Purdue University Department of Physics

Physics 22000

General Physics

Lecture 14 – Rotational Motion

Fall 2016 Semester

Prof. Matthew Jones

SUPPLEMENTAL INSTRUCTION

Free Study Sessions!

Rachel Hoagburg

Come to SI for more help in PHYS 220

Tuesday and Thursday	7:30-8:30PM Shreve C113

Office Hour

Tuesday 1:30-2:30 4th floor of Krach

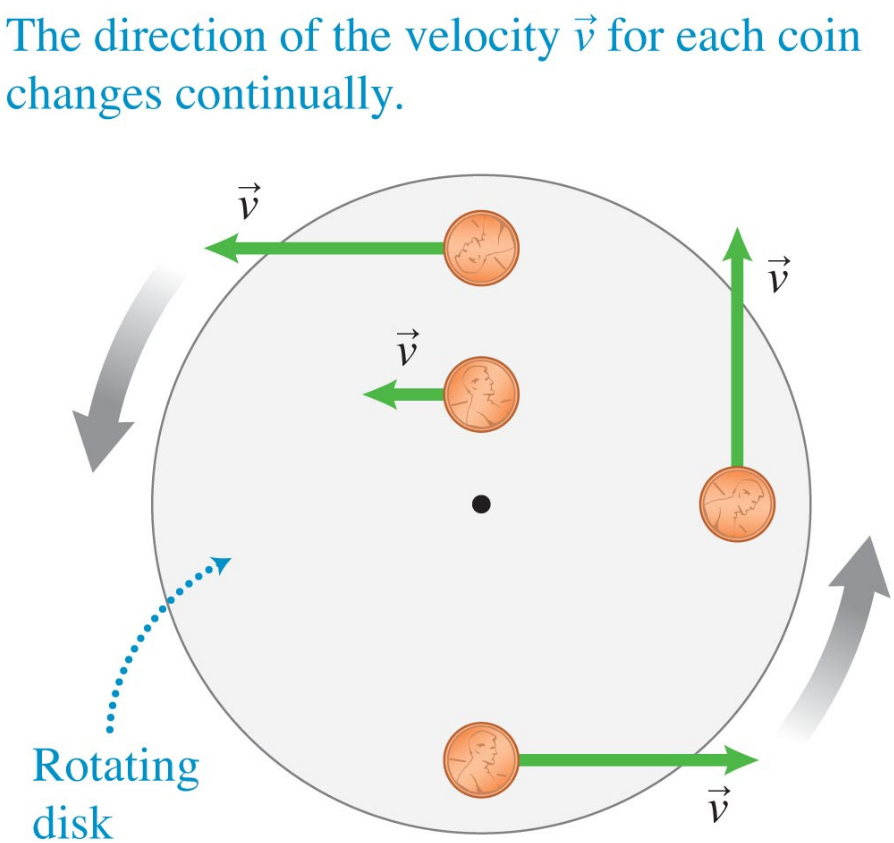
For other SI-linked courses and schedules, visit purdue.edu/si or purdue.edu/boilerguide

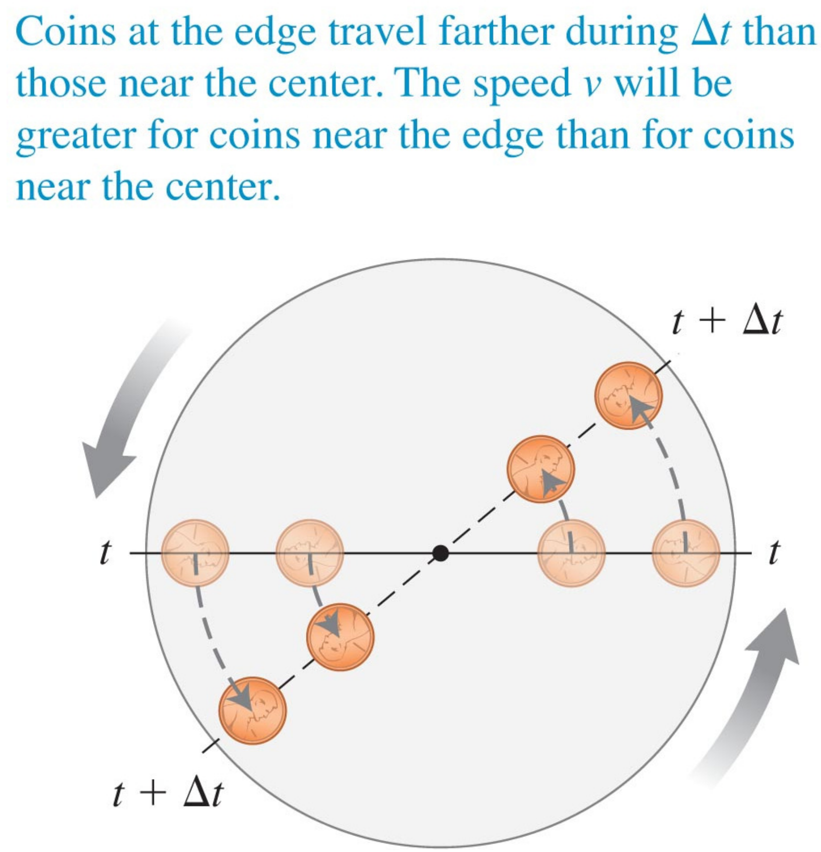
Static Equilibrium

- In the last lecture, we learned about the torque that a force can exert on a rigid body.
  - We analyzed only rigid bodies that were in static equilibrium.
- Today, we learn how to describe, explain, and predict motion for objects that rotate.
  - These objects are not in static equilibrium

Rotational Kinematics

Imagine placing several coins at different locations on a spinning disk.



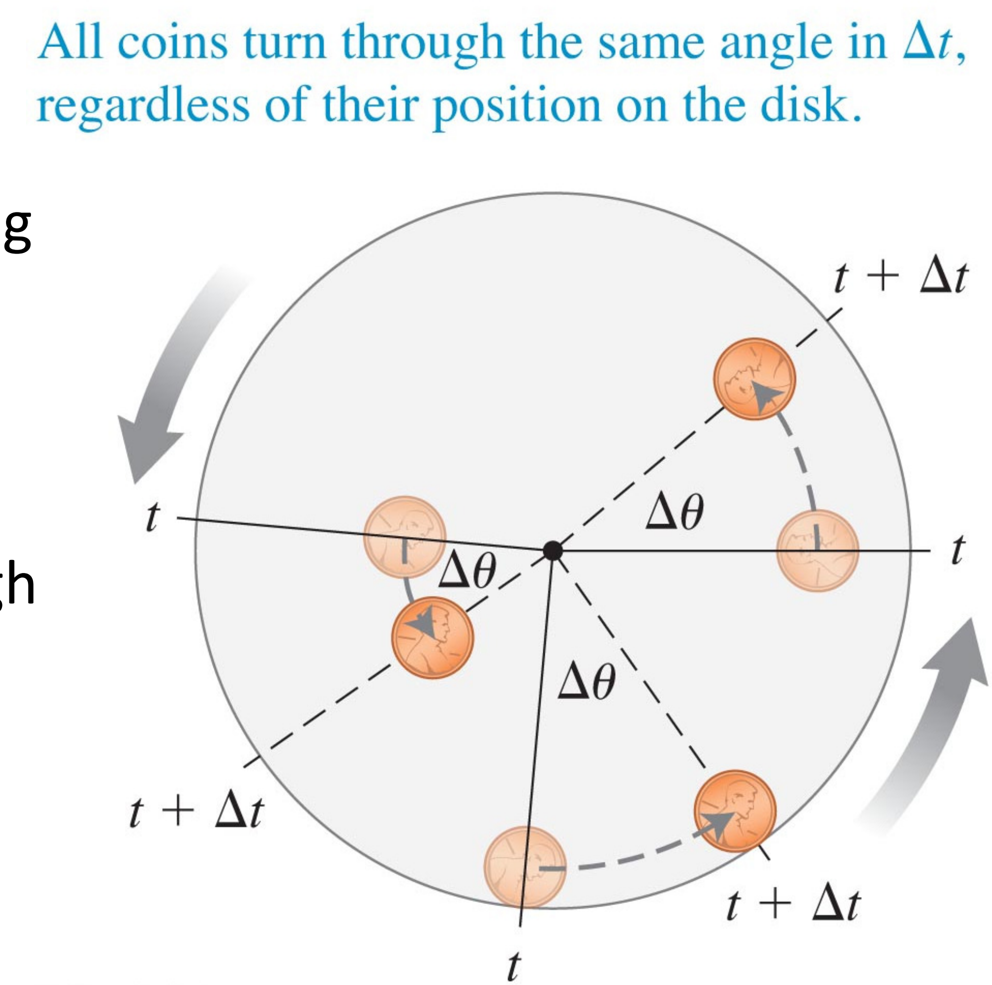


Rotational Kinematics

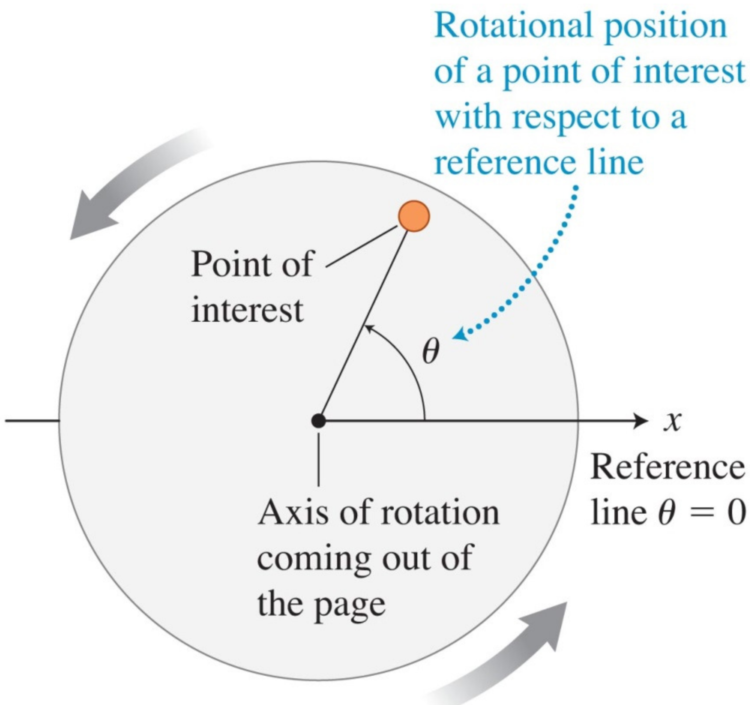
- Consider the various coins located at different places on a spinning disk…
  - The direction of the velocity of each coin changes continuously.
  - A coin that sits closer to the edge moves faster and covers a longer distance than a coin placed closer to the center.
- *Different parts of the disk move in different directions and at different speeds.*

Rotational Kinematics

- There are similarities between the motions of different points on a rotating rigid body.
  - During a particular time interval, all coins at the different points on the rotating disk turn through the same angle.
  - **Perhaps we should describe the rotational position of a rigid body using an angle.**



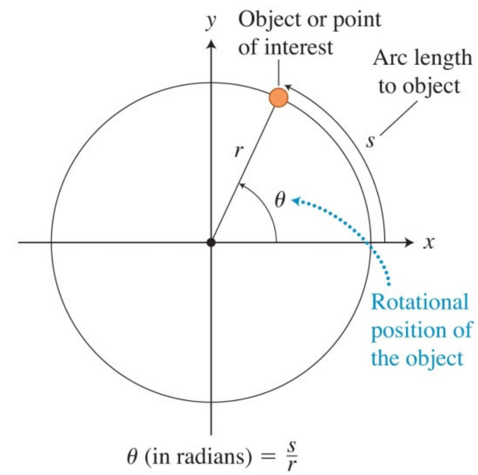
Rotational position, θ (or angular position)



Rotational position θ The rotational position θ of a point on a rotating object (sometimes called the angular position) is defined as an angle in the counterclockwise direction between a reference line (usually the positive x-axis) and a line drawn from the axis of rotation to that point. The units of rotational position can be either degrees or radians.

Units of Rotational Position

- The unit for rotational position is the radian (rad). It is defined in terms of:
  - The arc length *s*
  - The radius *r* of the circle
- The angle in units of radians is the ratio of *s* and *r*:



$$\theta = \frac{s}{r} \quad \text{(measured in radians)}$$

- The radian unit has no dimensions; it is the ratio of two lengths. The unit **rad** is just a reminder that we are using radians for angles.

Tips when Working in Radians

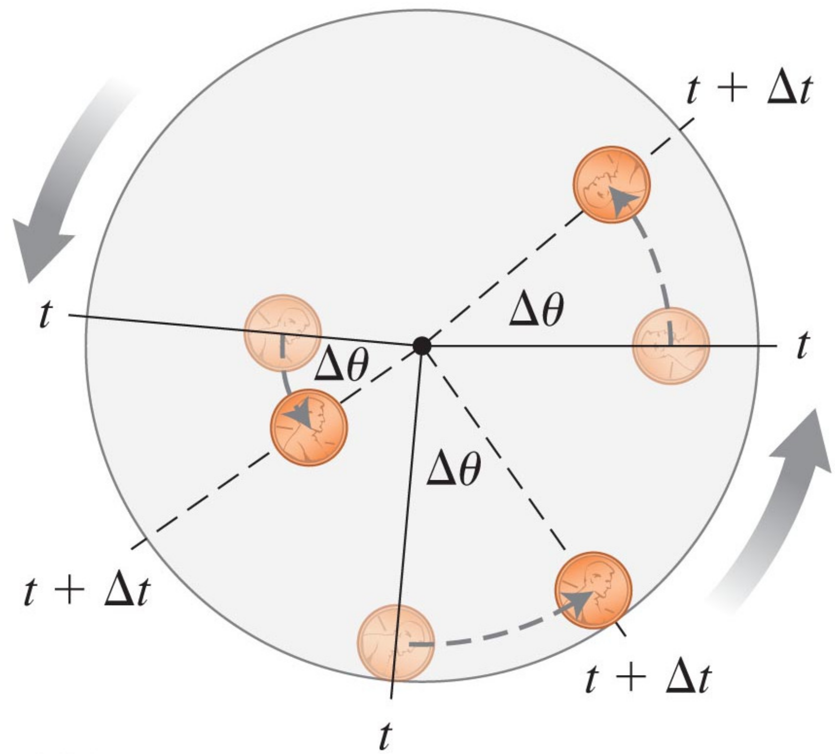
TIP You cannot calculate arc length using $s = r\theta$ when θ is measured in degrees. You must first convert θ to radians.

TIP From Eq. (8.1) we see that the arc length for a 1-rad angle equals the radius of the circle. For example, the 1-rad angle shown in **Figure 8.4** is the ratio of the 2-cm arc length and the 2-cm radius and is simply 1. If you use a calculator to work with radians, make sure it is in the radian mode.

Rotational (angular) velocity, ω

- Translational velocity is the rate of change of linear position.
- We define the rotational (angular) velocity *v* of a rigid body as the rate of change of each point's rotational position.
  - All points on the rigid body rotate through the same angle in the same time, so **each point has the same rotational velocity**.

All coins turn through the same angle in Δt, regardless of their position on the disk.



Rotational (angular) velocity, ω

Rotational velocity ω The average rotational velocity (sometimes called angular velocity) of a turning rigid body is the ratio of its change in rotational position $\Delta\theta$ and the time interval Δt needed for that change (see **Figure 8.5**):

$$\omega = \frac{\Delta\theta}{\Delta t} \tag{8.2}$$

The sign of ω (omega) is positive for counterclockwise turning and negative for clockwise turning, as seen looking along the axis of rotation. *Rotational (angular) speed* is the magnitude of the rotational velocity. The most common units for rotational velocity and speed are radians per second (rad/s) and revolutions per minute (rpm).

Rotational (angular) acceleration, α

- Translational acceleration describes an object's change in velocity for linear motion.
  - We could apply the same idea to the center of mass of a rigid body that is moving as a whole from one position to another.
- The rate of change of the rigid body's rotational velocity is its rotational acceleration.
  - When the rotation rate of a rigid body increases or decreases, it has a nonzero rotational acceleration.

Rotational (angular) acceleration, α

Rotational acceleration α The average rotational acceleration α (alpha) of a rotating rigid body (sometimes called angular acceleration) is its change in rotational velocity $\Delta\omega$ during a time interval Δt divided by that time interval:

$$\alpha = \frac{\Delta\omega}{\Delta t} \qquad (8.3)$$

The unit of rotational acceleration is $(\text{rad/s})/\text{s} = \text{rad/s}^2$.

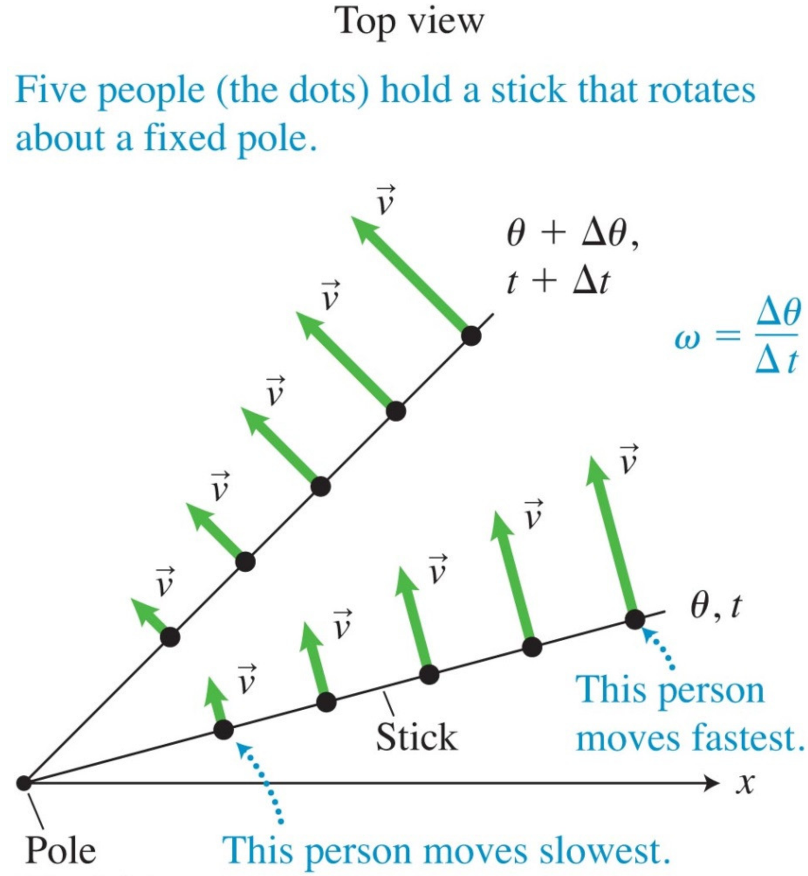
The sign of the rotational acceleration is the same as the sign of the rotational velocity when the object rotates increasingly faster.

The signs of the rotational acceleration and velocity are opposite if the object is rotating at an increasingly slower rate.

Relation Between Translational and Rotational Velocities

$$v_t = \frac{\Delta s}{\Delta t} = \frac{r\,\Delta\theta}{\Delta t} = r\left(\frac{\Delta\theta}{\Delta t}\right) = r\omega$$

$$a_t = \frac{\Delta v_t}{\Delta t} = \frac{r\,\Delta\omega}{\Delta t} = r\left(\frac{\Delta\omega}{\Delta t}\right) = r\alpha$$



TIP You get the familiar translational quantities for motion along the circular path by multiplying the corresponding angular rotational quantities by the radius r of the circle.

Example

- Determine the tangential speed of a stable gaseous cloud around a black hole.
- The cloud has a stable circular orbit at its innermost 30-km radius. This cloud moves in a circle about the black hole about 970 times per second.

$$v_t = r\omega$$
$$= (30 \times 10^3 m)(2\pi)(970\ s^{-1})$$
$$= 1.83 \times 10^8\ m/s$$

(60% the speed of light)

Rotational Motion at Constant Angular Acceleration

- θ_0 is an object's rotational position at $t_0 = 0$.
- ω_0 is an object's rotational velocity at $t_0 = 0$.
- θ and ω are the rotational position and the rotational velocity at some later time t.
- α is the object's constant rotational acceleration during the time interval from time 0 to time t.

Translational motion	Rotational motion	
$v_x = v_{0x} + a_x t$	$\omega = \omega_0 + \alpha t$	(8.6)
$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2$	$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	(8.7)
$2a_x(x - x_0) = v_x^2 - v_{0x}^2$	$2\alpha(\theta - \theta_0) = \omega^2 - \omega_0^2$	(8.8)

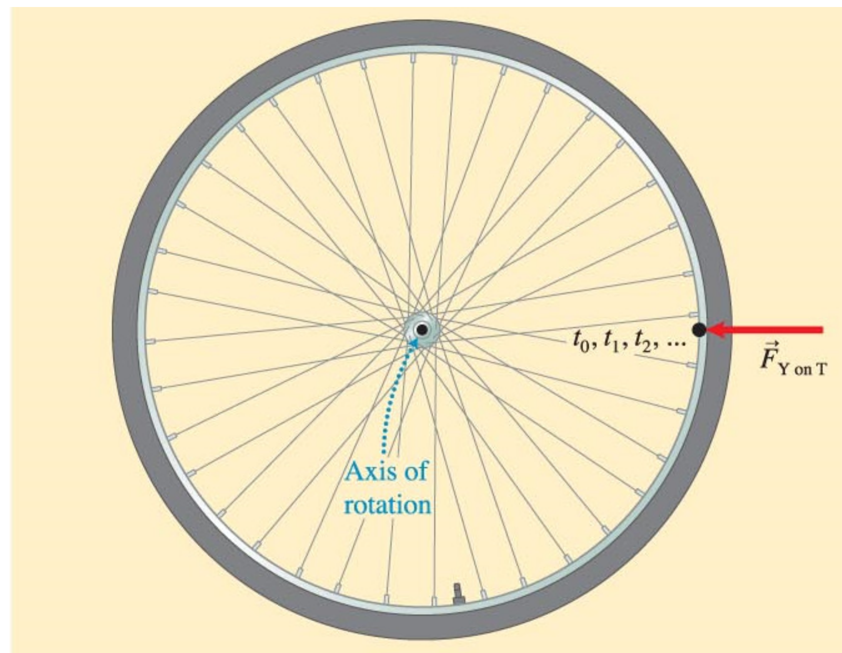
Rotational Motion at Constant Acceleration

- Rotational position θ is positive if the object has rotated counterclockwise and negative if it has rotated clockwise.
- Rotational velocity ω is positive if the object is rotating counterclockwise and negative if it is rotating clockwise.
- The sign of the rotational acceleration α depends on how the rotational velocity is changing:
  - α has the same sign as ω if the magnitude of ω is increasing.
  - α has the opposite sign of ω if the magnitude of ω is decreasing.

Torque and Rotational Acceleration

- When you push on the front of a bicycle tire, directly towards the axis of rotation, a force is applied but it has no effect on the rotation of the wheel.

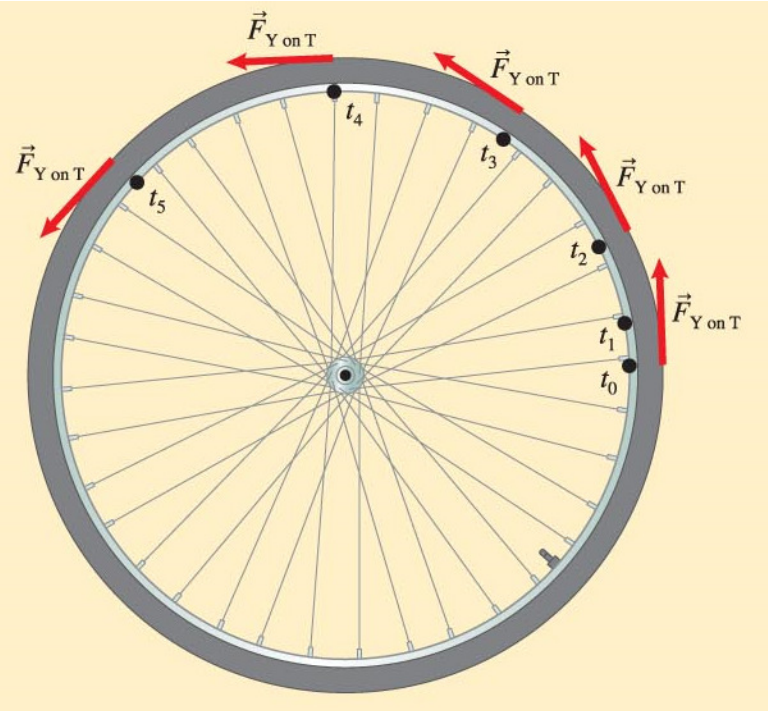
- There is no torque applied because the force points directly through the axis of rotation.



Torque and Rotational Acceleration

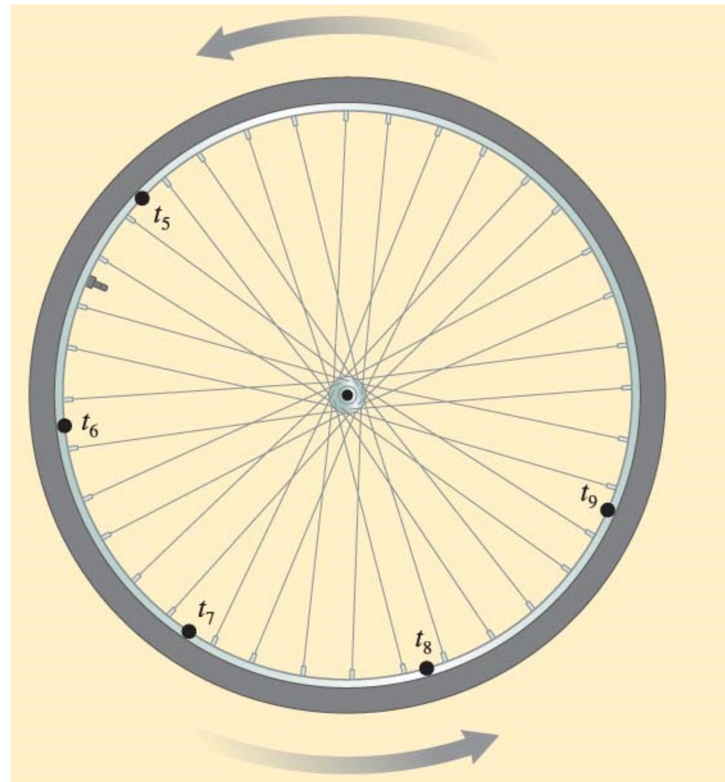
- When you push lightly and continuously on the outside of the tire in a counterclockwise (ccw) direction tangent to the tire, the tire rotates ccw faster and faster.

- The pushing creates a ccw torque. The tire has an increasingly positive rotational velocity and a positive rotational acceleration.



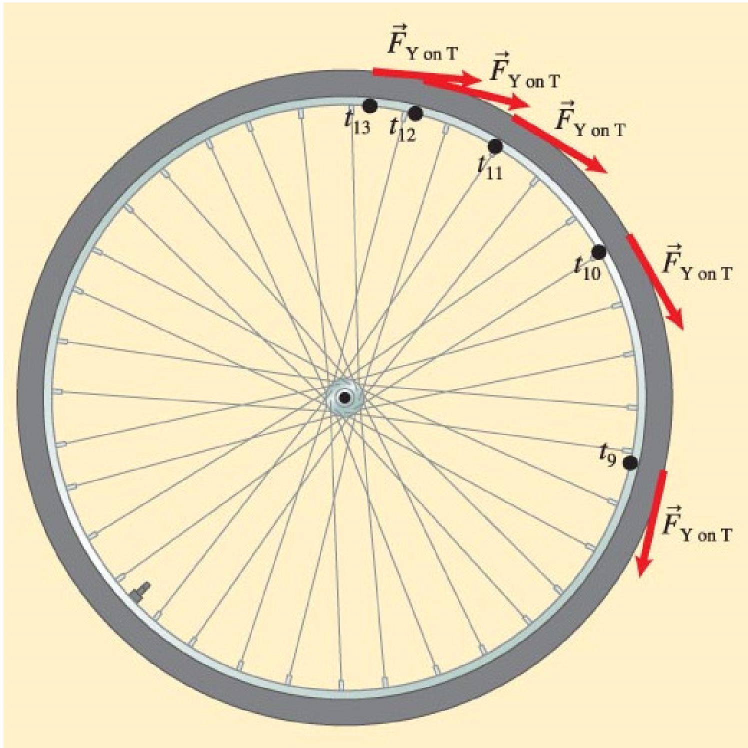
Torque and Rotational Acceleration

- When you stop pushing on the wheel, it continues to rotate ccw at a constant rate.
- The rotational velocity is constant and the rotational acceleration is zero.



Torque and Rotational Acceleration

- With the wheel rotating in the ccw direction, you gently and continuously push in the clockwise (cw) direction against tht tire.
- The rotational speed in the ccw direction decreases.
- This is a cw torque producing a negative rotational acceleration.



Torque and Rotational Acceleration

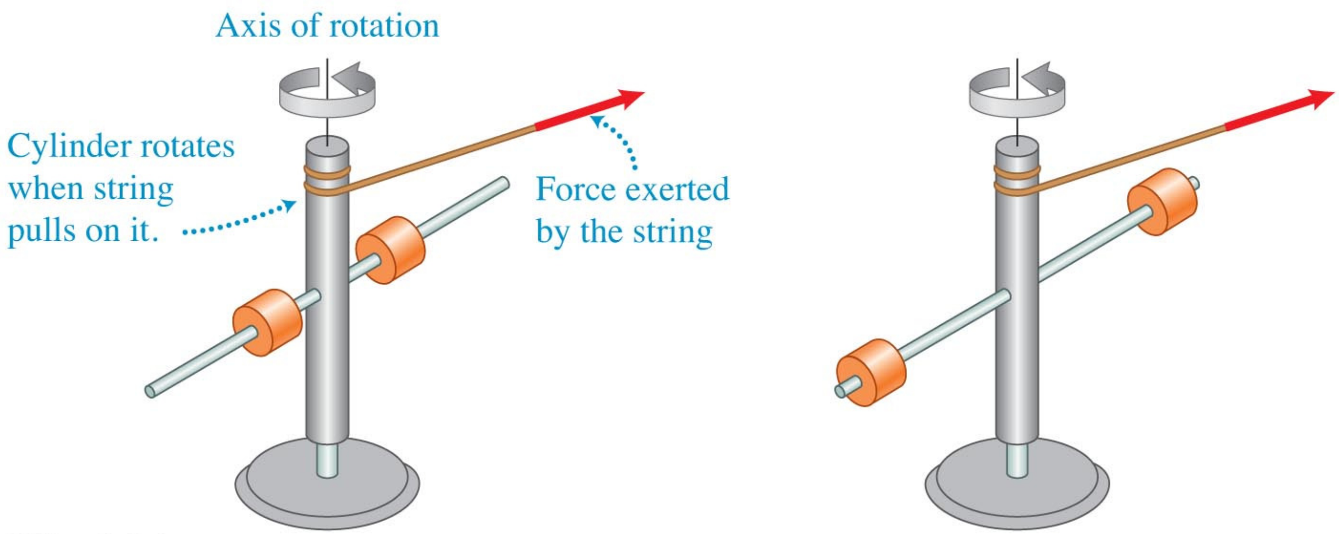
- The experiments indicate a zero torque has no effect on rotational motion but a nonzero torque does cause a change.
  - If the torque is in the same direction as the direction of rotation of the rigid body, the object's rotational speed increases.
  - If the torque is in the opposite direction, the object's speed decreases.

Torque and Rotational Acceleration

Changes in rotational velocity Rotational acceleration depends on net torque. The greater the net torque, the greater the rotational acceleration.

- This is similar to what we learned when studying translational motion.
- A non-zero net force needs to be exerted on an object to cause its velocity to change.
- The translational acceleration of the object was proportional to the net force.
  - The constant of proportionality was the mass
  - What is analogous to mass for rotational motion?

Rotational Inertia



- Pull each string and compare the rotational acceleration for the apparatus shown on the left and right.
  - The rotational acceleration is greater for the system on the left because the masses are closer to the axis of rotation.
  - The arrangement on the right has a greater rotational ineria
  - The same torque applied to the right results in a lesser rotational acceleration.

Rotational Inertia

- Rotational inertia is the physical quantity characterizing the location of the mass relative to the axis of rotation of the object.
  - The closer the mass of the object is to the axis of rotation, the easier it is to change its rotational motion and the smaller its rotational inertia.
  - The magnitude depends on both the total mass of the object and the distribution of that mass about its axis of rotation.

Rotational Inertia

- Rotational inertia for a point-like object of mass m rotating a distance r about a fixed axis is defined by

$$I = mr^2$$

- When subjected to a net torque, the resulting angular acceleration is

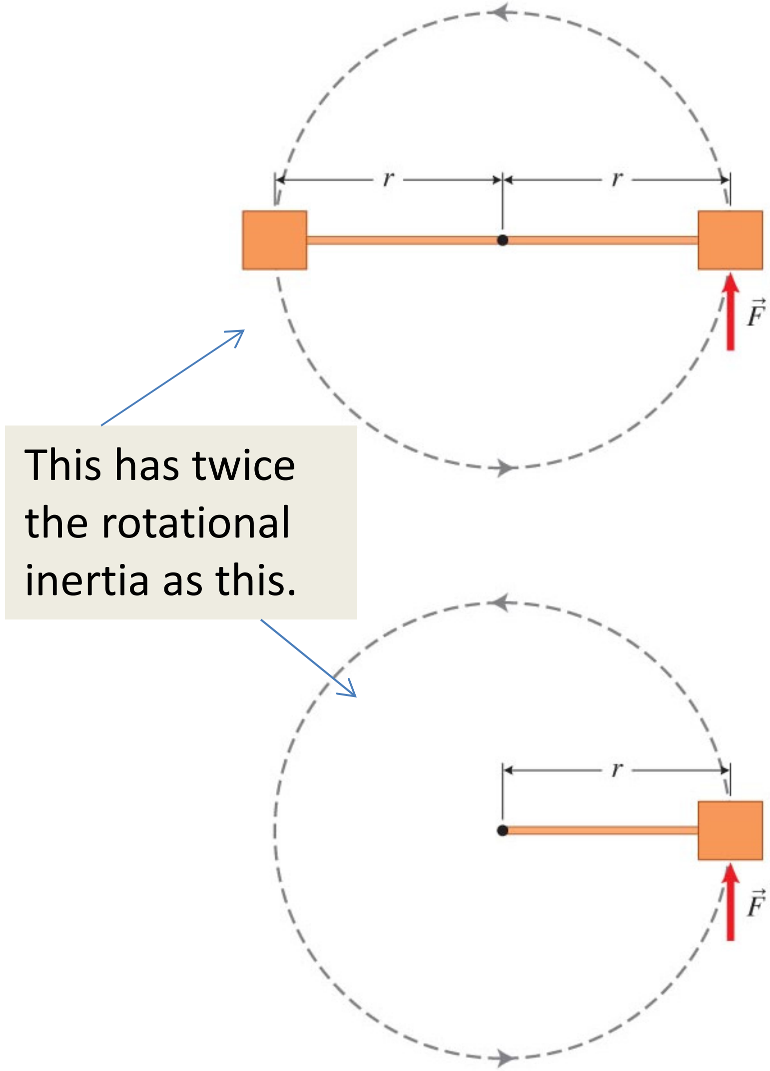
$$\alpha = \frac{1}{mr^2 \sum \tau} = \frac{1}{I \sum \tau}$$

- Notice how similar this is to Newton's second law for translational motion:

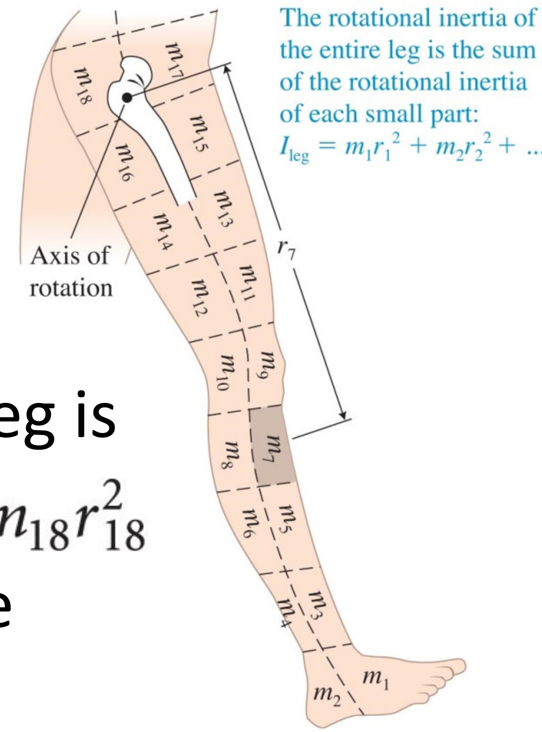
$$\vec{a} = \frac{1}{m \sum \vec{F}}$$

Newton's Second Law for Rotational Motion of Rigid Bodies

- The rotational inertia of a rigid body about some axis of rotation is the sum of the rotational inertias of the individual point-like objects that make up the rigid body.



Calculating Rotational Inertia



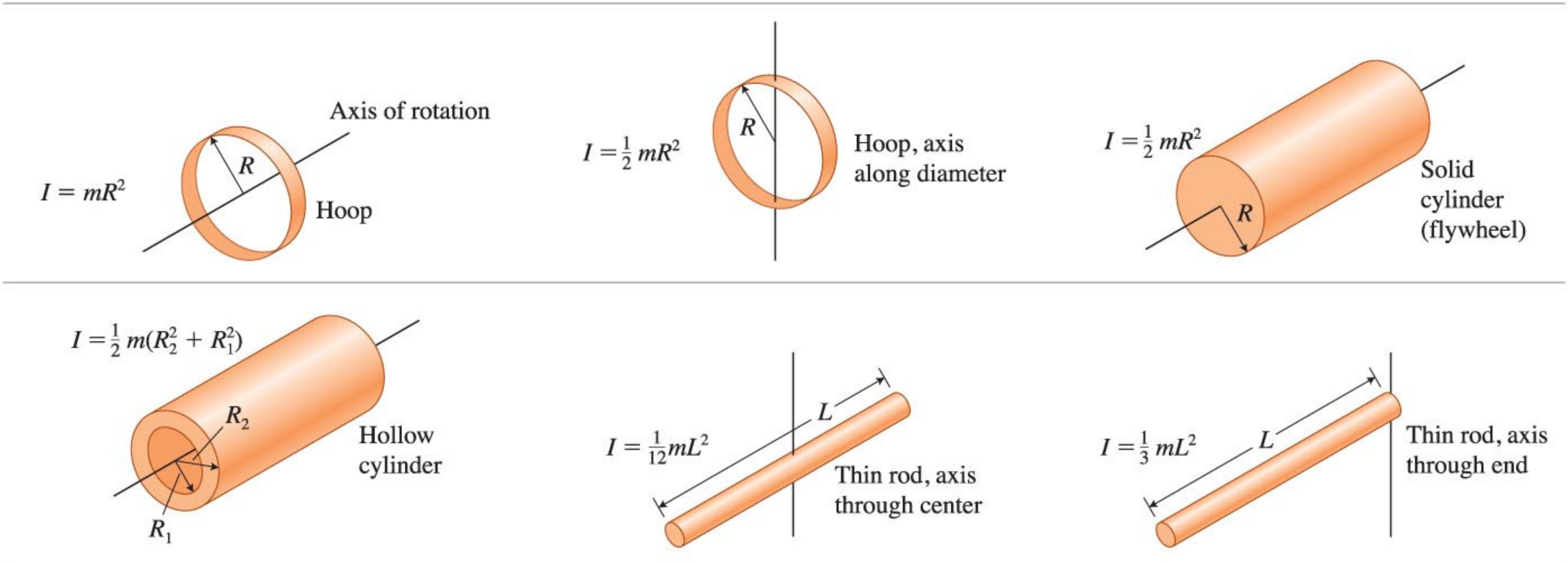
- The rotational inertia of the whole leg is

$$I = m_1 r_1^2 + m_2 r_2^2 + \ldots + m_7 r_7^2 + \ldots + m_{18} r_{18}^2$$

- There are other ways to perform the summation
  - Integral calculus
  - Numerical methods
  - Experimental measurement

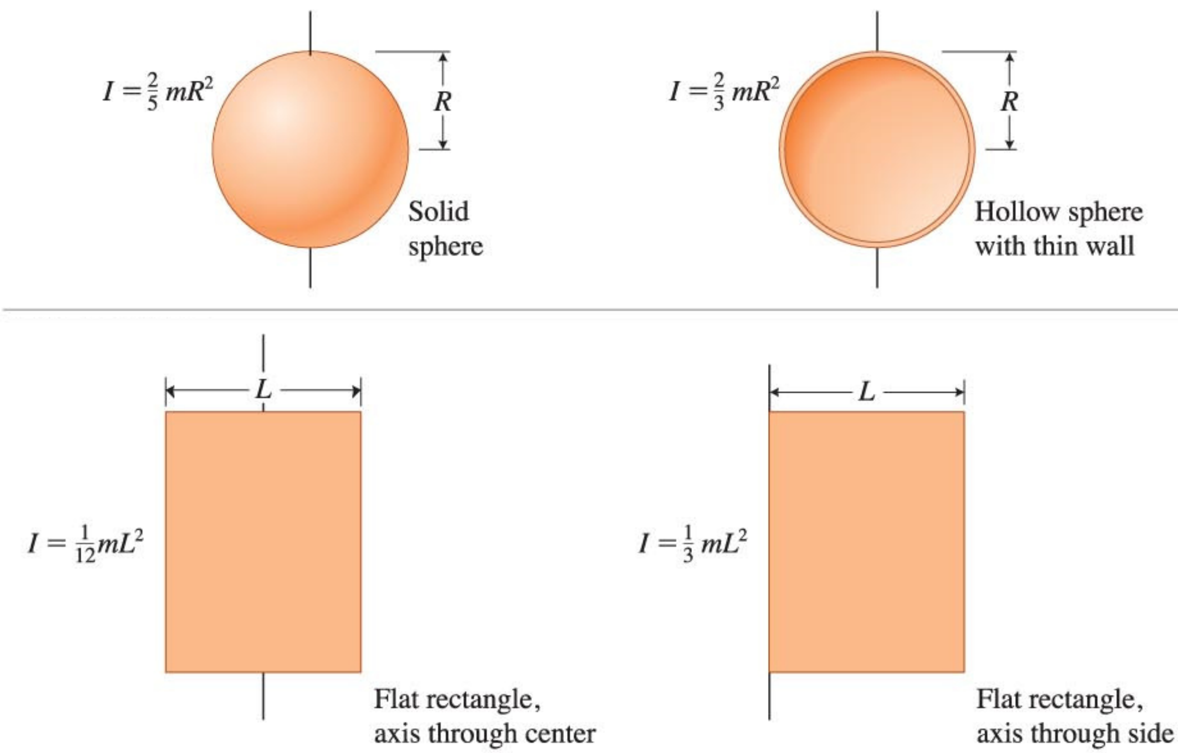
Expressions for the Rotational Inertia of Typical Objects

Table 8.6 **Expressions for the rotational inertia of standard shape objects.**



Expressions for the Rotational Inertia of Typical Objects

Table 8.6 **Expressions for the rotational inertia of standard shape objects.** ***(Continued)***



Rotational Form of Newton's Second Law

Rotational form of Newton's second law One or more objects exert forces on a rigid body with rotational inertia I that can rotate about some axis. The sum of the torques $\Sigma\tau$ due to these forces about that axis causes the object to have a rotational acceleration α:

$$\alpha = \frac{1}{I}\Sigma\tau \qquad (8.11)$$

Newton’s Second Law

TIP By writing Newton’s second law in the form

$$\vec{a}_S = \frac{1}{m_S}\Sigma\vec{F}_{\text{on S}} = \frac{\vec{F}_{O_1 \text{ on S}} + \vec{F}_{O_2 \text{ on S}} + \ldots + \vec{F}_{O_n \text{ on S}}}{m_S}$$

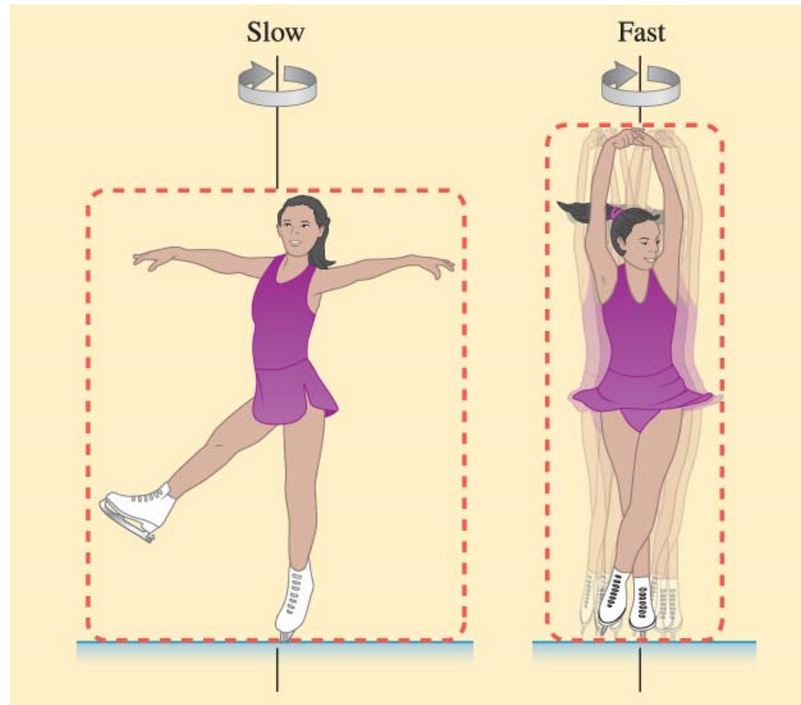
we see the cause-effect relationship between the net force $\Sigma\vec{F}_{\text{on S}}$ exerted on the system and the system’s resulting translational acceleration $\vec{a}_S$. The same idea is seen in Eq. (8.11), only applied to the rotational acceleration:

$$\alpha = \frac{1}{I}\Sigma\tau = \frac{\tau_1 + \tau_2 + \ldots + \tau_n}{I}$$

The net torque $\Sigma\tau$ produced by forces exerted on the system causes its rotational acceleration α.

Rotational Momentum

- A figure initially spins slowly with her leg and arms extended.
  - Initial rotational inertia is large and ω is small.
- She brings her arms and legs in towards the axis of rotation and spins much faster.
  - Final rotational inertia is small and ω is large.



Rotational Impulse and Rotational Momentum

- Rotational momentum is defined

$$L = I\omega$$

- Conservation of rotational momentum:

$$L_i + \sum \tau \ \Delta t = L_f$$

- When there is no external torque acting on the system,

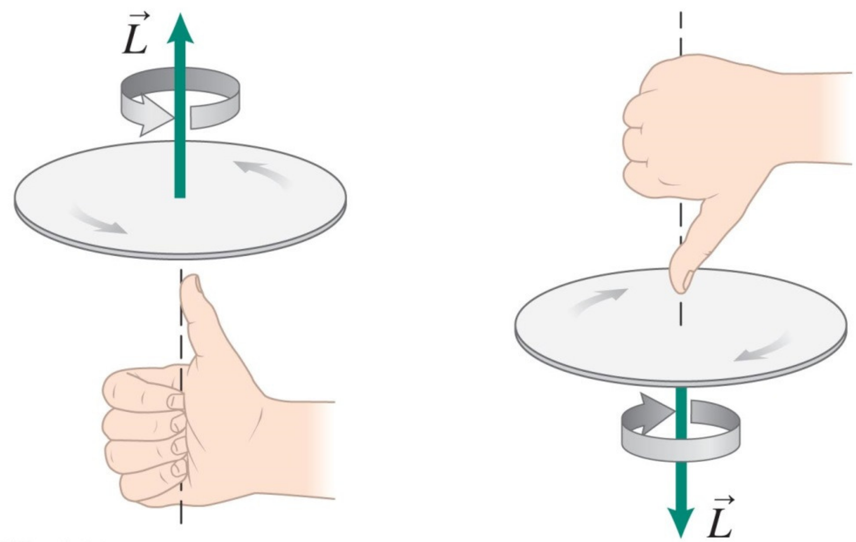
$$L_f = L_i \text{ or } I_f \omega_f = I_i \omega_i$$

- Rotational momentum is frequently also called *angular momentum.*

Rotational Momentum is a Vector

- Rotational velocity and momentum are vector quantities.
  - They have magnitude and direction
- The direction can be found by applying the right-hand rule:

Circle fingers in direction of rotation. Thumb points in the direction of rotational momentum.



Rotational Kinetic Energy

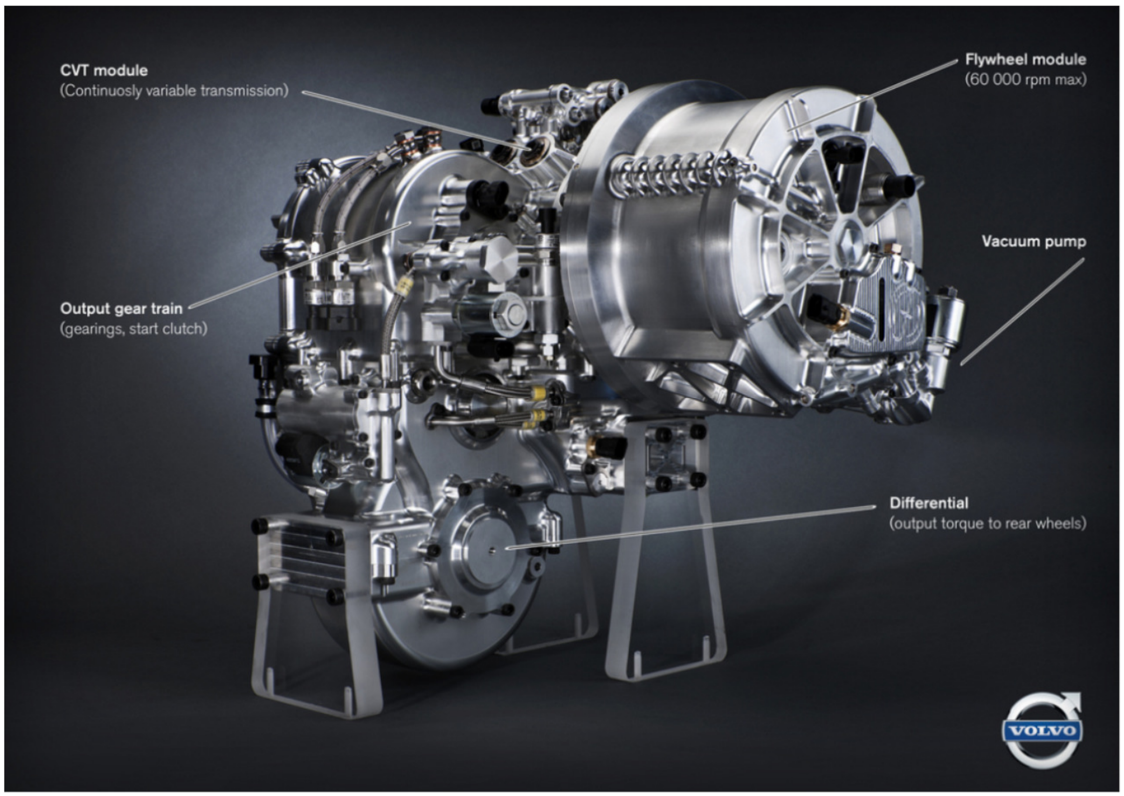
Rotational kinetic energy The rotational kinetic energy of an object with rotational inertia I turning with rotational speed ω is

$$K_{\text{rotational}} = \frac{1}{2}I\omega^2 \quad (8.14)$$

TIP When you encounter a new physical quantity, always check whether its units make sense. In this particular case the units for I are $\text{kg}\cdot\text{m}^2$ and the units for ω^2 are $1/\text{s}^2$. Thus, the unit for kinetic energy is $\text{kg}\cdot\text{m}^2/\text{s}^2 = (\text{kg}\cdot\text{m}/\text{s}^2)\text{m} = \text{N}\cdot\text{m} = \text{J}$, the correct unit for energy.

Flywheels for Storing and Providing Energy

- Some energy efficient cars have flywheels for temporarily storing energy.



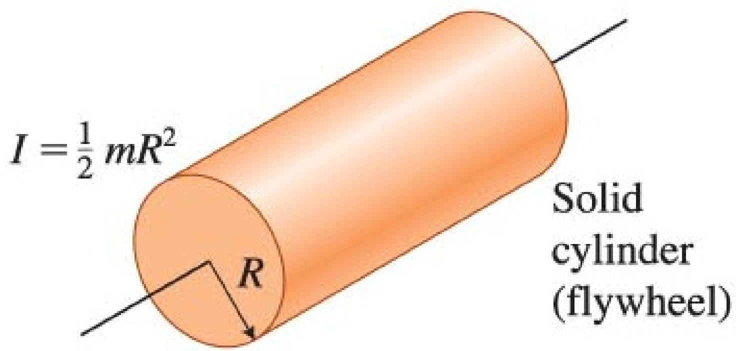
Claimed to improve fuel efficiency by 25%

Flywheels for Storing and Providing Energy

- Some energy efficient cars have flywheels for temporarily storing energy.
- In a car with a flywheel, instead of rubbing a brake pad against the wheel and slowing it down, the braking system converts the car's translational kinetic energy into the rotational kinetic energy of the flywheel.
- As the car's translational speed decreases, the flywheel's rotational speed increases. This rotational kinetic energy could then be used later to help the car start moving again.

Storing Energy in a Flywheel

- Suppose a car has a flywheel with a mass of 20 kg and a radius of 20 cm.
- Rotational inertia is: $I = \frac{1}{2}mR^2 = 0.4\ kg \cdot m^2$



Storing Energy in a Flywheel

- Suppose a 1600 kg car is travelling at a speed of 20 m/s and approaches a stop sign. If it transfers all its kinetic energy to the flywheel, what will its final rotational velocity be?

$$K = \frac{1}{2}mv^2 = \frac{1}{2}I\omega^2$$

$$\omega = v\sqrt{\frac{m}{I}} = (20\,m/s)\sqrt{\frac{1600\,kg}{0.4\,kg \cdot m^2}} = 1265\,s^{-1}$$

$$\approx 12000\,RPM$$