PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 22000

General Physics

Lecture 13 – Collisions and Extended Objects

Fall 2016 Semester

Prof. Matthew Jones

SUPPLEMENTAL INSTRUCTION

Free Study Sessions!

Rachel Hoagburg

Come to SI for more help in PHYS 220

Tuesday and Thursday	**7:30-8:30PM Shreve C113**

Office Hour

Tuesday 1:30-2:30 4th floor of Krach

For other SI-linked courses and schedules, visit purdue.edu/si or purdue.edu/boilerguide

Momentum and Energy

- Recall that momentum is always conserved

$$\vec{p} = m\vec{v}$$

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

(when there is no external impulse)

- Energy is also always conserved

$$U_i + W = U_f$$

$$\left(K_i + U_{gi} + U_{si}\right) + W = (K_f + U_{gf} + U_{sf} + \Delta U_{int})$$

$$\Delta U_{int} = U_{int\ f} - U_{int\ i}$$

Energy Conservation

- Energy is always conserved, but the different types of energy are not necessarily individually conserved.
- Kinetic energy can be transformed into internal energy (heating, or deforming an object).
- Examples include elastic and inelastic collisions of two objects…

Two Types of Collisions

- Suppose the two objects have equal mass ($m_1 = m_2$)

$$p_{1i} = m_1 v_{1i}$$
$$p_{2f} = m_2 v_{2f}$$

- Momentum is conserved so we conclude that

$$v_{2f} = v_{1i}$$

- Energy is also conserved:

$$K_i = \frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_2 v_{2f}^2 = K_f$$

Two Types of Collisions

- Suppose the two objects have equal mass ($m_1 = m_2$)

$$p_{1i} = m_1 v_{1i}$$
$$p_{2f} = m_2 v_{2f}$$

- Momentum is conserved so we conclude that

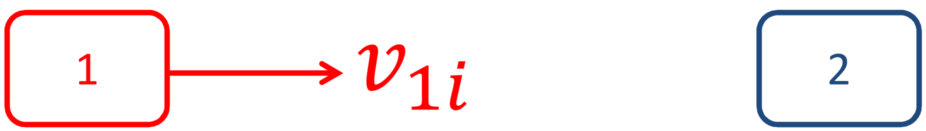
$$v_{2f} = v_{1i}$$

- Energy is also conserved:

$$K_i = \frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_2 v_{2f}^2 = K_f$$

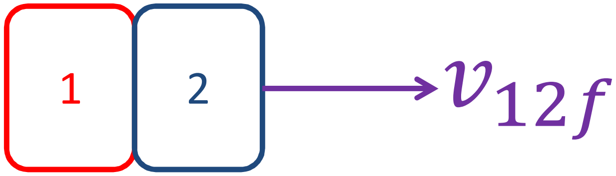
Two Types of Collisions

- Next, suppose the objects squish, and stick together:



Two Types of Collisions

- Next, suppose the objects squish, and stick together:



- Momentum is still conserved:

$$m_1 v_{1i} = (m_1 + m_2) v_{12f}$$

- Final kinetic energy is:

$$K_f = \frac{1}{2}(m_1 + m_2) v_{12f}^2 = \frac{1}{2}\frac{m_1^2}{m_1 + m_2} v_{1i}^2 < \frac{1}{2} m_1 v_{1i}^2 = K_i$$

- We still believe that energy was conserved, so where did the missing energy go?

Inelastic Collisions

- By measuring the final velocities, we can determine the amount of energy converted to internal energy due to deformation of the objects
- **In general, you can't predict this value ahead of time.**
- When objects deform, we cannot make any predictions about the amount of kinetic energy that is converted to internal energy
- Even in collisions where there is deformation, the momentum of the system remains constant.

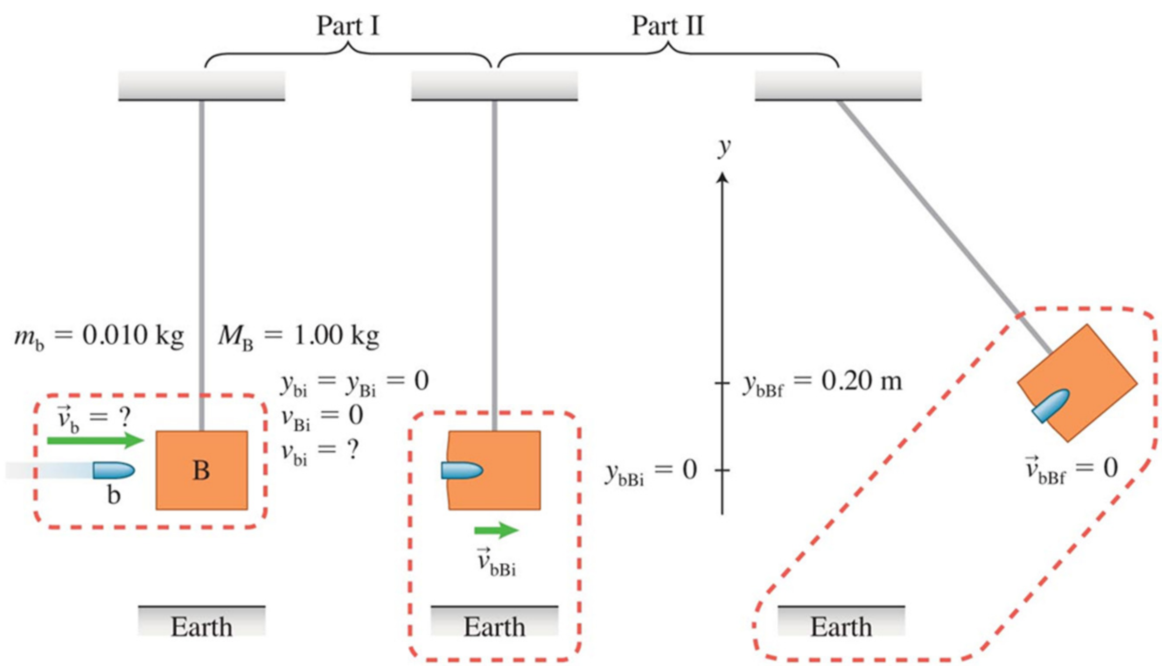
Types of Collisions

Table 6.7 **Types of collisions.**

Elastic collisions	Inelastic collisions	Totally inelastic collisions
Both the momentum and kinetic energy of the system are constant. The internal energy of the system does not change. The colliding objects never stick together. Examples: There are no perfectly elastic collisions in nature, although collisions between very rigid objects (such as billiard balls) come close. Collisions between atoms or subatomic particles are almost exactly elastic.	The momentum of the system is constant but the kinetic energy is not. The colliding objects do not stick together. Internal energy increases during the collisions. Examples: A volleyball bouncing off your arms, or you jumping on a trampoline.	These are inelastic collisions in which the colliding objects stick together. Typically, a large fraction of the kinetic energy of the system is converted into internal energy in this type of collision. Examples: You catching a football, or a car collision where the cars stick together.

Example: A Ballistic Pendulum

- A gun is several centimeters from a 1.0-kg wooden block hanging at the end of strings. The gun fires a 10-g bullet that becomes embedded in the block, which swings upward a height of 0.20 m.
- Determine the speed of the bullet…



A Ballistic Pendulum

- Initial momentum (just before collision):

$$p_i = m_b v_b$$

- Final momentum (just after collision):

$$p_f = (M_B + m_b) v_{bBi}$$

- Kinetic energy (just after collision):

$$K_f = \frac{1}{2}(M_B + m_b) v_{bBi}^2$$

- Final gravitational potential energy:

$$U_{gf} = (M_B + m_b) g y_{bBf}$$

A Ballistic Pendulum

- Kinetic energy is converted to gravitational potential energy:

$$K_f = \frac{1}{2}(M_B + m_b)v_{Bbi}^2 = (M_B + m_b)gy_{bBf} = U_g$$

$$v_{Bbi} = \sqrt{2gy_{bBf}}$$

- Momentum conservation:

$$m_b v_b = (M_B + m_b)v_{bBi}$$

$$v_b = \frac{M_B + m_b}{m_b} v_{bBi} = \frac{M_B + m_b}{m_b}\sqrt{2gy_{bBf}}$$

$$= \frac{1.010\ kg}{0.01\ kg}\sqrt{2(9.8\ m/s^2)(0.2\ m)} = 200\ m/s$$

Power

- Why is it more difficult for the same person to run up a flight of stairs than to walk if the change in gravitational potential energy of the person-Earth system is the same in both scenarios?
  - The amount of internal energy converted into gravitational energy is the same in both cases, but the rate of that conversion is not.
  - When you run upstairs, you convert the energy at a faster rate.
  - The rate at which the conversion occurs is called the power.

Power

Power The power of a process is the amount of some type of energy converted into a different type divided by the time interval Δt in which the process occurred:

$$\text{Power} = P = \left|\frac{\Delta U}{\Delta t}\right| \tag{6.9}$$

If the process involves external forces doing work, then power can also be defined as the magnitude of the work W done on or by the system divided by the time interval Δt needed for that work to be done:

$$\text{Power} = P = \left|\frac{W}{\Delta t}\right| \tag{6.10}$$

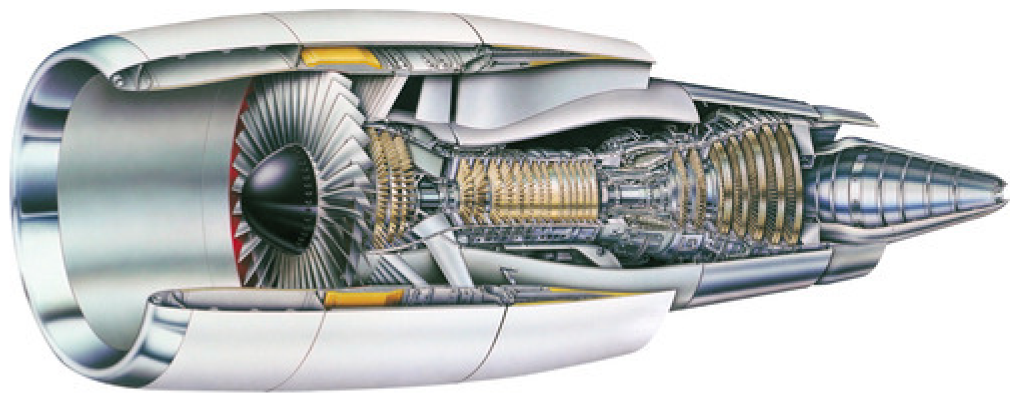
The SI unit of power is the watt (W). 1 watt is 1 joule/second $(1\text{ W} = 1\text{ J/s})$.

Power

- Power is sometimes expressed in horsepower (hp): 1 hp = 746 W.
- Horsepower is most often used to describe the power rating of engines or other machines.
  - A 50-hp gasoline engine (typical in cars) converts the internal energy of the fuel into other forms of energy at a rate of 50 x 746 W = 37,300 W, or 37.30 J/s.

Power of Engines

5 HP =3.73 kW

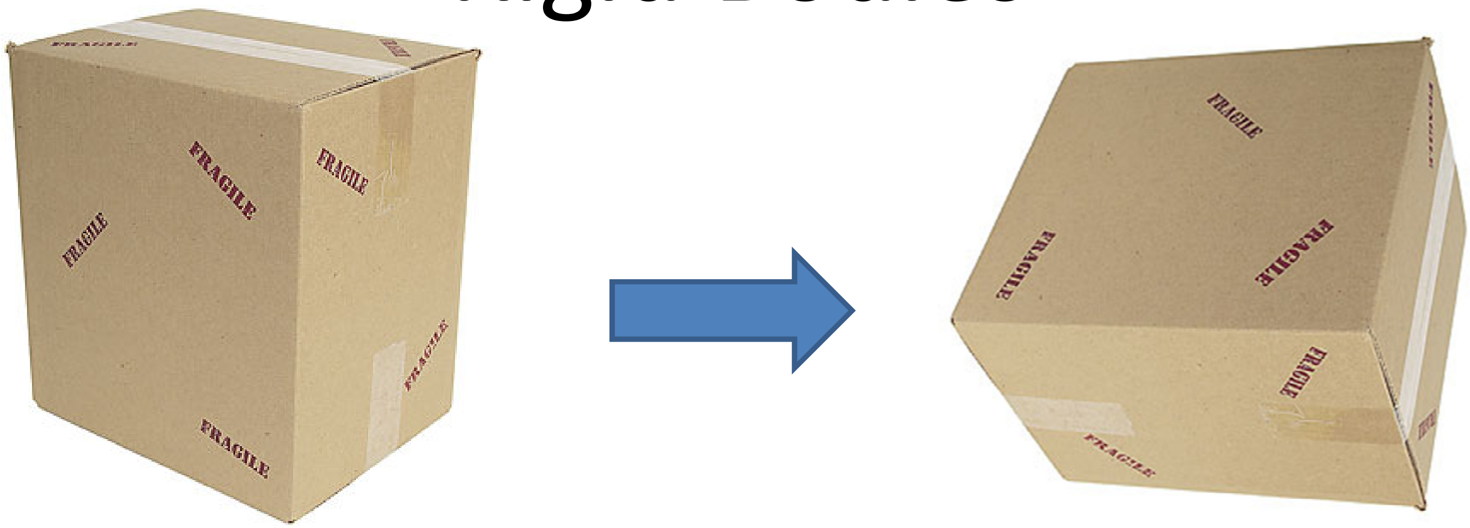


50,000 HP =37.3 MW

More Complex Objects

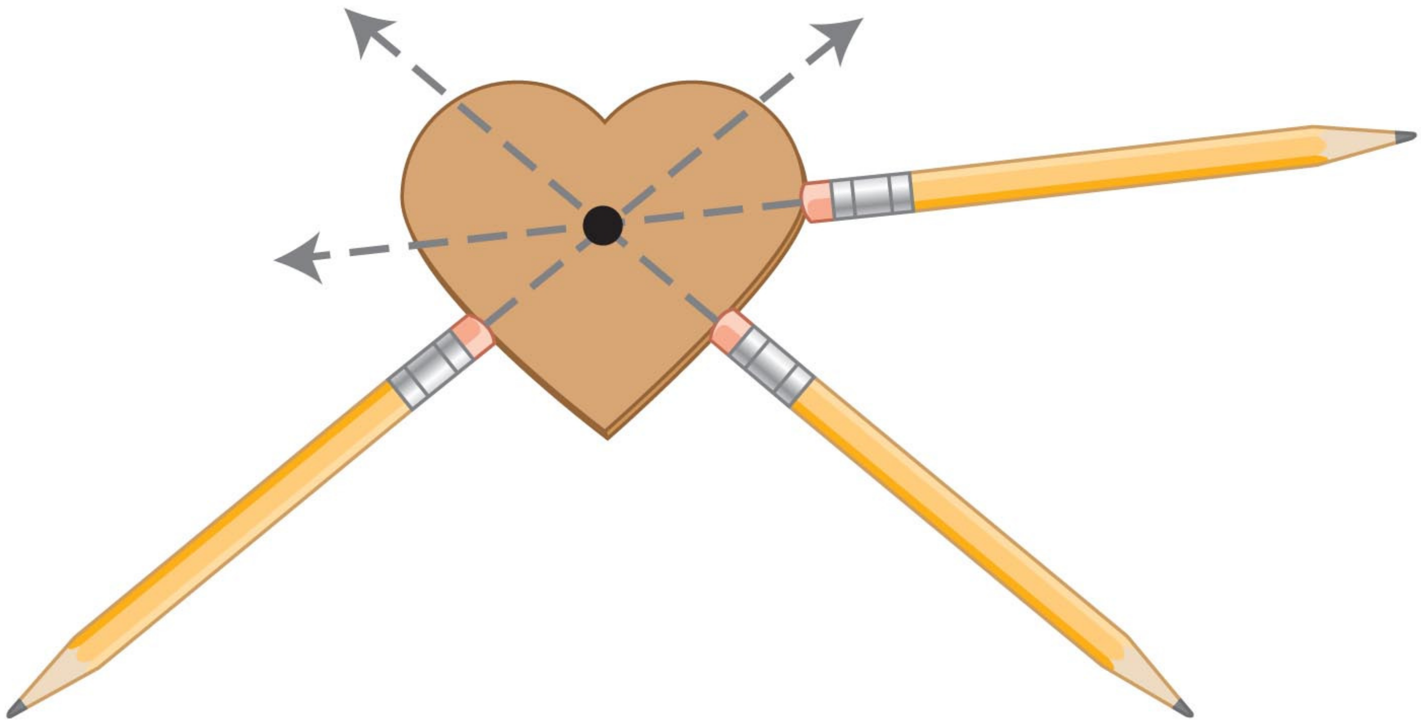
- So far we have been modeling objects as point-like with no internal structure.
- Objects are actually extraordinarily complex, with many internal parts that rotate and move relative to one another.
- To study complex structures, we need to develop a new way of modeling objects and of analyzing their interactions.

Rigid Bodies



- A rigid body is a model for an extended object.
- We assume that the object has a nonzero size but the distances between all parts of the object remain the same (the size and shape of the object do not change).

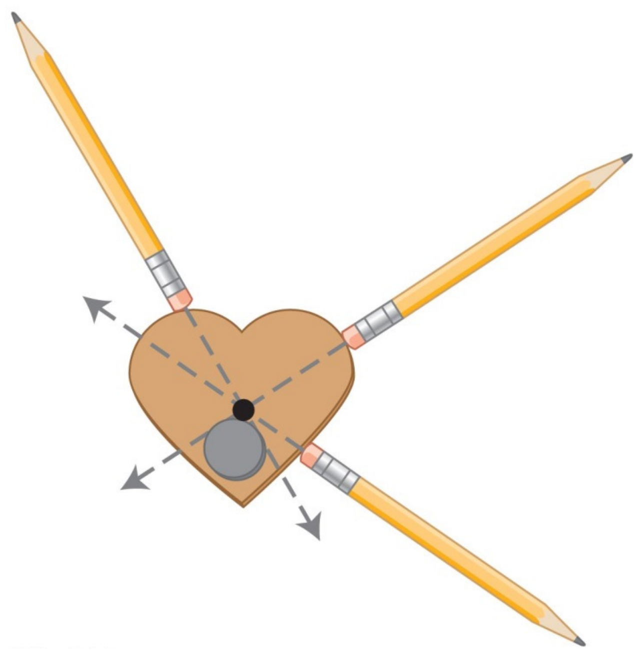
Center of Mass



- A rigid body possesses a special point such that if a force is exerted on that point, the object will not turn.
- We call this point the object's center of mass.

Center of Mass: Qualitative Definition

- The center of mass of an object is a point where a force exerted on the object pointing directly toward or away from that point will not cause the object to turn. The location of this point depends on the mass distribution of the object.

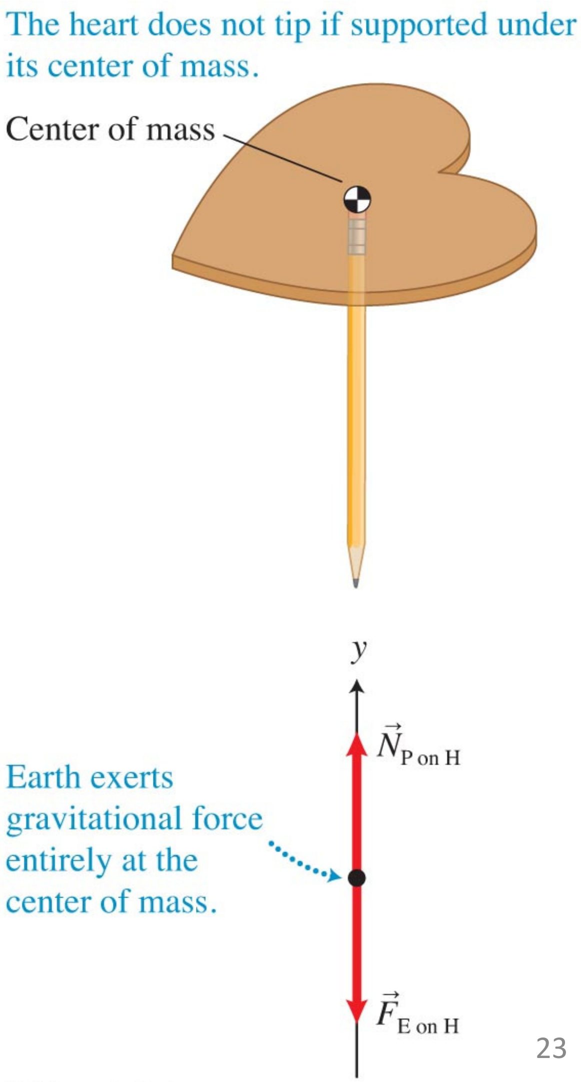


Center of Mass

- Although the location of the center of mass depends on the mass distribution of the object, the mass of the object is not necessarily evenly distributed around the center of mass.
- We will learn more about the properties of the center of mass; we just want to caution you about taking the name of this point literally.

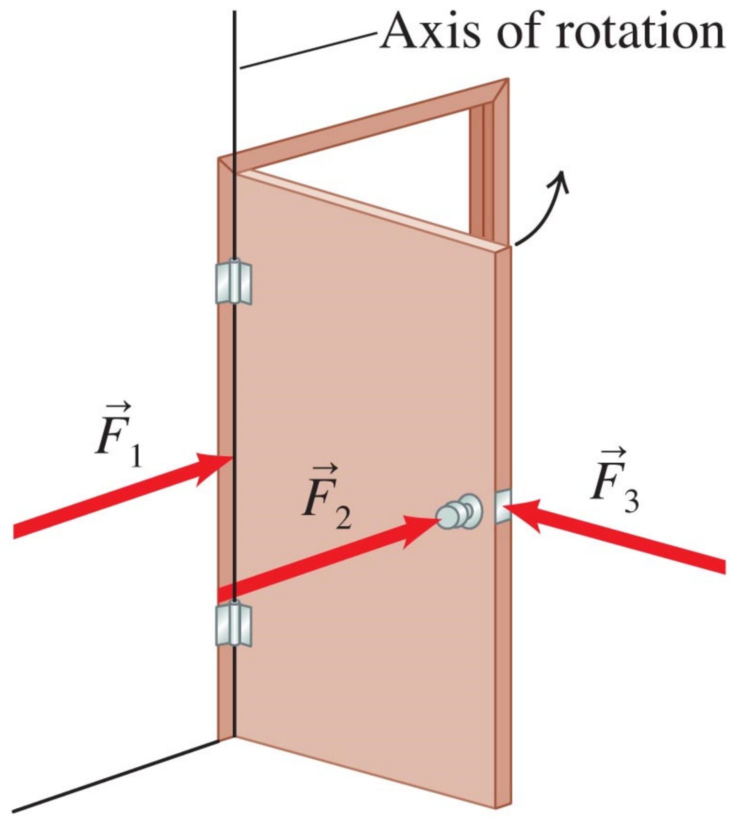
Where is the Gravitational Force Exerted on a Rigid Body?

- If the cardboard does not tip, all forces exerted on it pass through the center of mass.
- We can assume that the gravitational force exerted on an object is exerted at the location of its center of mass.
- That is why the object's center of mass is sometimes called the object's **center of gravity.**



Axis of Rotation

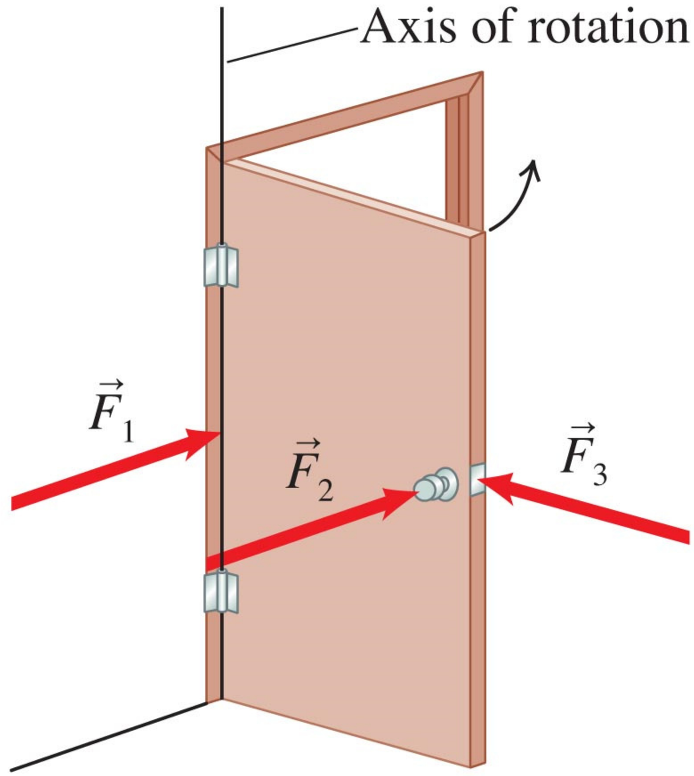
- When objects turn around an axis, physicists say that they undergo **rotational** motion.
- We call the imaginary line passing through the hinges the **axis of rotation.**



$\vec{F}_1$ and $\vec{F}_3$ do not rotate the door, whereas $\vec{F}_2$ moves it easily.

Causing Rotation

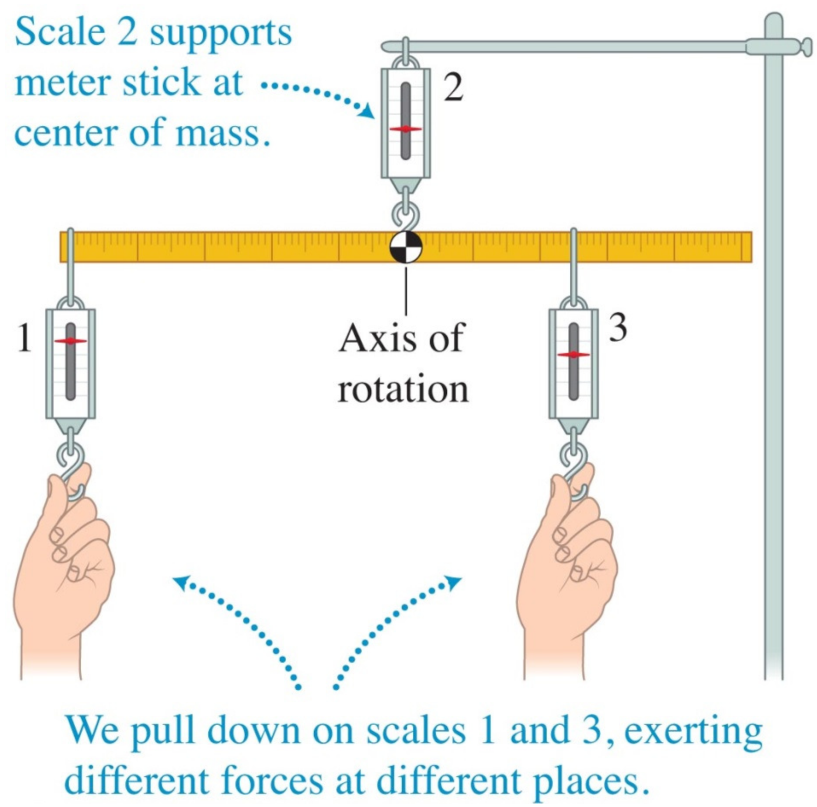
- Three factors affect the turning ability of a force:
  - The place where the force is exerted
  - The magnitude of the force
  - The direction in which the force is exerted



$\vec{F}_1$ and $\vec{F}_3$ do not rotate the door, whereas $\vec{F}_2$ moves it easily.

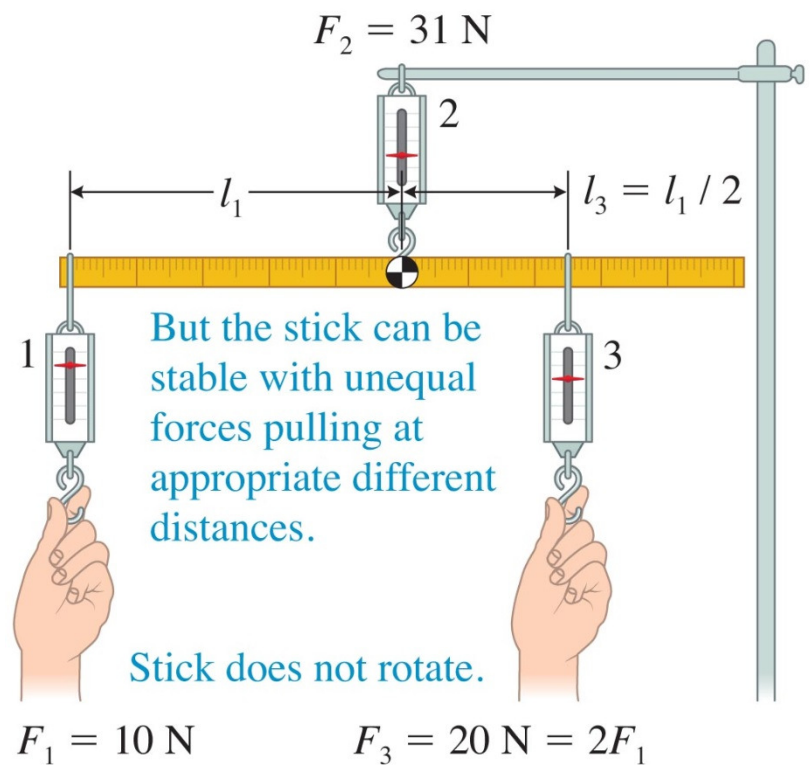
Qualitative Expressions for the Turning Ability of a Force

- If scale 1 is twice as far from the axis of rotation and pulls half as hard as scale 3, the stick remains in equilibrium.
- If scale 1 is three times farther from the axis of rotation and exerts one-third of the force of scale 3, the stick remains in equilibrium



Static Equilibrium

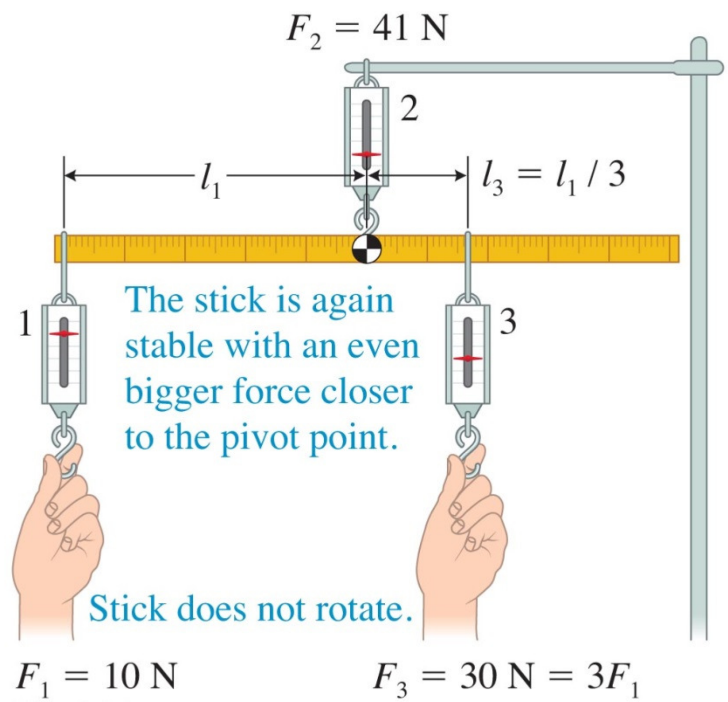
- An object is in static equilibrium when it remains at rest (does not undergo either translational or rotational motion) with respect to a particular observer in an inertial reference frame.



Static Equilibrium

- When the following condition is satisfied, an object will remain in static equililbrium:

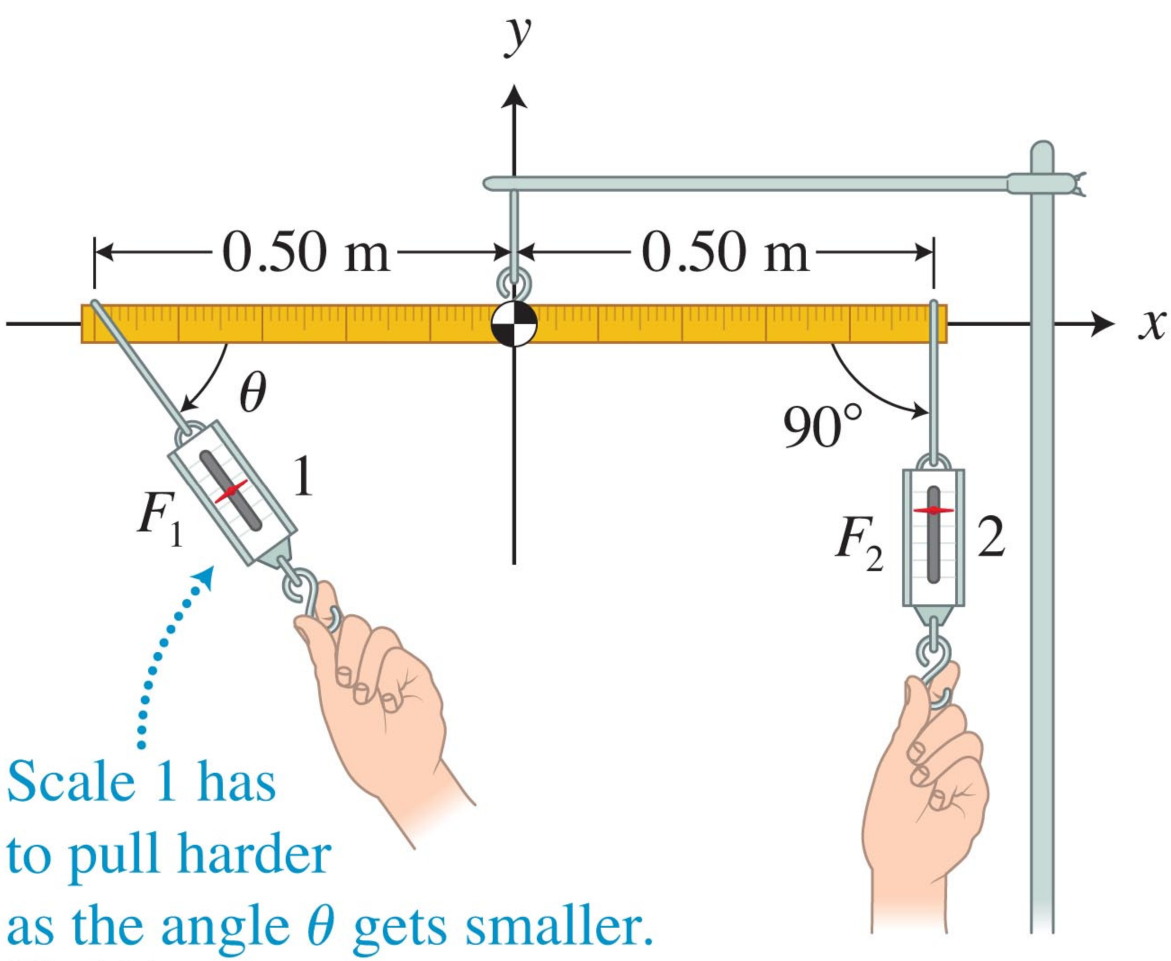
$$F_{left} l_{left} = F_{right} l_{right}$$



The "Turning Ability" of a Force

- It is equal to the product of the magnitude of the force and the distance the force is exerted from the axis of rotation.
- It is positive when the force tends to turn the object counterclockwise and negative when it turns the object clockwise.
- When one force tends to rotate an object counterclockwise and another force rotates it clockwise, their effects can cancel.

The Dependence of the "Turning Ability" of a force on an angle



Factors that Affect the "Turning Ability" of a Force

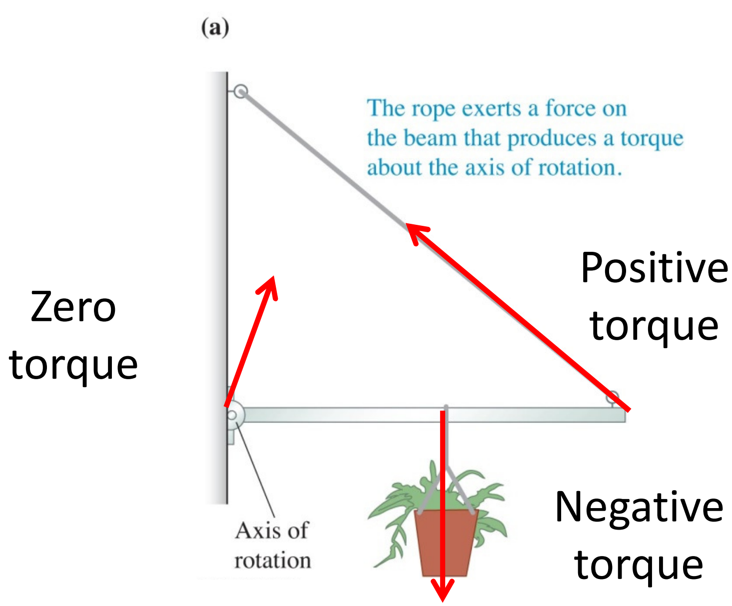
- The direction (counterclockwise or clockwise) that the force can potentially rotate the object
- The magnitude of the force F
- The distance l of the point of application of the force from the axis of rotation
- The angle that the force makes relative to a line from the axis of rotation to the point of application of the force

The Torque (τ) Produced by a Force

Torque τ produced by a force The torque produced by a force exerted on a rigid body about a chosen axis of rotation is

$$\tau = \pm Fl \sin\theta \qquad (7.1)$$

where F is the magnitude of the force, l is the magnitude of the distance between the point where the force is exerted on the object and the axis of rotation, and θ is the angle that the force makes relative to a line connecting the axis of rotation to the point where the force is exerted (see **Figure 7.10**).



The SI unit of torque is the Newton-meter (N·m)

Conditions for Static Equilibrium

- An object is in static equilibrium if an observer in an inertial reference frame does not observe it to move or to rotate.
- An object modeled as a rigid body is in translational static equilibrium when the net force acting on it is zero:

$$\sum F_{\text{on O } x} = F_{1 \text{ on O } x} + F_{2 \text{ on O } x} + \cdots + F_{n \text{ on O } x} = 0$$

$$\sum F_{\text{on O } y} = F_{1 \text{ on O } y} + F_{2 \text{ on O } y} + \cdots + F_{n \text{ on O } y} = 0$$

Conditions for Static Equilibrium

- A rigid body is in turning or rotational static equilibrium if it is at rest with respect to the observer and the sum of the torques about any axis of rotation produced by the forces exerted on the object is zero.
  - positive torques are counterclockwise
  - negative torques are clockwise

$$\Sigma\tau = \tau_1 + \tau_2 + \cdots + \tau_n = 0$$

Tip

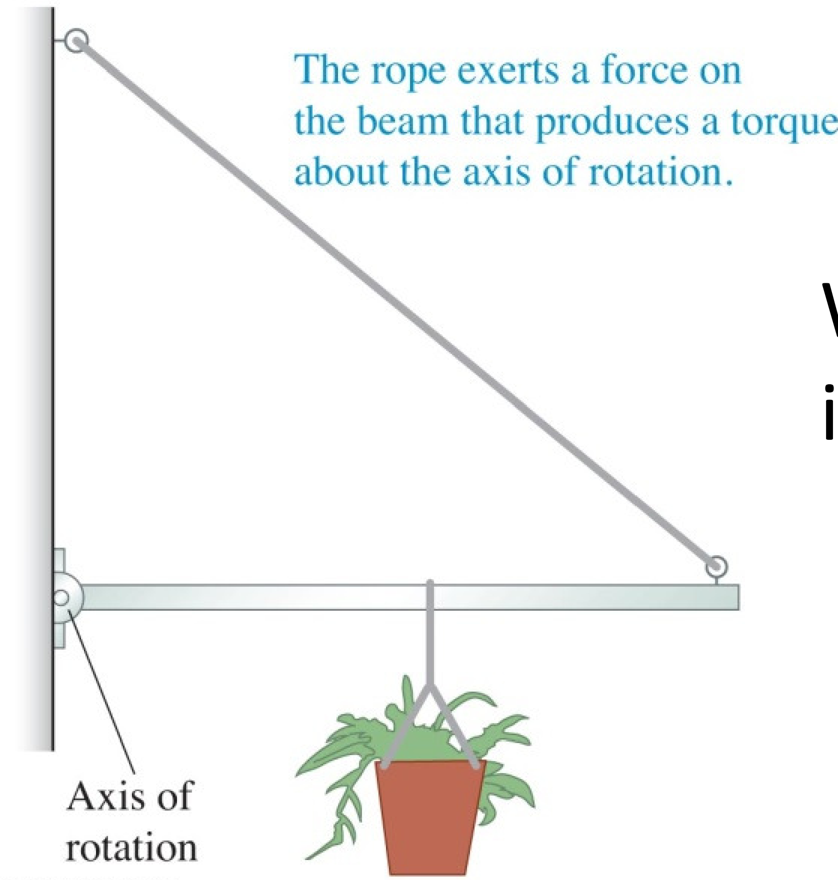
- All of the gravitational forces exerted by Earth on the different parts of a rigid body can be summarized as a single gravitational force being exerted on the center of mass of the rigid body.

Another Tip

- If a rigid body is in static equilibrium, the sum of the torques about any axis of rotation is zero. In problem solving, it is often helpful to place the imaginary axis at the point on the rigid body where the force you know least about is exerted. That force drops out of the second condition of the equilibrium, and you can then use that equation to solve for some other unknown quantity.

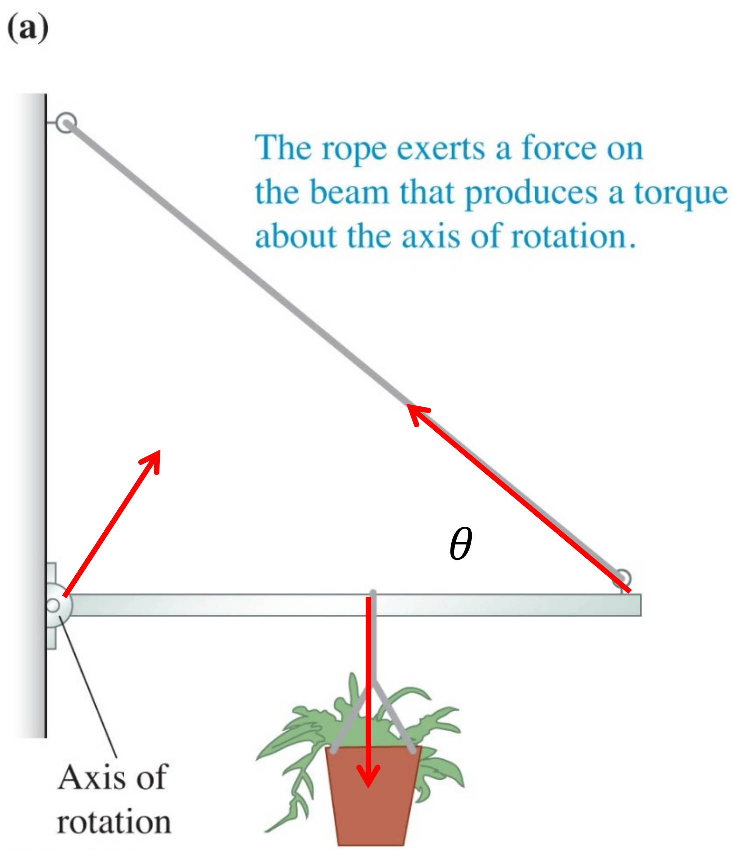
Example

(a)



What is the tension in the rope?

Example



There are three forces:

1. The weight of the plant

$$\tau = -mgl/2$$

2. The tension in the rope

$$\tau = Tl\sin\theta$$

3. The reaction of the pivot (no torque because it acts through the axis of rotation)

$$T = \frac{mg}{2\sin\theta}$$

Center of Mass

Center of mass (quantitative definition) If we consider an object as consisting of parts 1, 2, 3, . . . n whose centers of masses are located at the coordinates (x_1, y_1); (x_2, y_2); (x_3, y_3); . . . (x_n, y_n), then the center of mass of this whole object is at the following coordinates:

$$x_{cm} = \frac{m_1x_1 + m_2x_2 + m_3x_3 + \ldots + m_nx_n}{m_1 + m_2 + m_3 + \ldots + m_n}$$

$$y_{cm} = \frac{m_1y_1 + m_2y_2 + m_3y_3 + \ldots + m_ny_n}{m_1 + m_2 + m_3 + \ldots + m_n} \qquad (7.4)$$

- Using these equations for an object with a continuous mass distribution is difficult and involves calculus.