Purdue University Department of Physics

Physics 22000

General Physics

Lecture 15 – Rotational Motion

Fall 2016 Semester

Prof. Matthew Jones

SUPPLEMENTAL INSTRUCTION

Free Study Sessions!

Rachel Hoagburg

Come to SI for more help in PHYS 220

Tuesday and Thursday	7:30-8:30PM Shreve C113

Office Hour

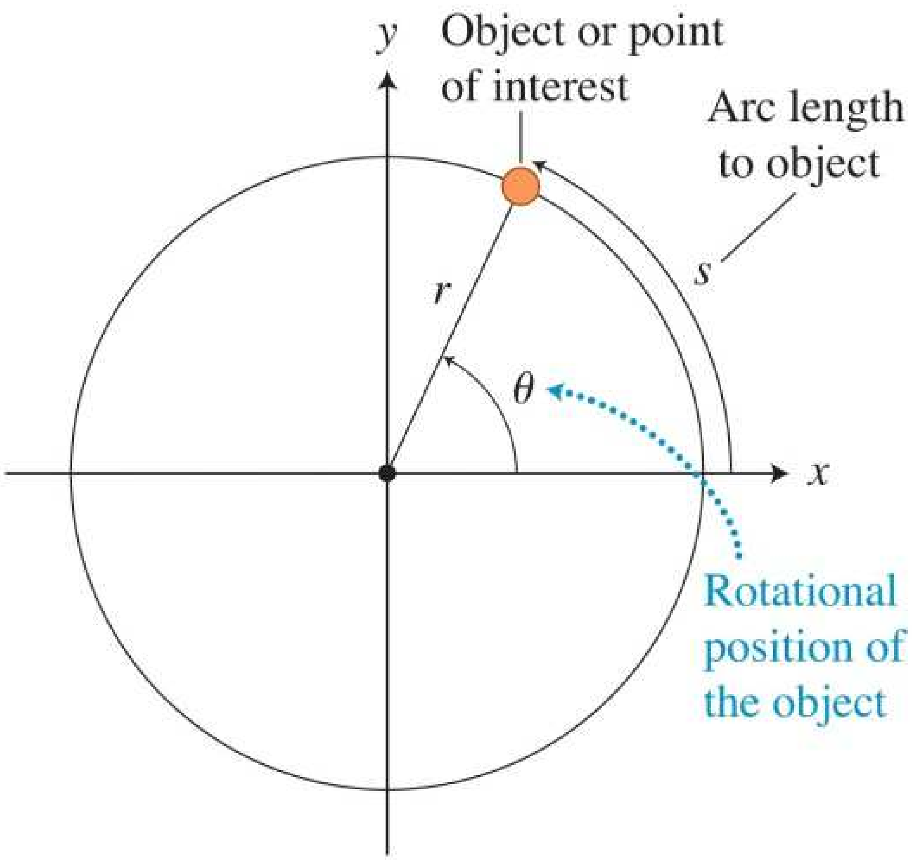
Tuesday 1:30-2:30 4th floor of Krach

For other SI-linked courses and schedules, visit purdue.edu/si or purdue.edu/boilerguide

Rotational Motion

- Instead of using the linear position, x, we use an angle, θ, to describe the orientation of an object.
- This is typical for an extended object that rotates about a fixed axis.
- The distance to a point on the object, r, is measured perpendicular to the fixed axis.

Rotational Motion



θ (in radians) $= \frac{s}{r}$

$$s = r\theta$$

The angle, θ, is measured in radians (which are dimensionless).

Rotational Motion

- The angular velocity describes how fast the object is rotating about the fixed axis.

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- A point located a distance r from the fixed axis moves with velocity

$$v = \frac{\Delta s}{\Delta t} = r\frac{\Delta\theta}{\Delta t} = r\omega$$

Rotational Motion

- Angular acceleration is defined as the rate of change of angular velocity:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

- A point located a distance r from the fixed axis will have linear acceleration

$$a = \frac{\Delta v}{\Delta t} = r\frac{\Delta\omega}{\Delta t} = r\alpha$$

Rotational Motion

- When an object rotates with constant angular acceleration, the angular velocity is

$$\omega(t) = \omega_0 + \alpha t$$

- The angle of a point on the object at any time is then

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$

Comparison with Linear Motion

Linear Motion	**Rotational Motion**
x	θ
$v(t) = v_0 + at$	$\omega(t) = \omega_0 + \alpha t$
$x(t) = x_0 + v_0 t + \frac{1}{2}at^2$	$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$2a(x - x_0) = v^2 - v_0^2$	$2\alpha(\theta - \theta_0) = \omega^2 - \omega_0^2$

$$s = r\theta$$

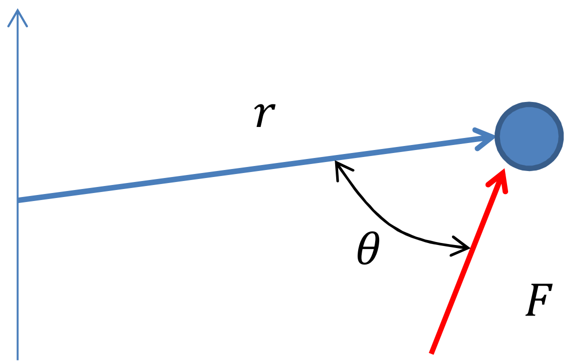
$$v = r\omega$$

$$a = r\alpha$$

Torque

- A force acting on a point, located a distance r from a fixed axis, produces a torque,

$$\tau = \pm Fr \sin\theta$$



$$\tau = Fr \text{ when } \theta = 90^o$$

- A positive torque causes an object to rotate counter-clockwise.

Newton's Second Law

- For linear motion, Newton's 2nd law is

$$a = \frac{\sum F}{m}$$

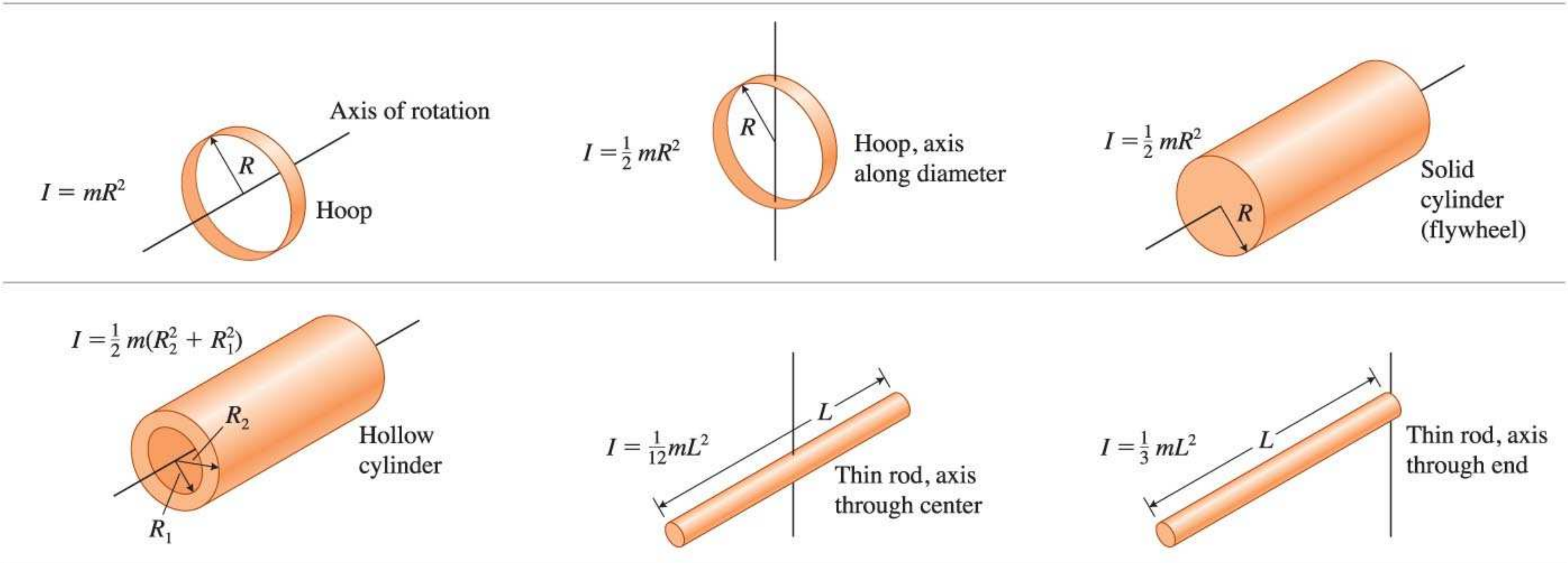
- For rotational motion, this implies that

$$\alpha = \frac{\sum \tau}{I}$$

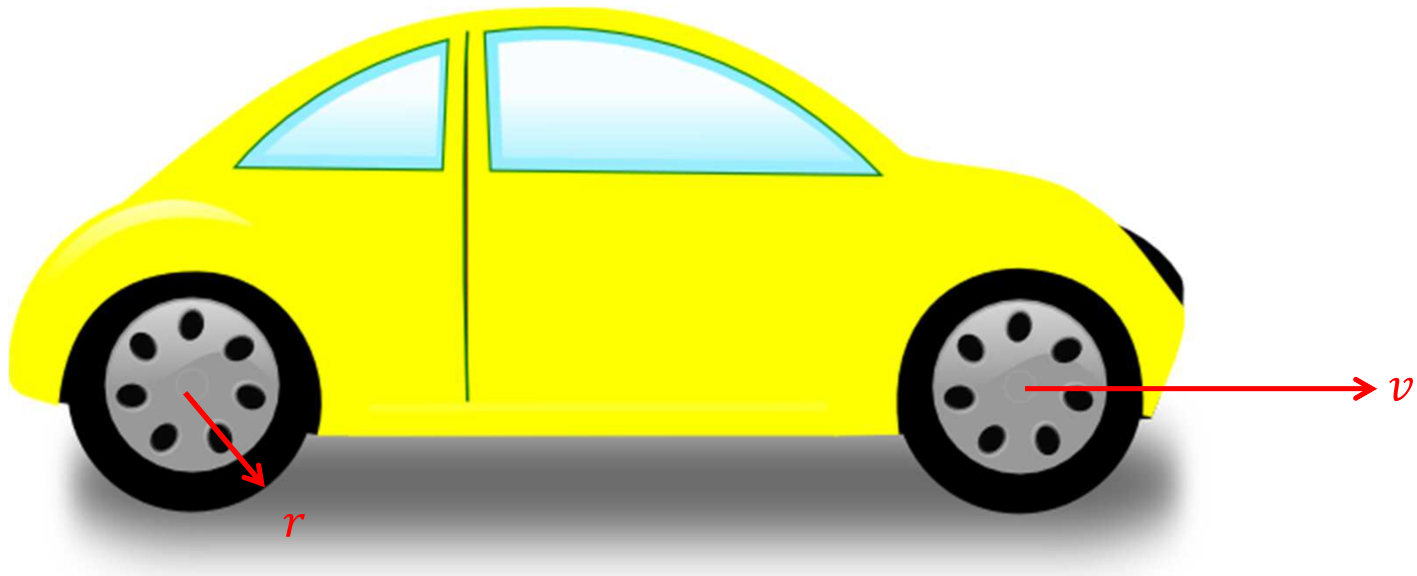
- The rotational inertia, I, depends on the mass of the object and on where its mass is distributed.

Rotational Inertia

Table 8.6 Expressions for the rotational inertia of standard shape objects.

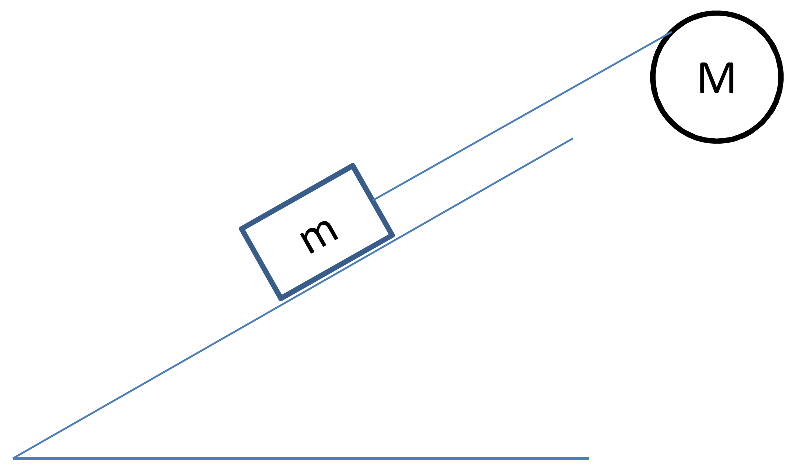


Some Examples



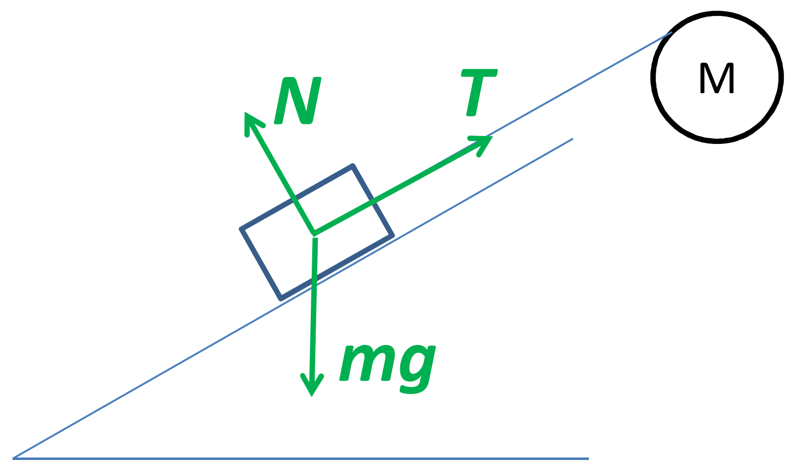
- The part of the wheel touching the pavement is stationary (unless the car skids).
- The angular velocity of the wheel is $\omega = -v/r$
- The negative sign indicates that the wheel rotates clockwise.

Some Examples

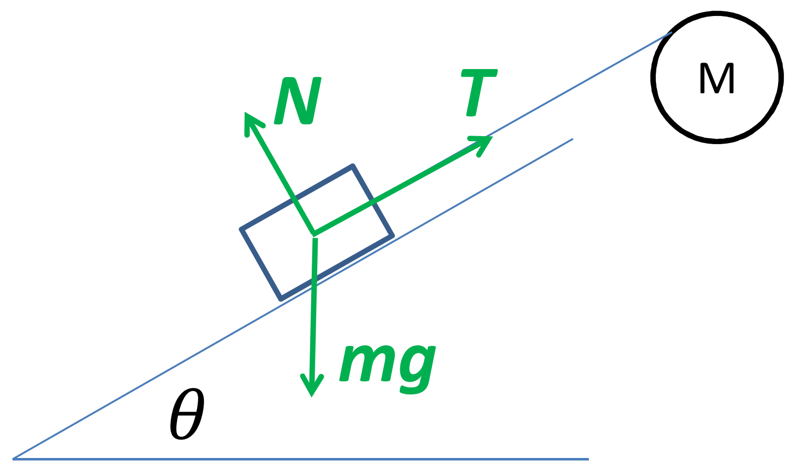


What is the acceleration of the block down the ramp?

Some Examples



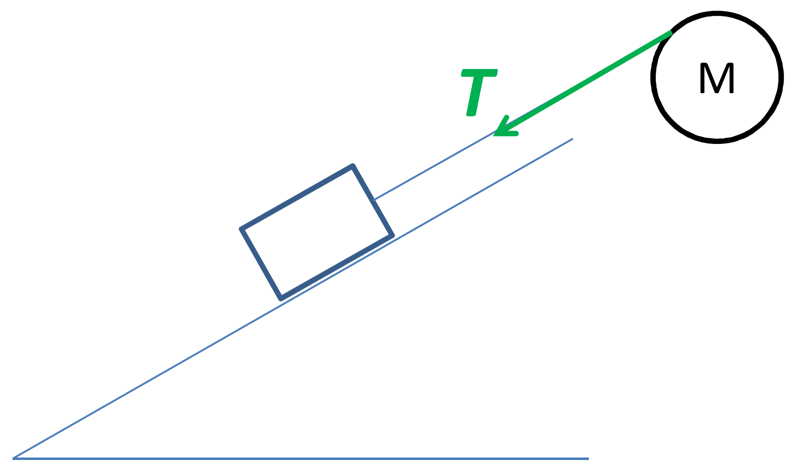
Some Examples



Acceleration down the ramp:

$$a = \frac{mg\sin\theta - T}{m}$$

Some Examples



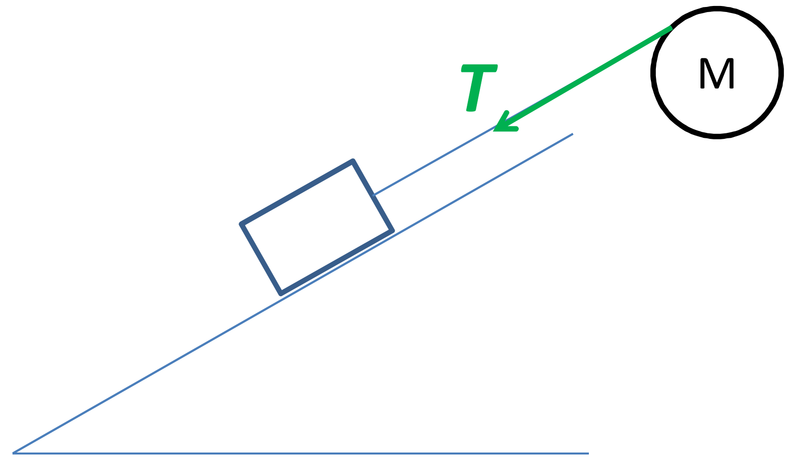
Torque on the wheel: $\tau = +Tr$

Angular acceleration: $\alpha = \tau / I$

Rotational inertia: $I = \frac{1}{2} M r^2$

$$\alpha = \frac{Tr}{\frac{1}{2} M r^2} = \frac{2T}{Mr}$$

Some Examples



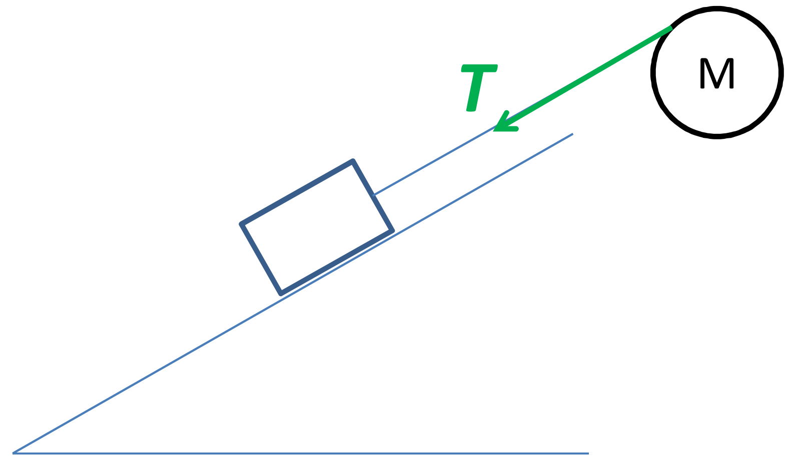
The rim of the wheel and the block have the same linear acceleration:

$$a = g\sin\theta - \frac{T}{m} = r\alpha = \frac{2T}{M}$$

Solve for *T*:

$$T\left(\frac{2}{M} + \frac{1}{m}\right) = g\sin\theta \rightarrow T = \frac{gMm\sin\theta}{2m+M}$$

Some Examples



Substitute back into the equation for acceleration:

$$a = g\sin\theta - T/m$$

$$T = \frac{gMm\sin\theta}{2m+M}$$

$$a = g\sin\theta - \frac{gM\sin\theta}{2m+M}$$

Some Examples

- Check the limiting cases:
  - What if M were very large? $M \gg m$
  - We expect $a \to 0$

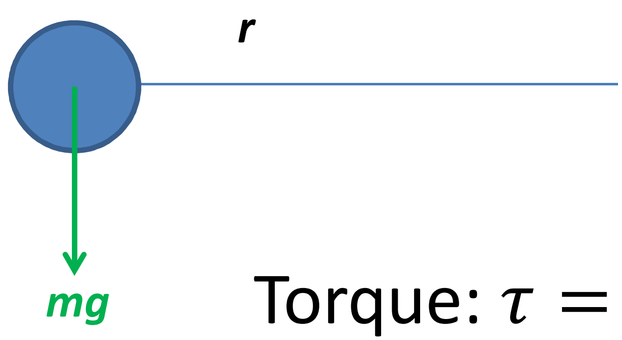
$$a = g\sin\theta - \frac{gM\sin\theta}{2m + M} \approx g\sin\theta - \frac{gM\sin\theta}{M} \to 0$$

- What if $M = m$?

$$a = g\sin\theta\left(1 - \frac{1}{3}\right) = \frac{2}{3}g\sin\theta$$

More Examples

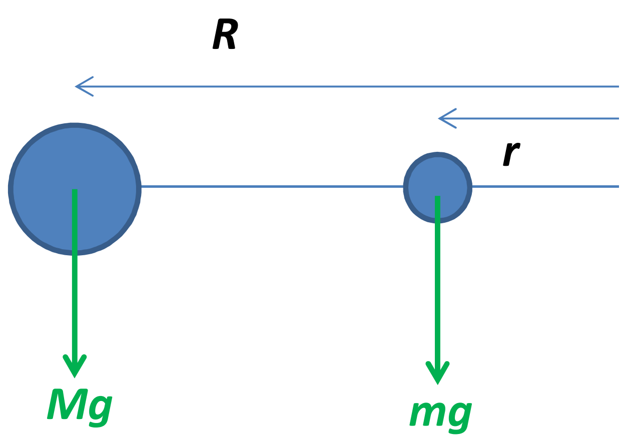
- Angular acceleration of a pendulum:



Torque: $\tau = +mgr$
Rotational inertia: $I = mr^2$
Angular acceleration: $\alpha = \tau/I = g/r$

More Examples

- Angular acceleration of a pendulum:



Total torque: $\tau = +MgR + mgr$

Rotational inertia: $I = MR^2 + mr^2$

Angular acceleration: $\alpha = \tau / I$

$$= g \frac{MR + mr}{MR^2 + mr^2}$$

Angular (rotational) Momentum

$$L = I\omega$$

When external torques are applied:

$$L_i + \sum \tau \ \Delta t = L_f$$

When no external torques are applied, angular momentum does not change:

$$I_i \omega_i = I_f \omega_f$$

Even More Examples

- A merry-go-round at the park has a radius of r=2 m and rotational inertia $I = 50\ kg \cdot m^2$
- It is initially rotating with $\omega = 1\ s^{-1}$ when a kid with mass $m = 50\ kg$ gets on.
- What is the final angular velocity?

Even More Examples

$$I_i = 50\ kg \cdot m^2$$
$$\omega_i = 1\ s^{-1}$$
$$L_i = I_i \omega_i = 50\ kg \cdot m^2/s$$

$$I_f = 50\ kg \cdot m^2$$
$$+ (50\ kg)(2\ m)^2$$
$$= 250\ kg \cdot m^2$$
$$\boldsymbol{\omega_f = \frac{L_i}{I_f} = 0.2\ s^{-1}}$$

Even More Examples

- The kid then moves to a radius of $r = 0.5\ m$
- What is the final angular velocity?

$$I_f = 50\ kg \cdot m^2$$
$$+(50\ kg) \cdot (0.25\ m)^2$$
$$= 62.5\ kg \cdot m^2$$

$$\boldsymbol{\omega_f = \frac{L_i}{I_f} = 0.8\ s^{-1}}$$

Comparison with Linear Motion

Linear Motion	**Rotational Motion**
$p = mv$	$L = I\omega$
$K = \frac{1}{2}mv^2$	$K = \frac{1}{2}I\omega^2$

Rotational momentum is always conserved.

Kinetic energy is not conserved in inelastic collisions.

Even More Examples

- The merry-go-round is initially at rest.
- A kid, with a mass of 50 kg is running with a speed of 2 m/s and jumps on at r=2 m.
- What is the final angular velocity?

$$\omega_f = \frac{L_i}{I_f}$$

$$L_i = mvr$$

$$= 200\ kg \cdot m^2/s$$

$$I_f = 50\ kg \cdot m^2$$

$$+ (50\ kg)(2\ m)^2$$

$$= 250\ kg\ \cdot m^2$$

$$\boldsymbol{\omega_f = 0.8\ s^{-1}}$$

A Final Example

- How much kinetic energy was lost?
- Initial $K_i = \frac{1}{2}mv^2 = \frac{1}{2}(50\,kg)(2\,m/s)^2 = 100\,J$
- Final moment of inertia is $I_f = 250\,kg \cdot m^2$

- Final angular velocity was $\omega_f = 0.8\,s^{-1}$
- Final kinetic energy is

$$K_f = \frac{1}{2}I_f\omega_f^2 = 80\text{ J}$$

$$\Delta K = 20\,J$$