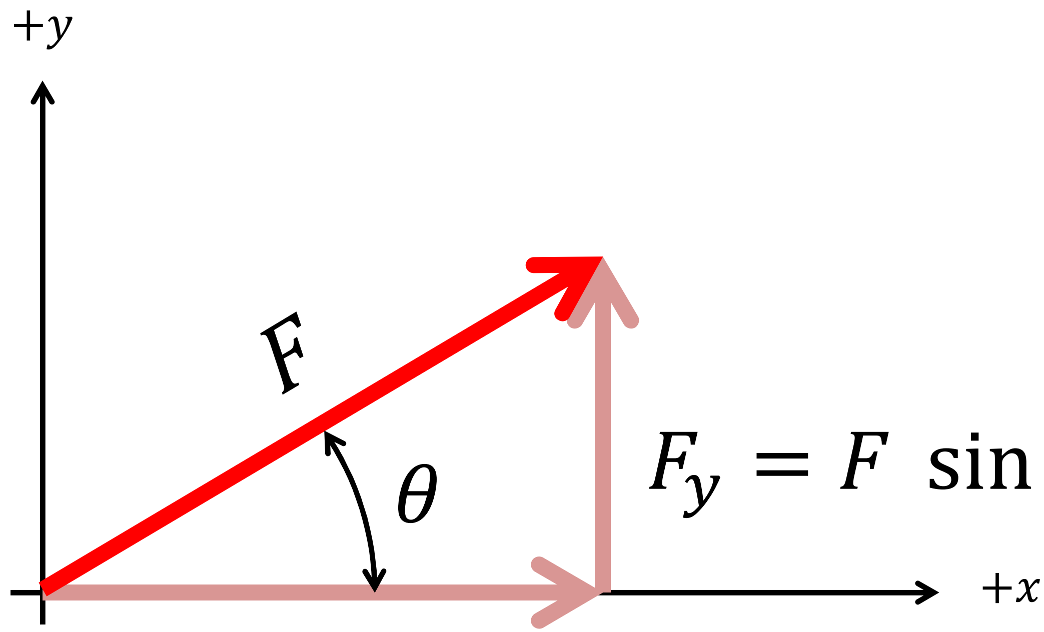
- Route 1 has the same displacement as route 2, showing how to add vectors graphically.
  - $\vec{A} + \vec{B} = \vec{C}$
  - $\vec{A}$ and $\vec{B}$ are perpendicular to each other.
- *The key to breaking a vector into its components is to pick vectors perpendicular to each other that, when added, are equivalent to the original vector.*

Forces in Two Dimensions

- A small box is being pulled by two springs scales.
- Each scale is at an angle θ with respect to the x-axis.
- How can we replace one scale with two scales that lie along the x- and y-axes?



Basic Trigonometry



$$F_y = F \sin\theta$$

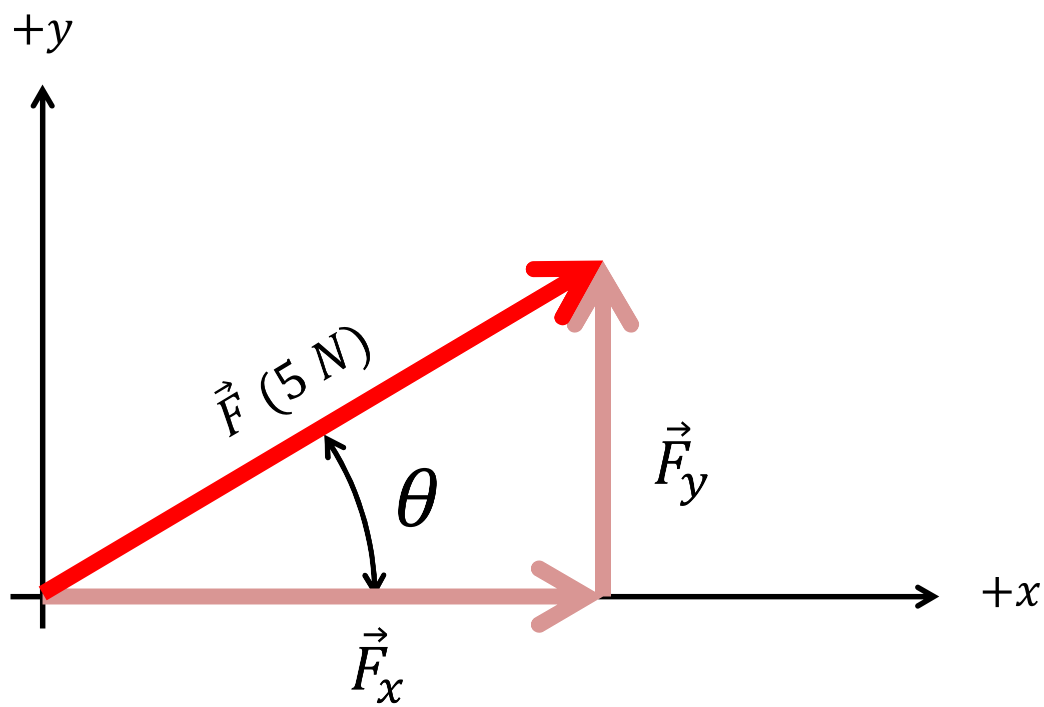
$$F_x = F \cos\theta$$

Pythagoras' Theorem:

$$F^2 = F_x^2 + F_y^2 \quad \Rightarrow \quad F = \sqrt{F_x^2 + F_y^2}$$

Forces in Two Dimensions

- We need to find the lengths of the perpendicular sides of the triangle:



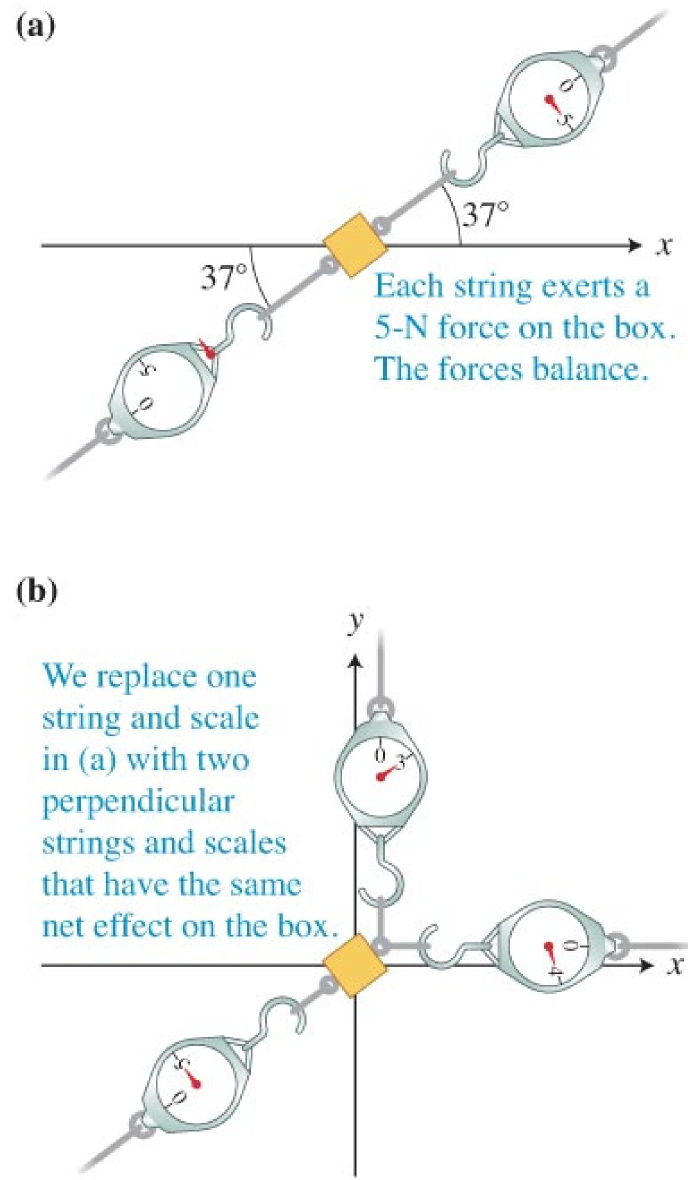
Forces in Two Dimensions

Suppose $\theta = 37°$…

$$F = 5\ N$$
$$F_x = (5\ N)\cos 37° = 4\ N$$
$$F_y = (5\ N)\sin 37° = 3\ N$$

It works out nicely because this is a 3-4-5 triangle.

The net force is the same!
(ZERO)



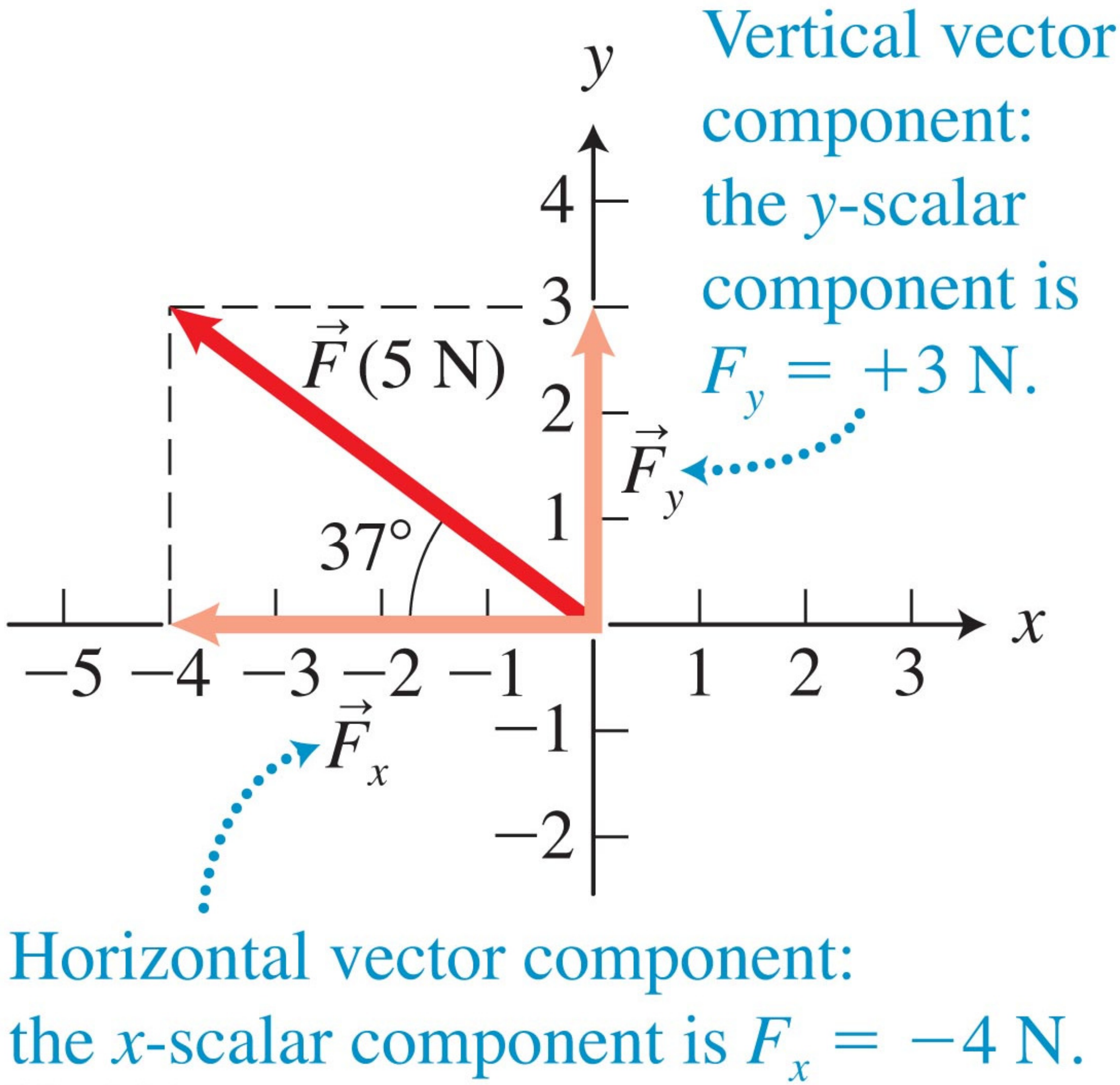
Breaking Forces into Scalar Components

- We can replace any force $\vec{F}$ into two perpendicular forces along the x- and y-axes, so long as they add graphically to equal $\vec{F}$.
- We represent these two forces by scalar components:

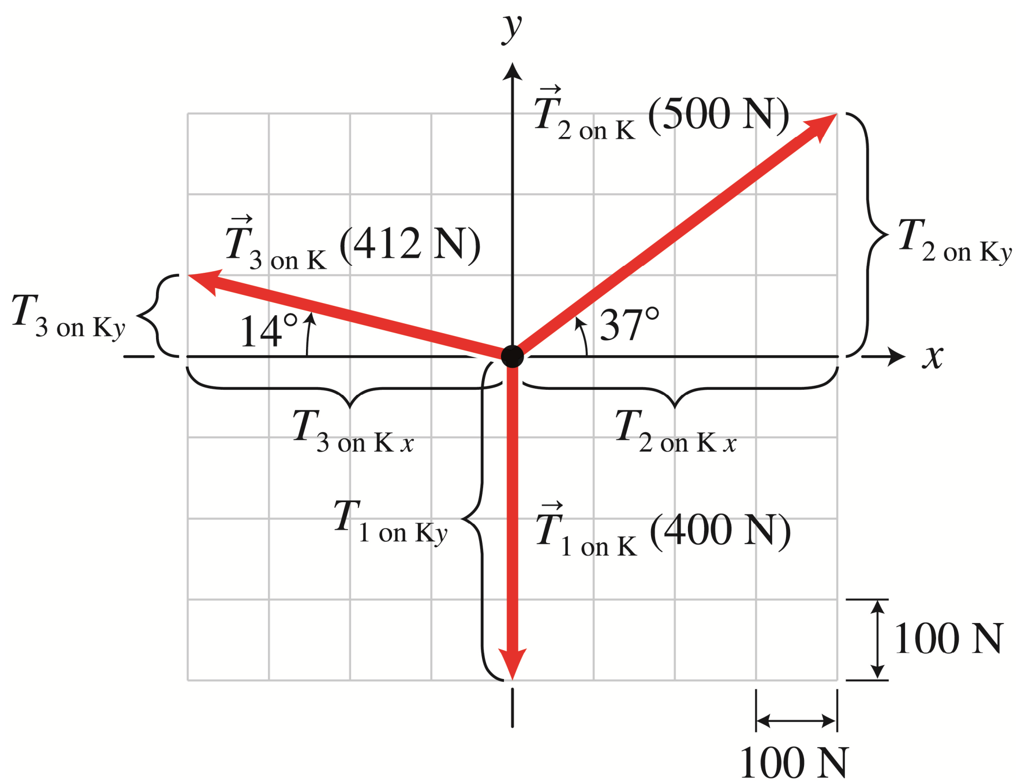
  F_x is the component along the x-axis

  F_y is the component along the y-axis
- Components can be positive or negative, depending on whether they point in the + or – direction.

Breaking Forces into Scalar Components



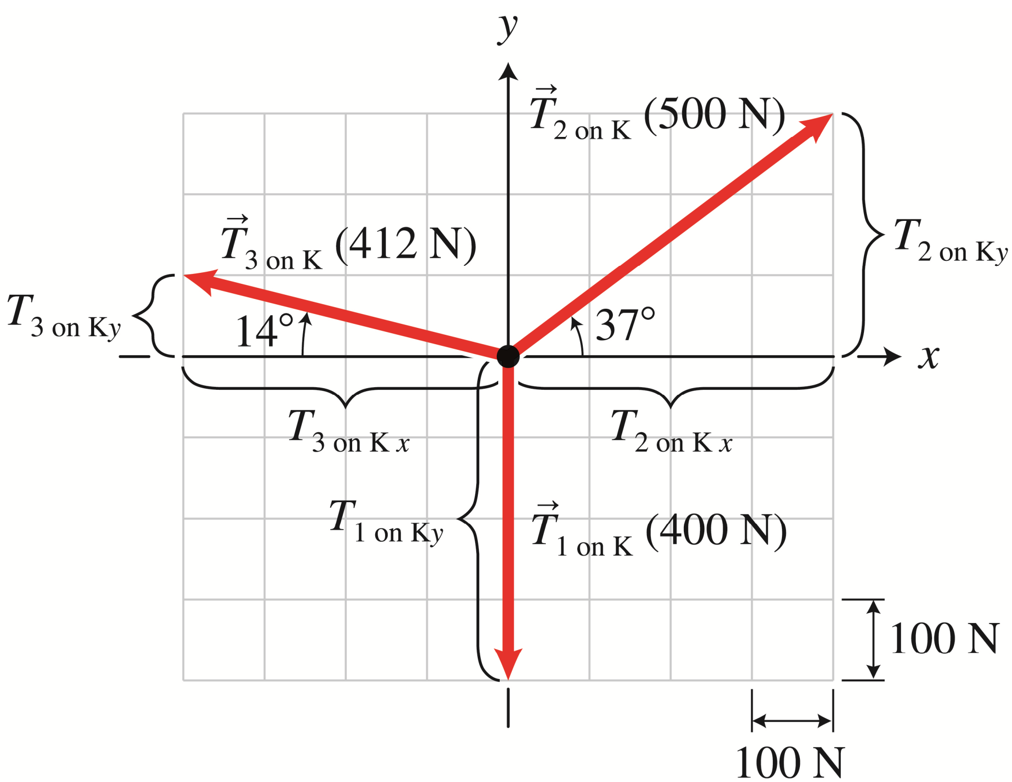
Newton's Second Law in Two Dimensions



- We can apply Newton's second law to situations where the forces are not along only one axis.
- First, find the scalar components of the forces.

- The sum of the scalar components of the force along the **x-axis** equals *mass times acceleration* along the **x-axis**.

Newton's Second Law in Two Dimensions



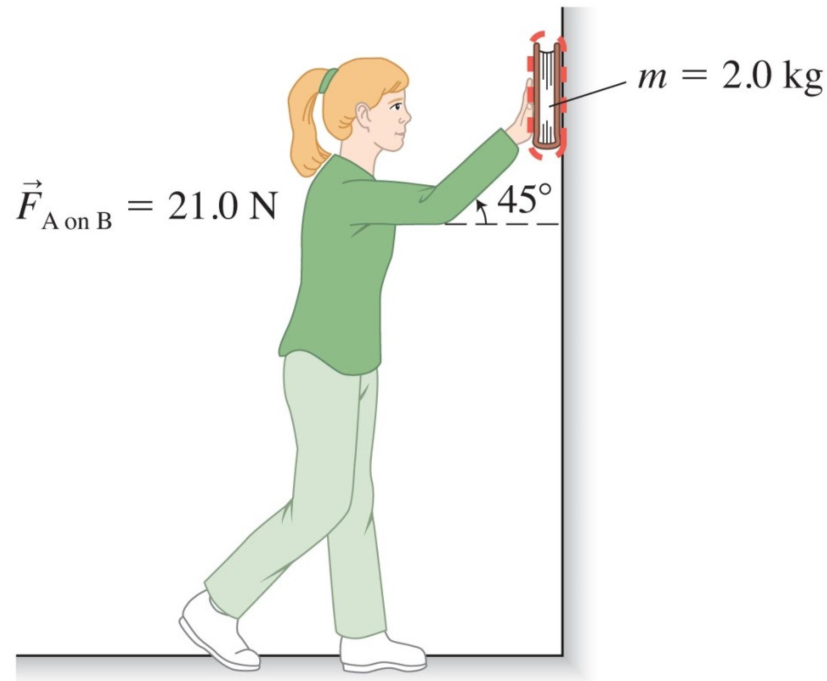
- We can apply Newton's second law to situations where the forces are not along only one axis.
- First, find the scalar components of the forces.
- The sum of the scalar components of the force along the **y-axis** equals *mass times acceleration* along the **y-axis**.

Newton's Second Law in Two Dimensions

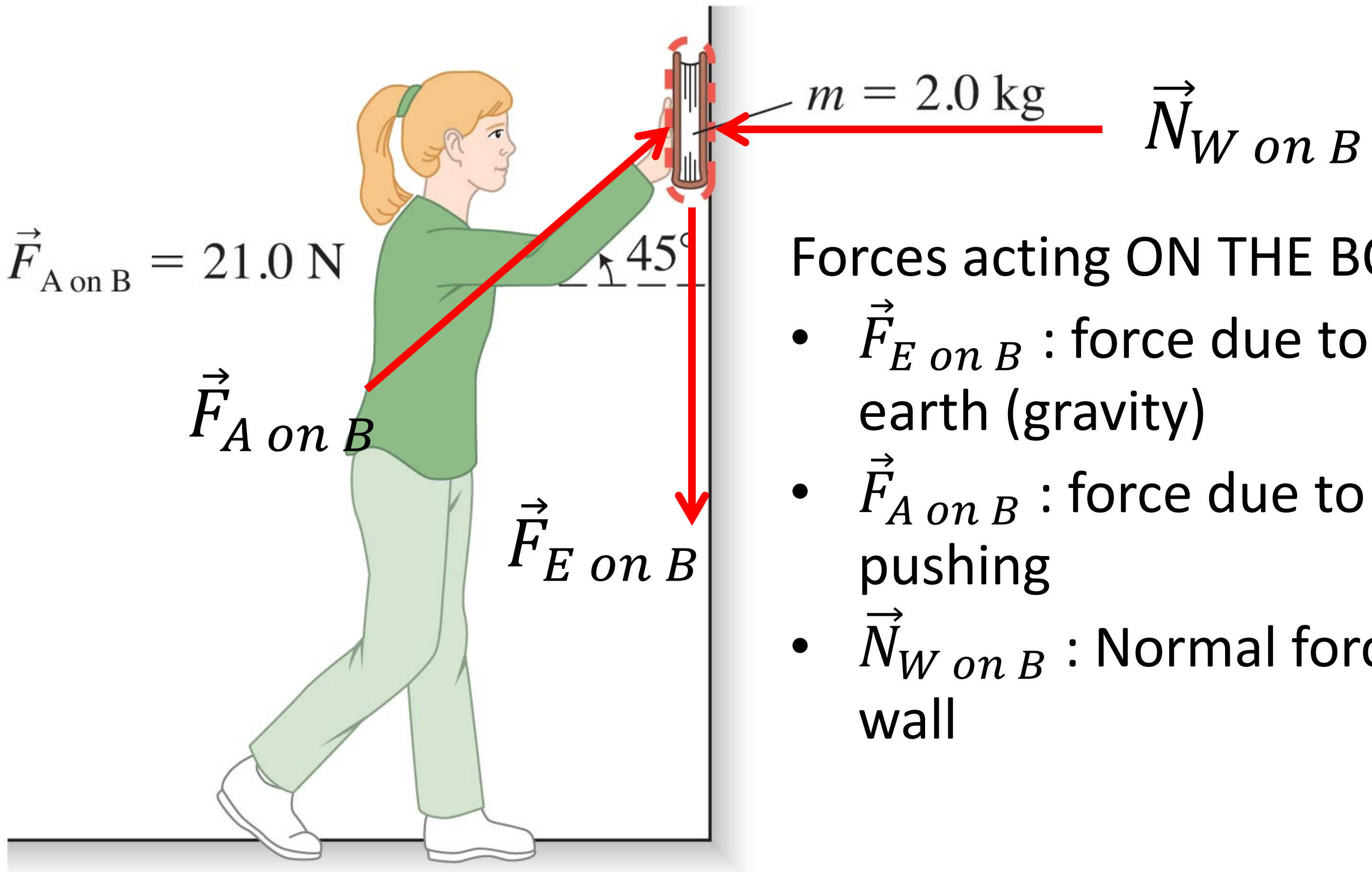
- If possible, we might pick the direction of the x-axis to be in the direction of the acceleration.
- Then the component of the acceleration along the other axis will be zero.
- This would simplify the problem, but might not always be possible.
- *Think carefully about which choice of geometry makes the problem easier to analyze!*

Example: Determining Forces an Acceleration

- Princess Anna Arkadievna Karenina pushes a 2.0 kg book onto a slippery wall, exerting a force of 21.0 N at an angle of 45° above the horizontal.
- What is the normal force of the wall on the book?
- Does the book slide up or down the wall?



Determining Forces and Acceleration



Forces acting ON THE BOOK:

- $\vec{F}_{E\ on\ B}$: force due to the earth (gravity)
- $\vec{F}_{A\ on\ B}$: force due to Anna pushing
- $\vec{N}_{W\ on\ B}$: Normal force of the wall

- These are the forces at this particular instant in time… at a later time, the angle might change…

Determining Forces and Acceleration

- The acceleration in the x-direction is zero
  - the book can only slide up and down the wall

$$a_{B,x} = \frac{\sum F_x}{m_B} = \frac{F_{A\ on\ B,x} + N_{W\ on\ B,x} + F_{E\ on\ B,x}}{m_B} = 0$$

  - But $F_{E\ on\ B,x} = 0$ because gravity only has a component in the vertical direction.

$$F_{A\ on\ B,x} + N_{W\ on\ B,x} = 0$$

$$F_{A\ on\ B,x} = (21\ N)\cos 45° = 14.8\ N$$

$$N_{W\ on\ B,x} = -14.8\ N$$

(it points in the –x direction)

Determining Forces and Acceleration

- What is the acceleration in the y-direction?

$$a_{B,y} = \frac{F_{A\ on\ B,y} + N_{W\ on\ B,y} + F_{E\ on\ B,y}}{m_B}$$

  - There is no vertical component from the normal force:

  $$N_{W\ on\ B,y} = 0.$$

  - Gravity exerts a force in the –y direction:

  $$F_{E\ on\ B,y} = -(2.0\ kg)(9.8\ N/kg) = -19.6\ N$$

  - Vertical component due to Anna:

  $$F_{A\ on\ B,y} = (21\ N)\sin 45° = 14.8\ \mathrm{N}$$

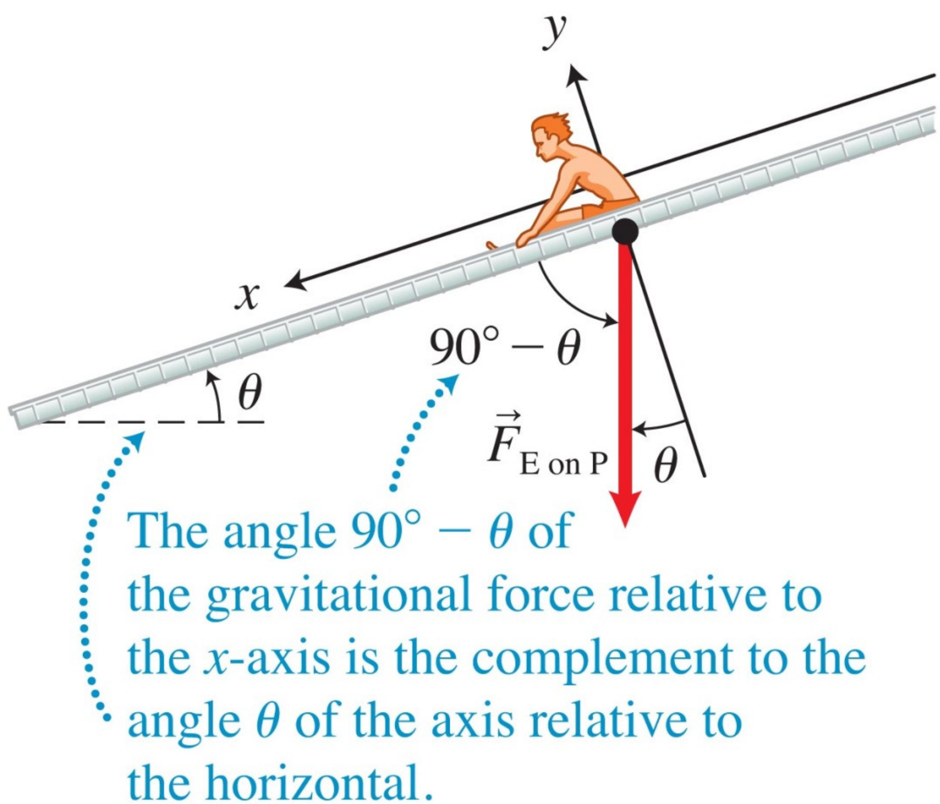
  - Acceleration:

  $$a_{B,y} = \frac{-19.6\ N + 14.8\ N}{2.0\ kg} = -2.4\ m/s^2$$

  - The book slides *down* the wall.

Incline Planes

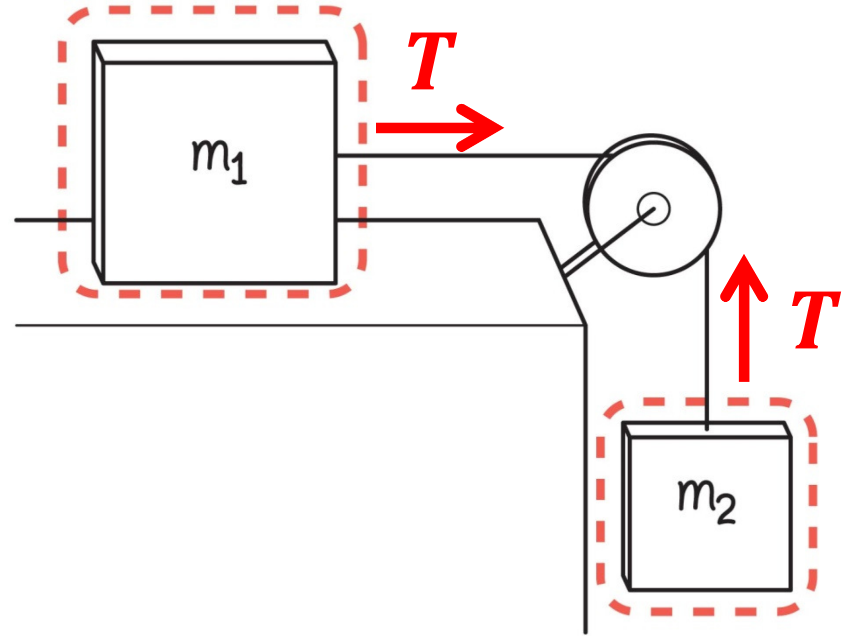
- Gravity is always in the downward vertical direction
- The normal force is always perpendicular to a surface
- The motion is along the surface of the plane
- Sometimes it is easier to align the x-axis with the direction of motion rather than the usual horizontal direction:



$$a_x = \frac{\sum F_x}{m} = \frac{(mg)\sin\theta}{m} = g\sin\theta$$

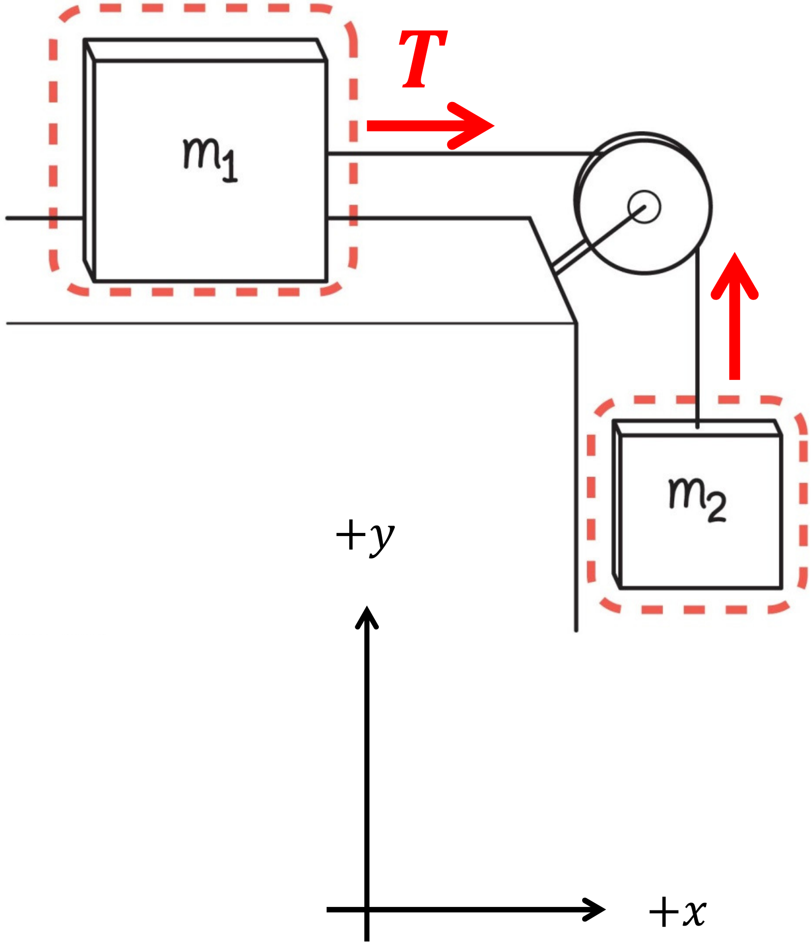
Tension in Strings

- We assume that strings will not stretch
  - Their length remains constant
- Strings can only pull… they can't push.
- The tension in a string, T, is the same everywhere.



Tension in Strings

- m_1 accelerates in the +x direction.
- m_2 accelerates in the –y direction.
- The magnitudes of the acceleration are equal because the string can't stretch.



Tension in Strings

- $a_1 = T/m_1$
- $a_1 = -a_2 = \frac{T - F_{E\ on\ m_2}}{m_2}$

$$= \frac{T - m_2 g}{m_2} = -\frac{T}{m_1}$$

- Solve for the tension:

$$T\left(\frac{1}{m_2} + \frac{1}{m_1}\right) = g$$

$$T = g\left(\frac{m_1 m_2}{m_1 + m_2}\right)$$

- Substitute it in to solve for a:

$$a_1 = \frac{T}{m_1} = g\left(\frac{m_2}{m_1 + m_2}\right) = -a_2$$