Physics 22000

General Physics

Lecture 11 – Work and Energy

Fall 2016 Semester

Prof. Matthew Jones

First Midterm Exam

Tuesday, October 4th, 8:00-9:30 pm
Location: PHYS 112 and WTHR 200.

Covering material in chapters 1-6
(but probably not too much from chapter 6)

Multiple choice, probably about 25 questions, 15 will be conceptual, 10 will require simple computations.
A formula sheet will be provided.
You can bring one page of your own notes.

I put a couple exams from previous years on the web page.

SUPPLEMENTAL INSTRUCTION

Free Study Sessions!

Rachel Hoagburg

Come to SI for more help in PHYS 220

Tuesday and Thursday	**7:30-8:30PM Shreve C113**

Office Hour

Tuesday 1:30-2:30 4th floor of Krach

For other SI-linked courses and schedules, visit purdue.edu/si or purdue.edu/boilerguide

Work and Energy

- The total energy U of a system is the sum:

$$\text{Total Energy} = U = K + U_g + U_s + U_{int}$$

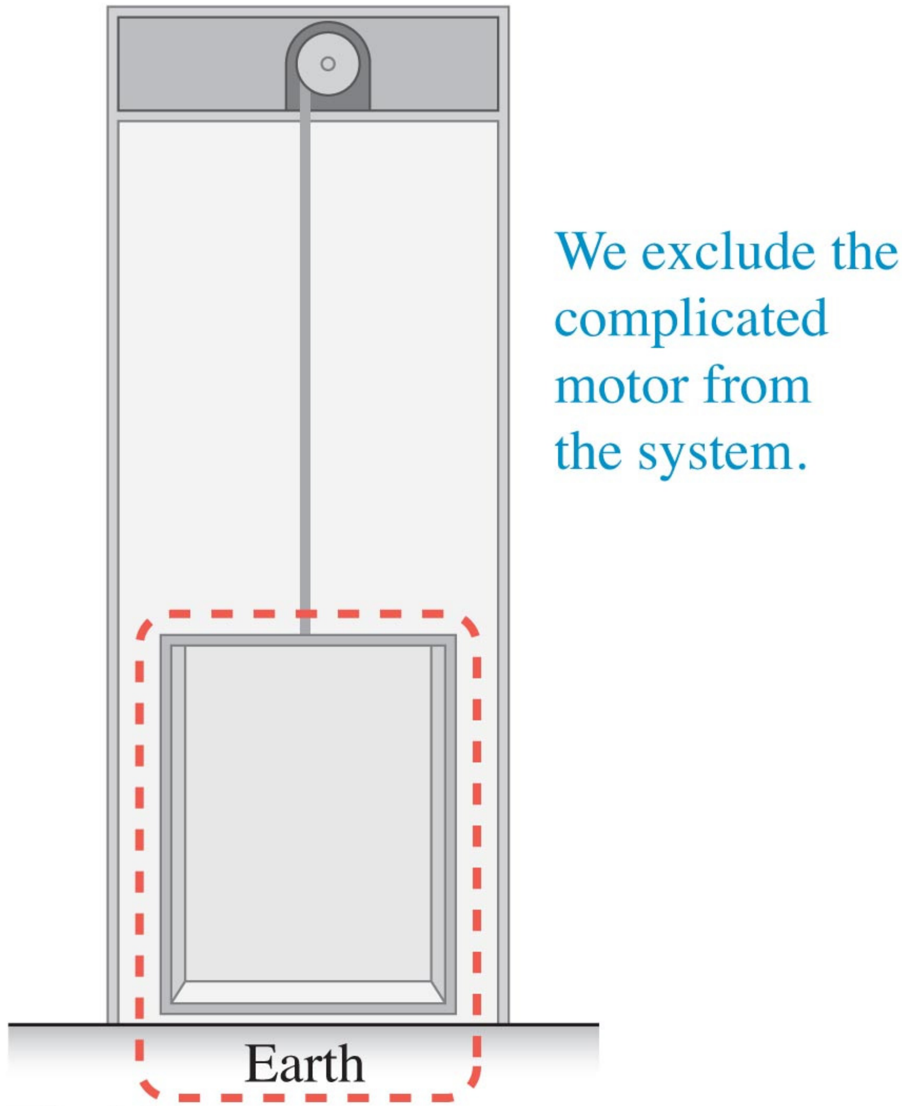
  - K is the kinetic energy
  - U_g is the gravitational potential energy
  - U_s is the elastic potential energy
  - U_{int} is the internal energy

- An external force does work on a system and can change its total energy:

$$U_i + W = U_f$$

Choosing What to Include in a System

- It is preferable to have a larger system so that the changes occurring can be included as energy changes within the system rather than as the work done by external forces.
- Even so, often it is best to exclude something like a motor from a system because its energy changes are complex.



Gravitational Potential Energy

- A rope lifts a heavy box upward at a constant negligible velocity. The box is the system.
  - The box moves at constant velocity; the upward tension force of the rope on the box is equal in magnitude to the downward gravitational force Earth exerts on the box: $m_{box}g$.
  - The rope does work on the box, changing its vertical position from y_i to y_f:

$$W_{R\ on\ B} = T_R d \cos\theta = T_R(y_f - y_i)\cos 0°$$

$$= mg\left(y_f - y_i\right)$$

$$U_{gi} + mg\left(y_f - y_i\right) = U_{gf}$$

Gravitational Potential Energy

- Changes in gravitational potential energy only depend on changes in the height of an object.
- It is convenient to define U_g to be zero when the object is at the origin of the vertical axis.
- Then, at an time, the object has gravitational potential energy

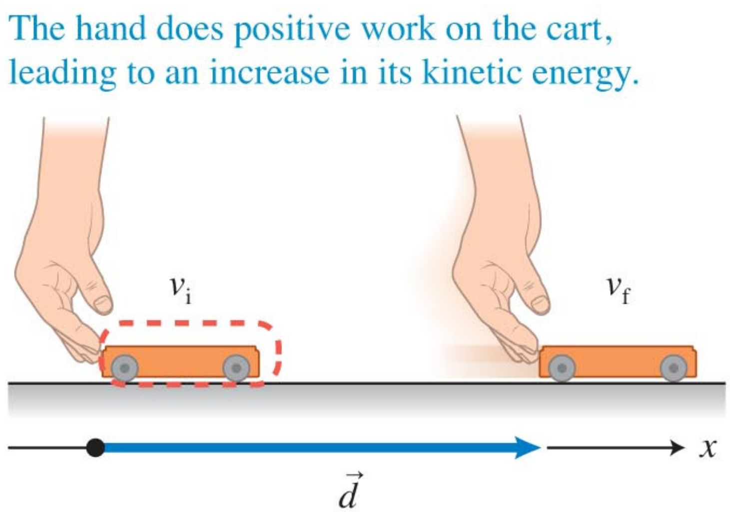
$$U_g = mgy$$

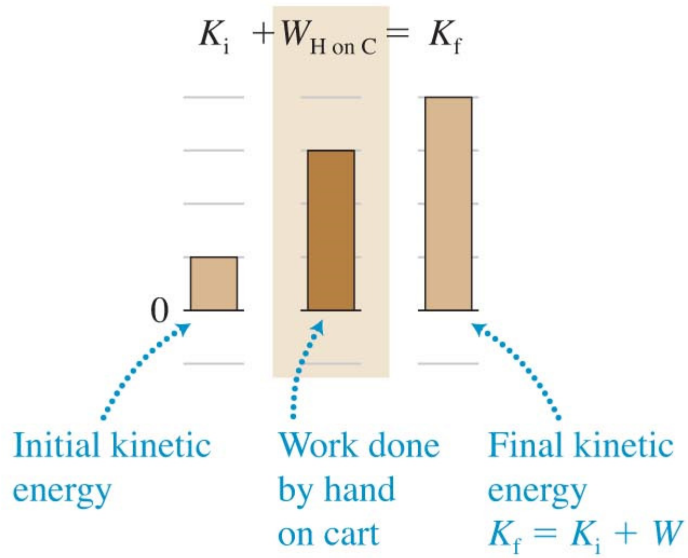
- Units for gravitational potential energy are

$$kg \cdot (N/kg) \cdot m = N \cdot m \equiv J \text{ (joule).}$$

Kinetic Energy

- The cart is the system. Your hand exerts a force on the cart while pushing it to the right on a horizontal frictionless surface.





$$W_{H\ on\ C} = K_f - K_i$$
$$F_{H\ on\ C}\, d = K_f - K_i$$

Kinetic Energy

- The equation we found can be put in terms of properties of the cart using dynamics and kinematics:

$$2a_C d = v_f^2 - v_i^2$$

$$a_C d = \frac{v_f^2 - v_i^2}{2}$$

$$\boldsymbol{m_C a_C} d = \frac{1}{2} m_C v_f^2 - \frac{1}{2} m_C v_i^2$$

$$\boldsymbol{m_C\ a_C} = F_{H\ on\ C}$$

$$F_{H\ on\ C}\ d = \boldsymbol{K_f} - \boldsymbol{K_i} = \boldsymbol{\frac{1}{2} m_C v_f^2 - \frac{1}{2} m_C v_i^2}$$

Kinetic Energy

- The kinetic energy of an object is

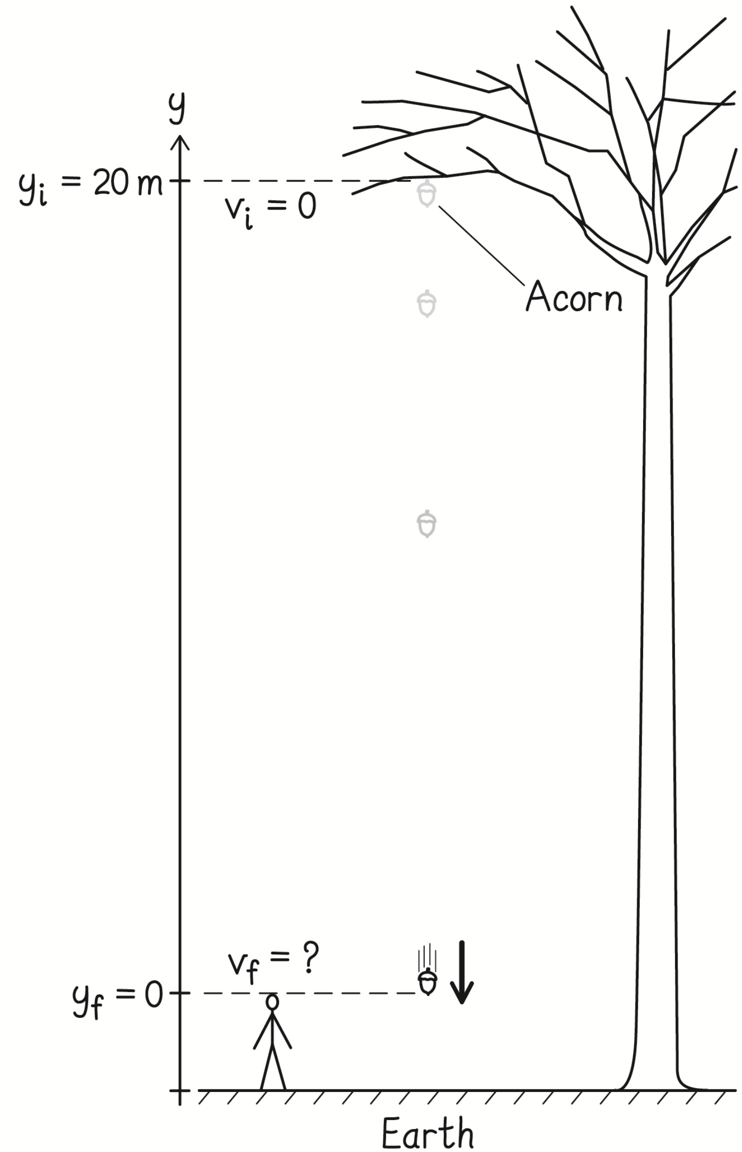
$$K = \frac{1}{2}mv^2$$

- Kinetic energy also has units of joules:

$$\frac{kg \cdot m^2}{s^2} = \left(\frac{kg \cdot m}{s^2}\right) m = N \cdot m = J$$

Example

- You sit on the deck behind your house. Several 5-g acorns fall from the trees high above, just missing your chair and head.
- Use the work-energy equation to estimate how fast one of these acorns is moving just before it reaches the level of your head.



Example

- The initial potential energy is:

$$U_{gi} = mgy_i$$

- The initial kinetic energy is $K_i = 0$.
- The final potential energy is $U_{gf} = 0$.
- The final kinetic energy is

$$K_f = \frac{1}{2} m\, v^2 = U_{gi} = mgy_i$$

- Solve for v:

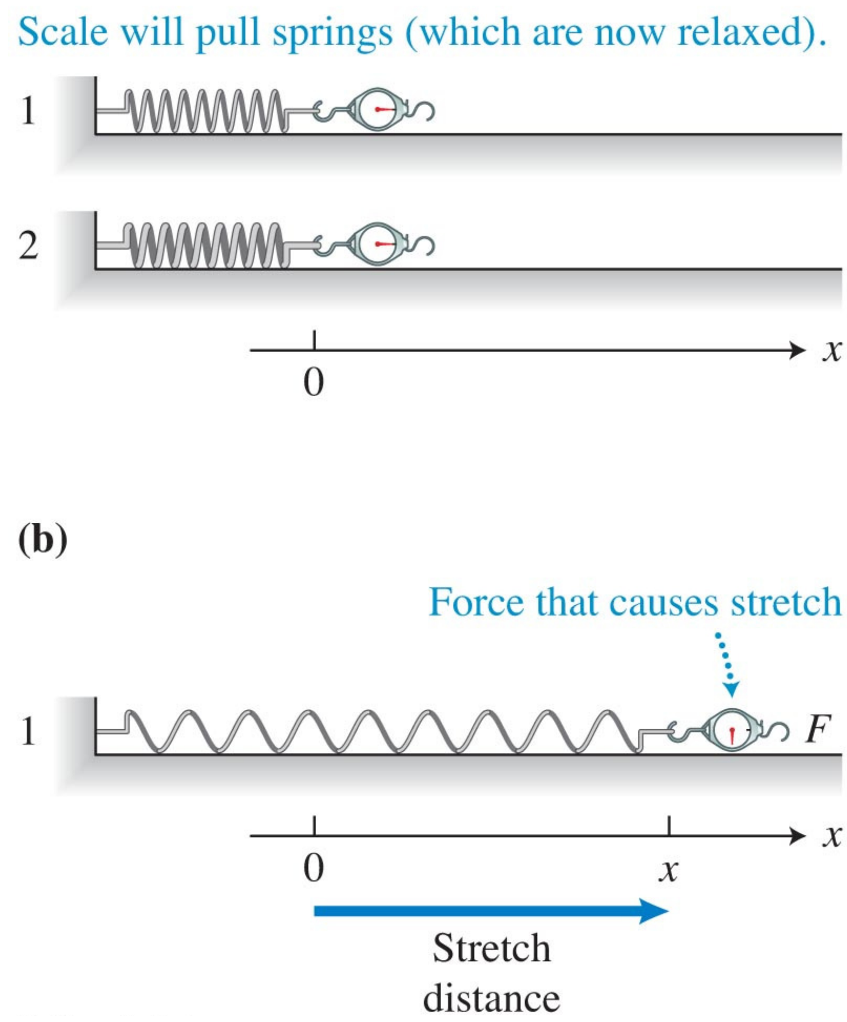
$$v = \sqrt{2gy_i}$$

Elastic Potential Energy

- When you stretch or compress an elastic spring-like object, you have to pull or push harder as the object is stretched or compressed more.
  - **The force is not constant.**
- This factor makes it more difficult to find a mathematical expression for the elastic potential energy stored by an elastic object when it has been stretched or compressed.

Hooke's Law

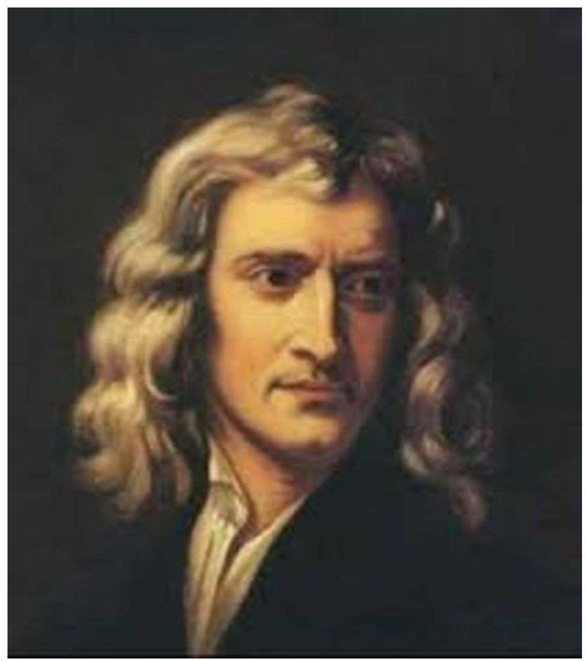
- Two springs of the same length—one less stiff and the other more stiff—are pulled, and the magnitude of the force and the distance that each spring stretches from its unstretched position are measured.



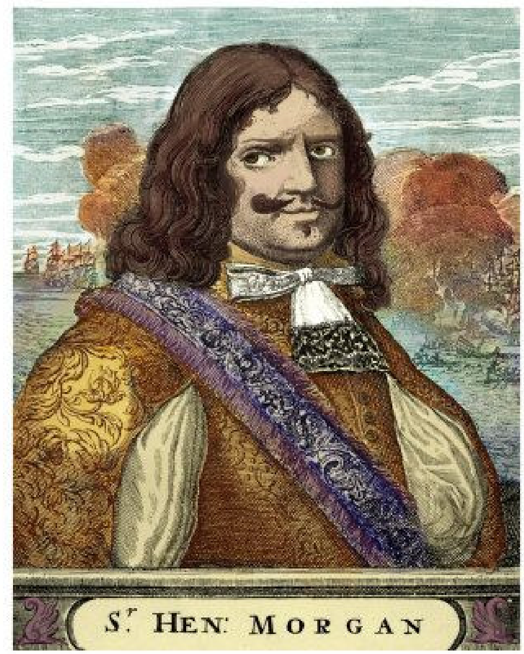
The Study of Springs and Things

- Robert Hooke was a contemporary of Newton (and Henry Morgan)

Robert Hooke, 1635-1703



Isaac Newton, 1643-1727



Henry Morgan, 1635-1688

Hooke’s Law

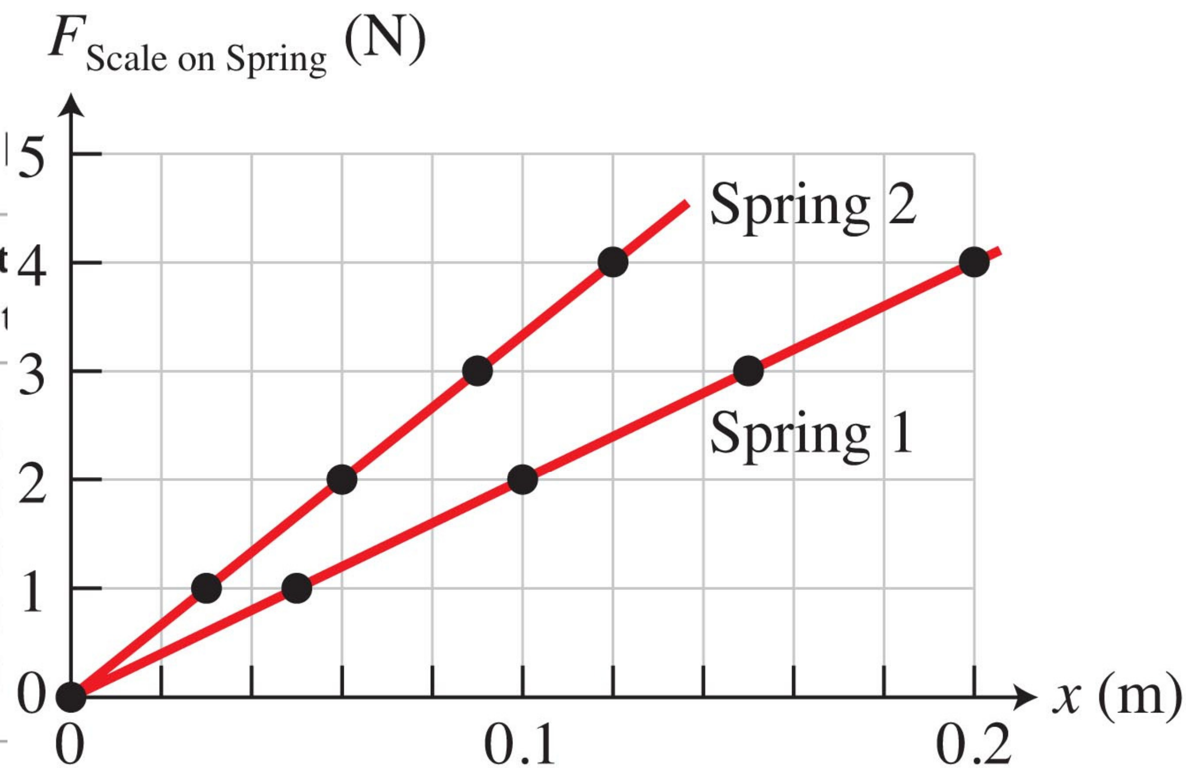


Table 6.5 ... ng force.

Force *F* exert ... the scale on ...	... stretch ... *x*
0.00 N	...0 m
1.00 N	...0 m
2.00 N	...0 m
3.00 N	...0 m
4.00 N	...0 m

We need to pull harder on spring 2 than on spring 1 to stretch it 0.1 m.

Hooke's Law

- The magnitude of the force exerted by the scale on each spring is proportional to the distance that each spring stretches:

$$F_{Scale\ on\ Spring} = kx$$

- From Newton's third law:

$$F_{Spring\ on\ Scale} = -kx$$

Elastic Force (Hooke's Law)

- If any object causes a spring to stretch or compress, the spring exerts an elastic force on that object.
- If the object stretches the spring along the x-direction, the x-component of the force that the spring exerts on the object is:

$$F_{Spring\ on\ Object} = -kx$$

- The spring constant, k, is a property of the spring and is measured in N/m.

Elastic Potential Energy

- To calculate the work done on the spring by such a variable force, we can replace this variable force with the average force:

$$(F_{H\ on\ S})_{average} = \frac{0 + kx}{2}$$

- Thus the work done by this force on the spring to stretch it a distance *x* is:

$$W = (F_{H\ on\ S})_{average}\ x = \left(\frac{1}{2}k\ x\right)x = \frac{1}{2}kx^2$$

Elastic Potential Energy

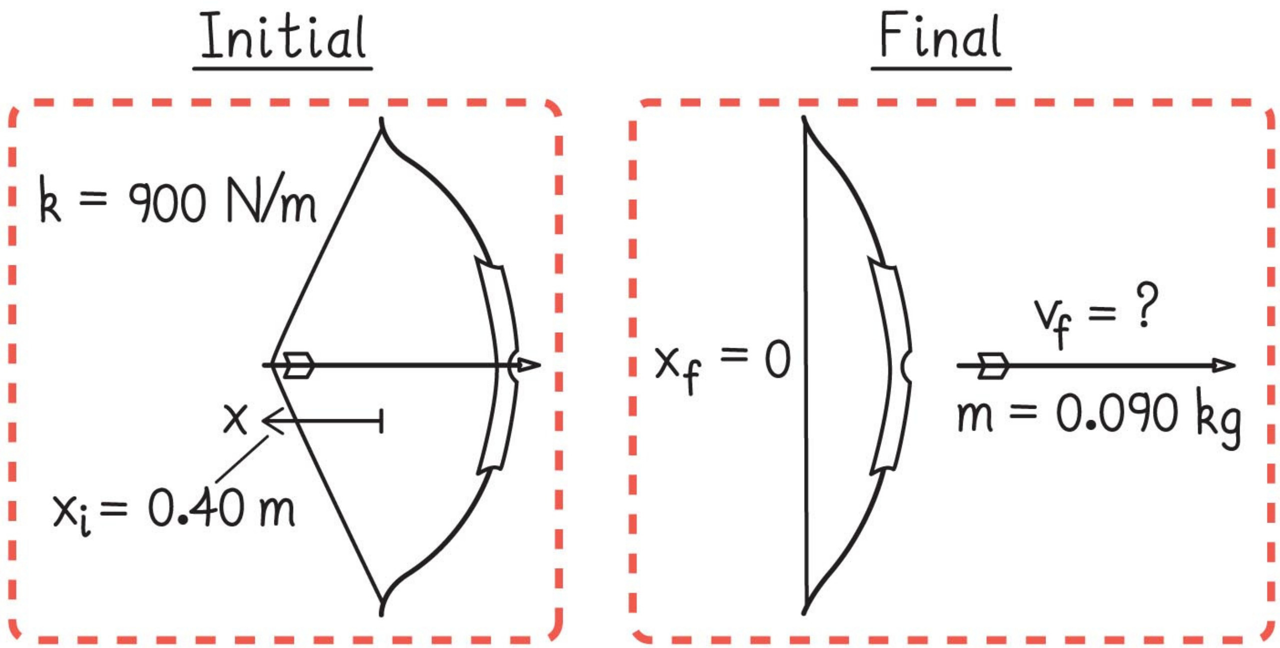
- The elastic potential energy of a spring-like object with spring constant k that has been stretched or compressed a distance x from its unstretched position is

$$U_s = \frac{1}{2}kx^2$$

- This assumes that the elastic potential energy of the unstretched spring is zero.

Example: Shooting an arrow

- You load an arrow (mass = 0.090 kg) into a bow and pull the bowstring back 0.50 m. The bow has a spring constant k = 900 N/m.
- Determine the arrow's speed as it leaves the bow.



Example: Shooting an arrow

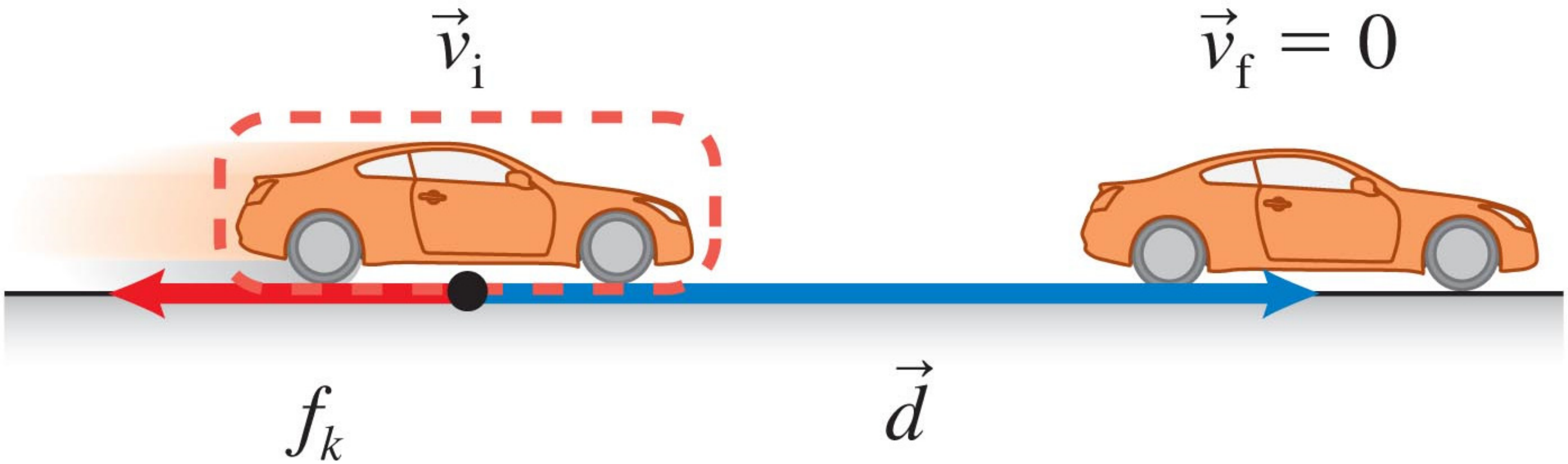
- Initial displacement: $x_i = 0.4\,m$
- $k = 900\,N/m\,, m = 0.09\,kg$
- Initial elastic potential energy: $U_{si} = \frac{1}{2}kx_i^2$
- Initial kinetic energy: $K_i = 0$
- Final elastic potential energy: $U_{sf} = 0$
- Final kinetic energy: $K_f = \frac{1}{2}mv_f^2 = U_{si}$
- Final velocity: $\boldsymbol{v_f = x_i\sqrt{k/m} = 40\,m/s}$

Friction and Energy Conservation

- In nearly every mechanical process, objects exert friction forces on each other.
- Sometimes the effect of friction is negligible, but most often friction is important.
- Our next goal is to investigate how we can incorporate friction into work and energy concepts.

Can friction do work?

- Consider a car skidding on a road…



- The kinetic friction force exerted by the road surface on the car does work on the car and slows it to a stop.

$$W_{friction} = f_k d\ \cos 180° = -f_k d$$

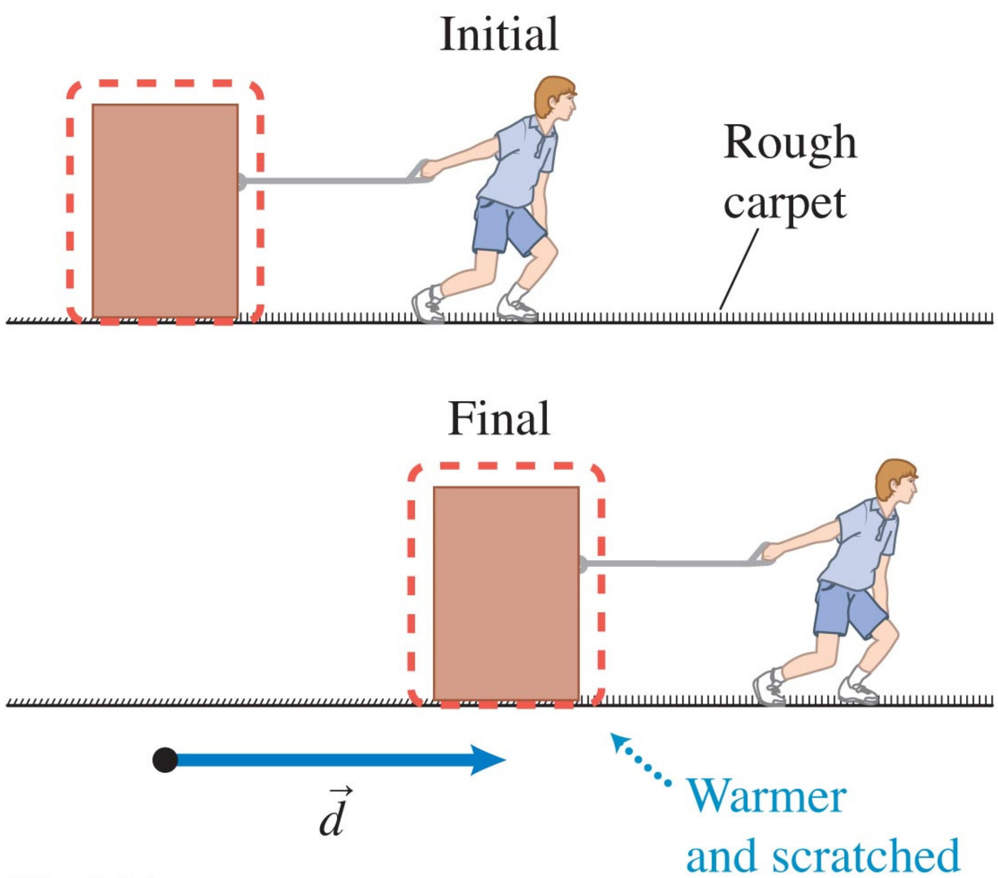
- The negative work done indicates that the car's energy is decreasing.

Can friction do work?

- The car lost energy because $W_{friction} = -f_k d$.
- And yet the breaks became warmer, so the internal energy of the system increased.
  - Did we just violate energy conservation?
  - Did we create free energy?
  - Can we patent this?
  - Can we get rich defrauding people who haven't taken Physics 220?

(we really shouldn't because that wouldn't be nice)

Can the force of friction do work?



The effect of friction as a change in internal energy

- If we choose the box or car as our system object, our model does not account for the change in internal energy.
- If we pick the surface and the box or car as our system, then the friction force between the surface and the box or car is internal and does no work.
- This choice of systems allows us to construct an expression for the change in internal energy of a system.

Increase in the system's internal energy due to friction

$$\Delta \boldsymbol{U_{int}} = +\boldsymbol{f_k d}$$

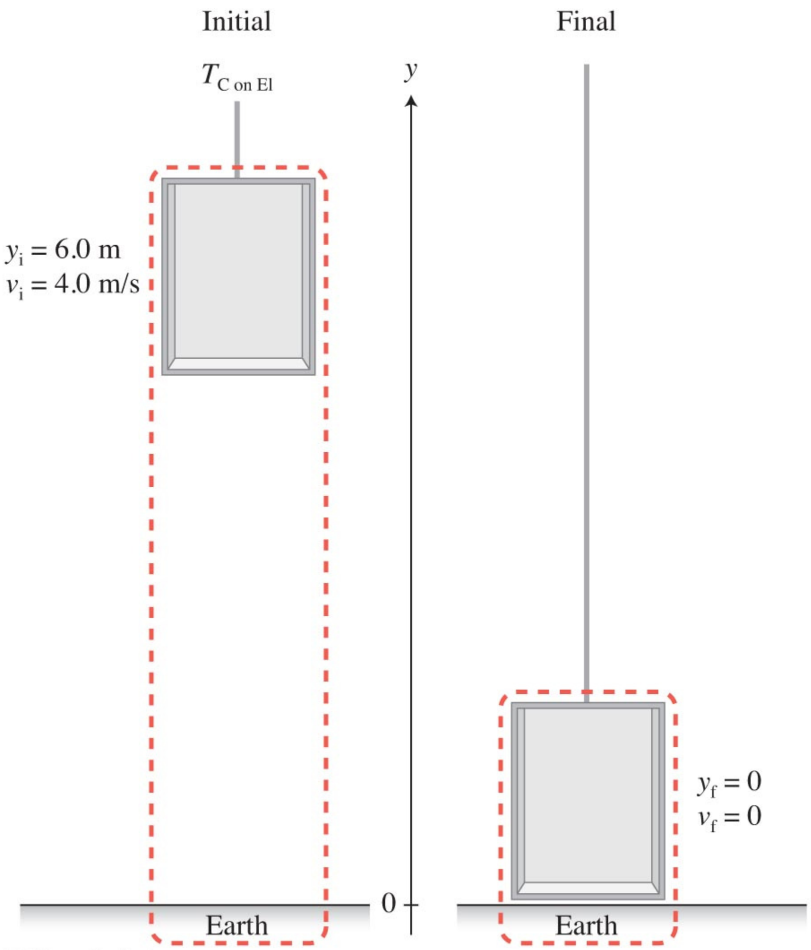
where f_k is the magnitude of the average friction force exerted on the object moving relative to the surface and d is the distance that the object moves across the surface.

- *The increase in internal energy is shared between the moving object and the surface.*

- Including friction in the work-energy equation as an increase in the system's internal energy produces the same result as calculating the work done by the friction force.

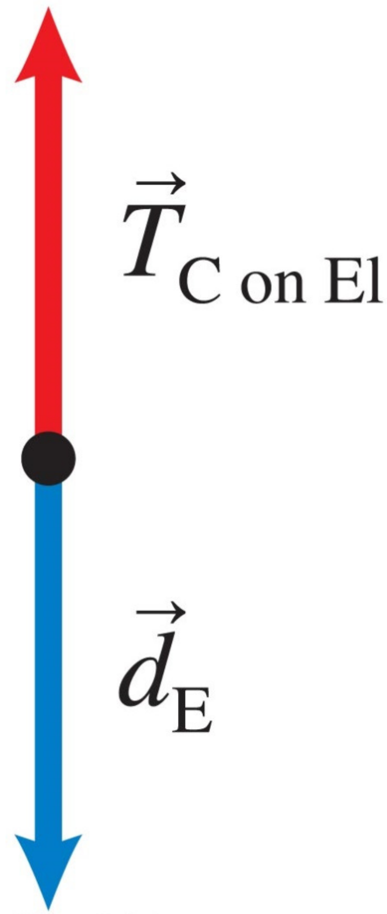
Skills for analyzing processes using the work-energy principle

- Sketch and translate
  - Sketch the initial and final states of the process, labeling the known and unknown information.
  - Choose the system of interest.
  - Include the object of reference and the coordinate system.



Skills for analyzing processes using the work-energy principle

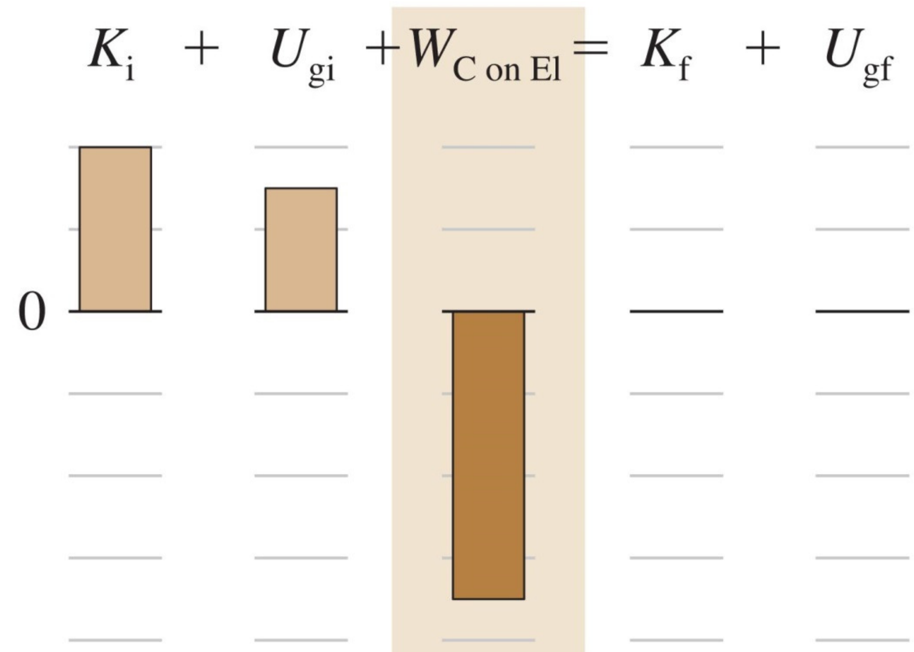
- Simplify and diagram
  - Which simplifications can you make to the objects, interactions, and processes?
  - Decide which energy types are changing.
  - Are external objects doing work?
  - Use the initial and final sketches to help draw a work-energy bar chart.



Skills for analyzing processes using the work-energy principle

- Represent mathematically
  - Convert the bar chart into a mathematical description of the process.
  - Each bar in the chart will appear as a single term in the equation

$$U_i + W = U_f$$



Skills for analyzing processes using the work-energy principle

- Solve and evaluate
  - Solve for the unknown and evaluate the result.
  - Does it have the correct units? Is its magnitude reasonable? Do the limiting cases make sense?