PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 22000

General Physics

Lecture 15 – Rotational Motion

Fall 2016 Semester

Prof. Matthew Jones

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SUPPLEMENTAL INSTRUCTION

Free Study Sessions!

Rachel Hoagburg

Come to SI for more help in **PHYS 220**

Tuesday and Thursday	7:30-8:30PM Shreve C113

Office Hour

Tuesday 1:30-2:30 4th floor of Krach

For other SI-linked courses and schedules, visit purdue.edu/si or purdue.edu/boilerguide

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Rotational Motion

- Instead of using the linear position, x, we use an angle, θ, to describe the orientation of an object.
- This is typical for an extended object that rotates about a fixed axis.
- The distance to a point on the object, r, is measured perpendicular to the fixed axis.

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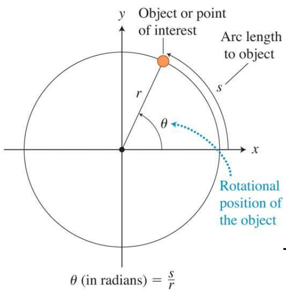
Rotational Motion



$$s = r\theta$$

The angle, θ, is measured in radians (which are dimensionless).

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Rotational Motion

- The angular velocity describes how fast the object is rotating about the fixed axis.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- A point located a distance r from the fixed axis moves with velocity

$$v = \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t} = r\omega$$

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Rotational Motion

- Angular acceleration is defined as the rate of change of angular velocity:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

- A point located a distance r from the fixed axis will have linear acceleration

$$a = \frac{\Delta v}{\Delta t} = r\frac{\Delta\omega}{\Delta t} = r\alpha$$

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Rotational Motion

- When an object rotates with constant angular acceleration, the angular velocity is

$$\omega(t) = \omega_0 + \alpha t$$

- The angle of a point on the object at any time is then

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

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Comparison with Linear Motion

Linear Motion	**Rotational Motion**
x	θ
$v(t) = v_0 + at$	$\omega(t) = \omega_0 + \alpha t$
$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$2a(x - x_0) = v^2 - v_0^2$	$2\alpha(\theta - \theta_0) = \omega^2 - \omega_0^2$

$$s = r\theta$$
$$v = r\omega$$
$$a = r\alpha$$

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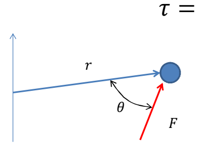
Torque

- A force acting on a point, located a distance r from a fixed axis, produces a torque,

$$\tau = \pm Fr\sin\theta$$



$\tau = Fr$ when $\theta = 90^o$

- A positive torque causes an object to rotate counter-clockwise.

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Newton's Second Law

- For linear motion, Newton's 2nd law is
$$a = \frac{\sum F}{m}$$
- For rotational motion, this implies that
$$\alpha = \frac{\sum \tau}{I}$$
- The rotational inertia, I, depends on the mass of the object and on where its mass is distributed.

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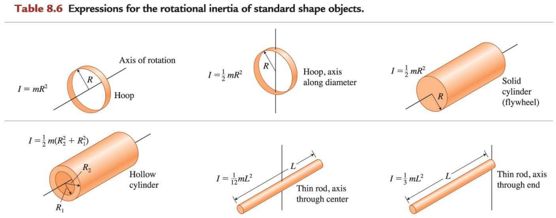
Rotational Inertia

Table 8.6 Expressions for the rotational inertia of standard shape objects.



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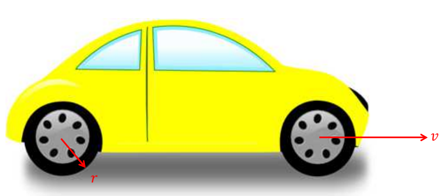
Some Examples



- The part of the wheel touching the pavement is stationary (unless the car skids).
- The angular velocity of the wheel is $\omega = -v/r$
- The negative sign indicates that the wheel rotates clockwise.

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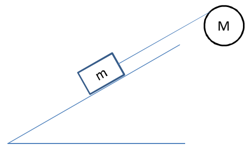
Some Examples



What is the acceleration of the block down the ramp?

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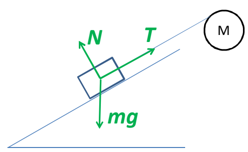
Some Examples



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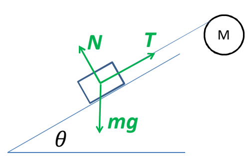
Some Examples



Acceleration down the ramp:

$$a = \frac{mg\sin\theta - T}{m}$$

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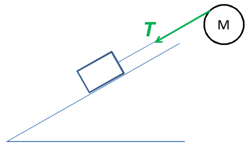
Some Examples



Torque on the wheel: $\tau = +Tr$
Angular acceleration: $\alpha = \tau / I$
Rotational inertia: $I = \frac{1}{2}Mr^2$

$$\alpha = \frac{Tr}{\frac{1}{2}Mr^2} = \frac{2T}{Mr}$$

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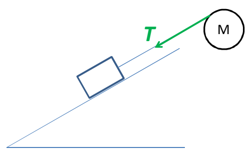
Some Examples



The rim of the wheel and the block have the same linear acceleration:

$$a = g \sin\theta - \frac{T}{m} = r\alpha = \frac{2T}{M}$$

Solve for *T*:

$$T\left(\frac{2}{M} + \frac{1}{m}\right) = g \sin\theta \rightarrow T = \frac{gMm \sin\theta}{2m+M}$$

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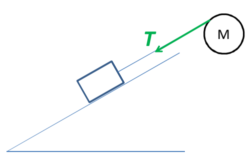
Some Examples



Substitute back into the equation for acceleration:

$$a = g \sin\theta - T/m$$

$$T = \frac{gMm \sin\theta}{2m + M}$$

$$a = g \sin\theta - \frac{gM \sin\theta}{2m + M}$$

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Some Examples

- Check the limiting cases:
  - What if M were very large? $M \gg m$
  - We expect $a \to 0$

$$a = g\sin\theta - \frac{gM\sin\theta}{2m+M} \approx g\sin\theta - \frac{gM\sin\theta}{M} \to 0$$

- What if $M = m$?

$$a = g\sin\theta\left(1 - \frac{1}{3}\right) = \frac{2}{3}g\sin\theta$$

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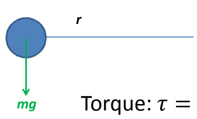
More Examples

- Angular acceleration of a pendulum:



Torque: $\tau = +mgr$
Rotational inertia: $I = mr^2$
Angular acceleration: $\alpha = \tau/I = g/r$

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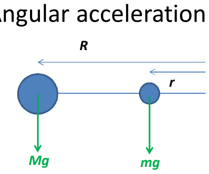
More Examples

- Angular acceleration of a pendulum:



Total torque: $\tau = +MgR + mgr$
Rotational inertia: $I = MR^2 + mr^2$
Angular acceleration: $\alpha = \tau/I$

$$= g\frac{MR + mr}{MR^2 + mr^2}$$

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Angular (rotational) Momentum

$$L = I\omega$$

When external torques are applied:

$$L_i + \sum \tau \, \Delta t = L_f$$

When no external torques are applied, angular momentum does not change:

$$I_i\omega_i = I_f\omega_f$$

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Even More Examples

- A merry-go-round at the park has a radius of r=2 m and rotational inertia $I = 50\ kg \cdot m^2$
- It is initially rotating with $\omega = 1\ s^{-1}$ when a kid with mass $m = 50\ kg$ gets on.
- What is the final angular velocity?

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Even More Examples

$$I_i = 50\ kg \cdot m^2$$

$$\omega_i = 1\ s^{-1}$$

$$L_i = I_i\omega_i = 50\ kg \cdot m^2/s$$

$$I_f = 50\ kg \cdot m^2 + (50\ kg)(2\ m)^2 = 250\ kg \cdot m^2$$

$$\boldsymbol{\omega_f = \frac{L_i}{I_f} = 0.2\ s^{-1}}$$

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Even More Examples

- The kid then moves to a radius of $r = 0.5\ m$
- What is the final angular velocity?

$$I_f = 50\ kg \cdot m^2 + (50\ kg) \cdot (0.25\ m)^2 = 62.5\ kg \cdot m^2$$

$$\boldsymbol{\omega_f = \frac{L_i}{I_f} = 0.8\ s^{-1}}$$

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Comparison with Linear Motion

Linear Motion	Rotational Motion
$p = mv$	$L = I\omega$
$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$

Rotational momentum is always conserved.

Kinetic energy is not conserved in inelastic collisions.

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Even More Examples

- The merry-go-round is initially at rest.
- A kid, with a mass of 50 kg is running with a speed of 2 m/s and jumps on at r=2 m.
- What is the final angular velocity?

$$\omega_f = \frac{L_i}{I_f}$$

$$L_i = mvr = 200\ kg \cdot m^2/s$$

$$I_f = 50\ kg \cdot m^2 + (50\ kg)(2\ m)^2 = 250\ kg \cdot m^2$$

$$\boldsymbol{\omega_f = 0.8\ s^{-1}}$$

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A Final Example

- How much kinetic energy was lost?
- Initial $K_i = \frac{1}{2}mv^2 = \frac{1}{2}(50\ kg)(2\ m/s)^2 = 100\ J$
- Final moment of inertia is $I_f = 250\ kg \cdot m^2$

- Final angular velocity was $\omega_f = 0.8\ s^{-1}$
- Final kinetic energy is

$$K_f = \frac{1}{2} I_f \omega_f^2 = 80\ \text{J}$$

$$\Delta K = 20\ J$$

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