PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 21900

General Physics II

Electricity, Magnetism and Optics

Lecture 9 – Chapter 16.7-10

Kirchhoff's Laws

Fall 2015 Semester

Prof. Matthew Jones

Reminder

- The first mid-term exam will be tomorrow, Thursday, September 24th at 8:00 pm.
- Material to be covered is chapters 14 and 15
  - Coulomb's law
  - Electric potential energy
  - Electric field
  - Electric potential
  - Capacitors

DC Circuit Analysis

- An electric circuit is a combination of elements connected together in such a way that there are continuous paths through which charge is able to move.
- Calculating the current in a circuit is called ***circuit analysis***.
- DC stands for ***direct current*** – the current is constant.
- The current is viewed as the motion of ***positive charges*** moving through the circuit.
- Exact values of current and voltage depend on where in the circuit you are measuring.

DC Circuit Analysis

- So far we just used Ohm's law to analyze simple circuits.
- This can get complicated and confusing without a systematic method for analyzing circuits.
- Kirchhoff's rules for circuit analysis just make use of two fundamental concepts:
  - Energy conservation (charges can't gain more and more energy each time they go around a loop)
  - Charge conservation (charges can't be created or destroyed)

Kirchhoff's Rules

Kirchhoff's loop rule The sum of the electric potential differences ΔV across the circuit elements that make up a closed path (called a loop) in a circuit is zero.

$$\sum_{\text{Loop}} \Delta V = 0 \qquad (16.6)$$

Kirchhoff's junction rule The total rate at which electric charge enters a junction equals the total rate at which electric charge leaves the junction:

Sum of currents into junction = Sum of currents out

In symbols:

$$\sum_{\text{In}} I = \sum_{\text{Out}} I \qquad (16.7)$$

Kirchhoff's Rules

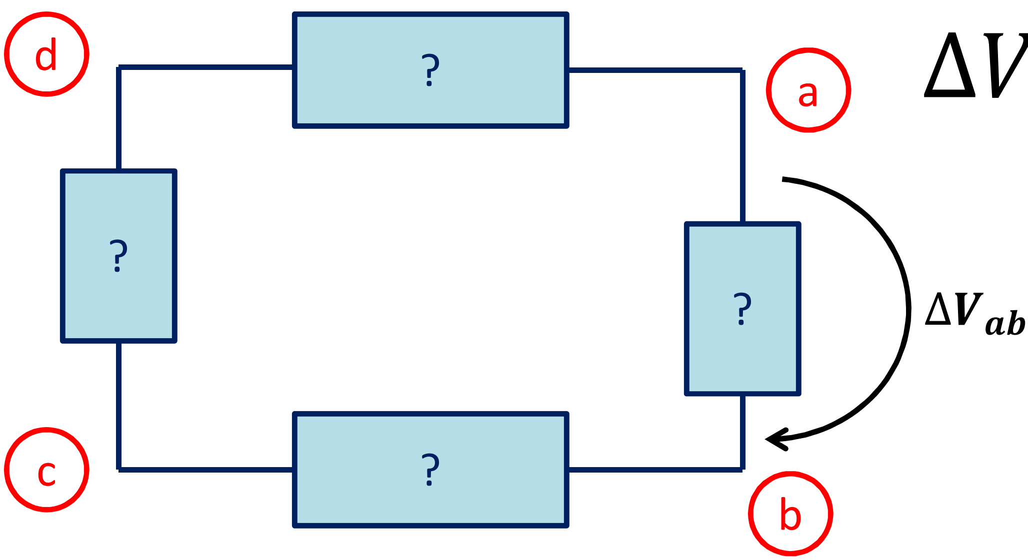
- General problem:
  - Calculate the currents that flow in an electric circuit composed of voltage sources and resistors connected by wires.
  - Recall that work done to move a charge q from point a to point b is $W = -q\,\Delta V_{ab}$
  - If a and b are the same point then $W = 0$

$$\underbrace{\Delta V_{ac} + \Delta V_{cd} + \cdots + \Delta V_{xb}}_{\Delta V_{ab}} = 0$$

Kirchhoff's Loop Rule

$$\sum \Delta V = 0$$

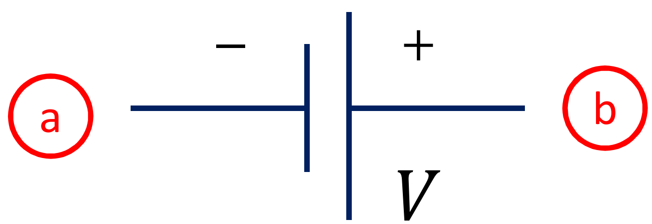
"The sum of the potential differences around a closed loop is zero."



$$\Delta V_{ab} + \Delta V_{bc} + \Delta V_{cd} + \Delta V_{da} = 0$$

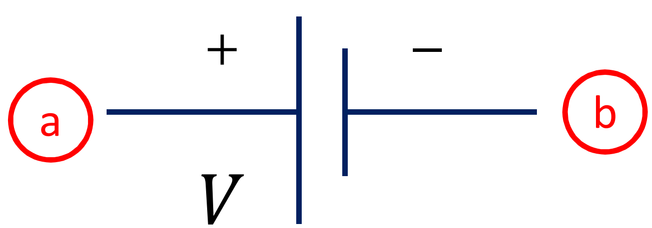
Circuit Elements

- Voltage sources (like batteries):



$$V_b = V_a + V$$

- Make sure you get the sign right!

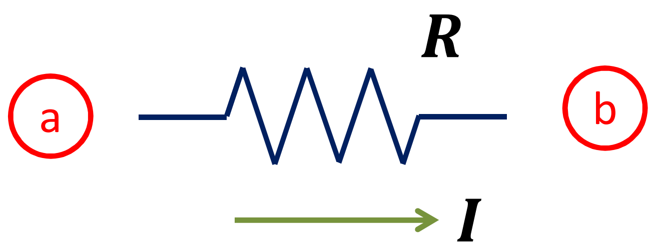


$$V_b = V_a - V$$

Circuit Elements

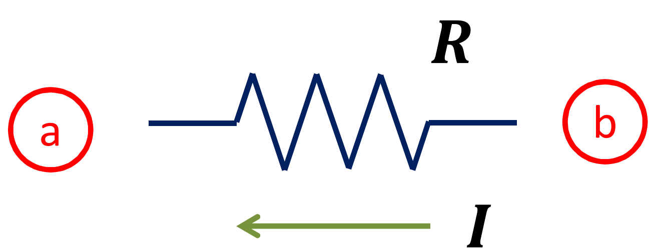
- Resistors:

The charges lose energy as they are pushed through the resistor.



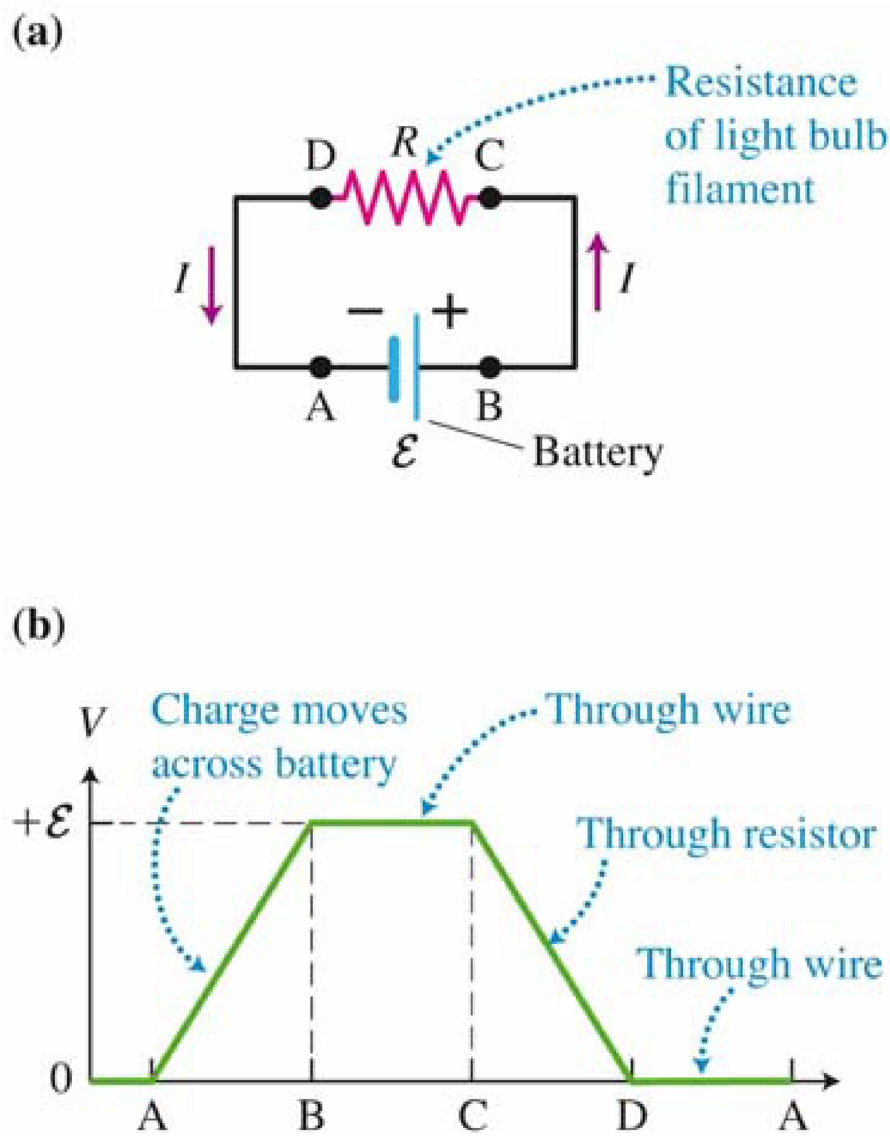
$$V_b = V_a - IR$$

- Make sure you get the sign right!



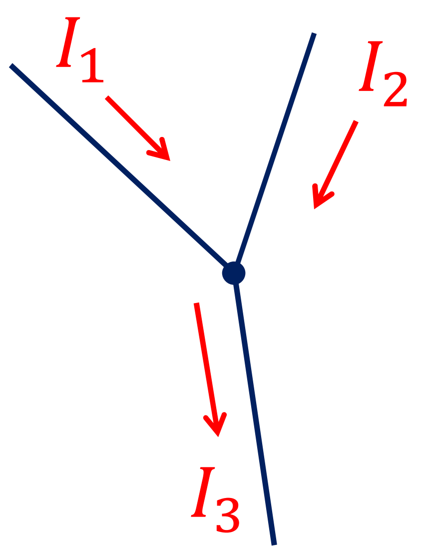
$$V_b = V_a + IR$$

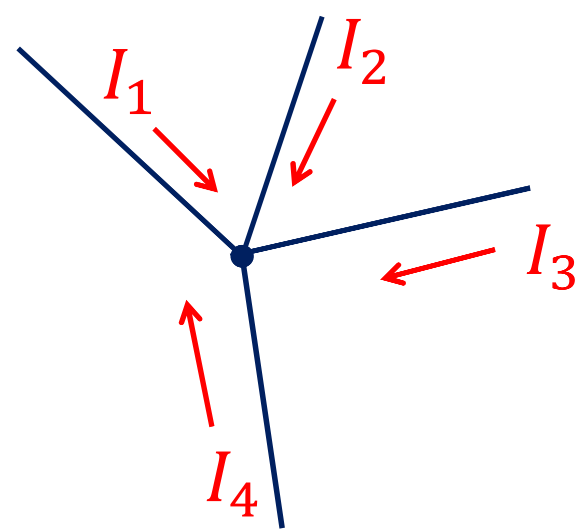
Example


(a)
Resistance
of light bulb
filament
D
R
C
I
I
− +
A
B
ℰ
Battery
(b)
V
Charge moves
across battery
Through wire
+ℰ
Through resistor
Through wire
0
A
B
C
D
A

Kirchhoff's Node Rule

- The sum of the currents entering a node must equal the sum of the currents leaving.





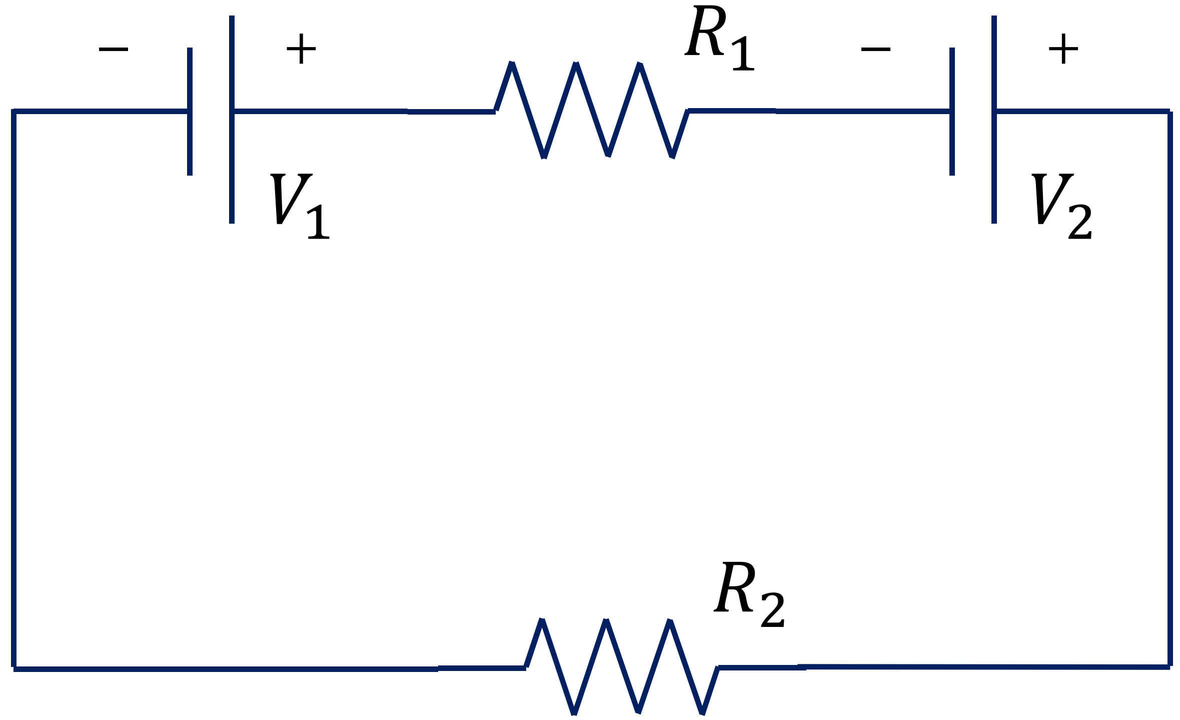
$$I_1 + I_2 = I_3$$

$$I_1 + I_2 + I_3 + I_4 = 0$$

(at least one of these must be negative)

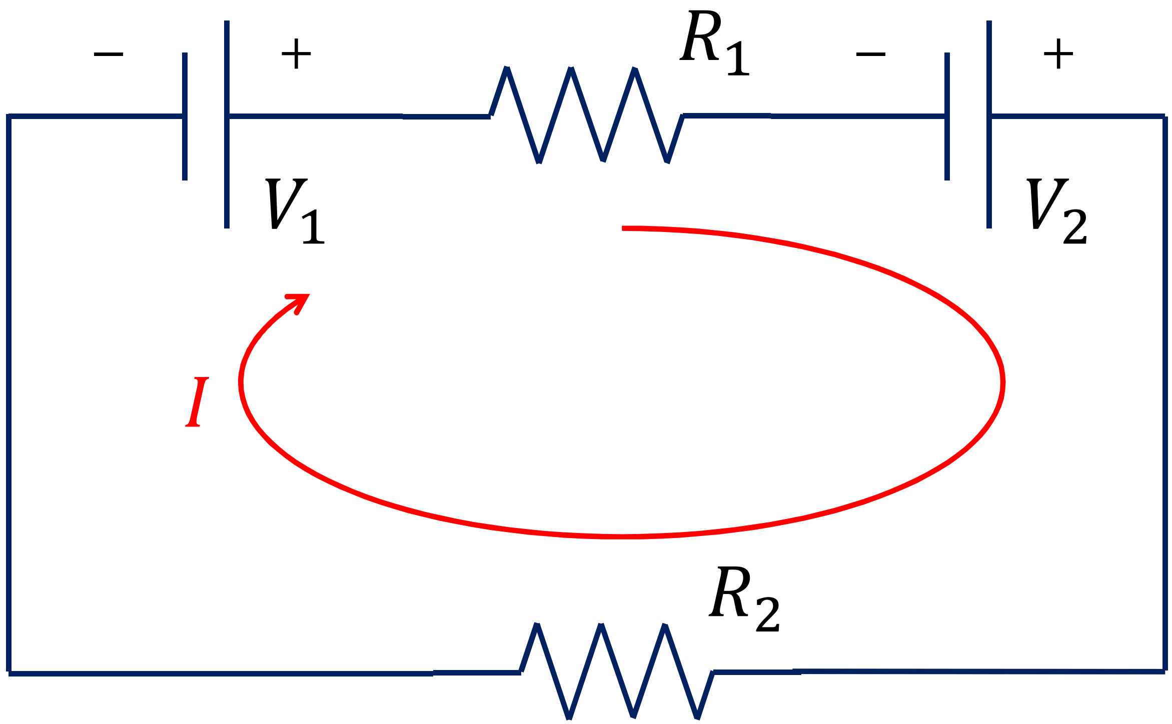
Circuit Analysis

- Find the current in the following circuit:



Circuit Analysis

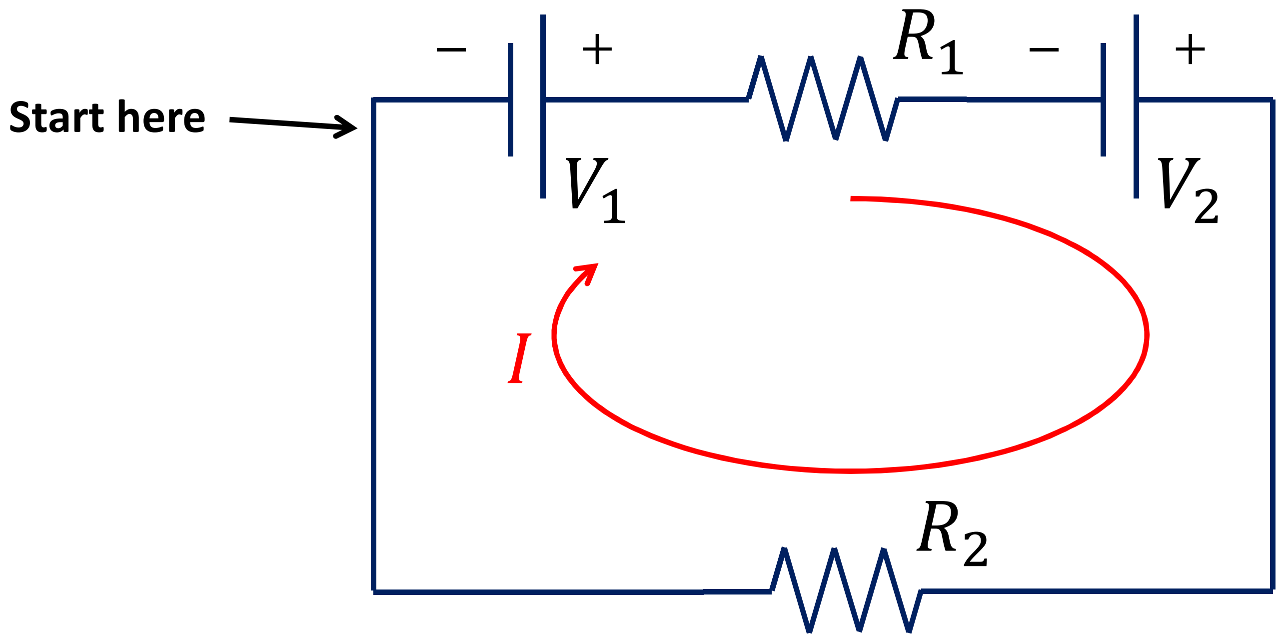
- Step 1: Draw a loop to represent the current.



Which direction? It doesn't matter, but let's ALWAYS pick clockwise to avoid confusion.

Circuit Analysis

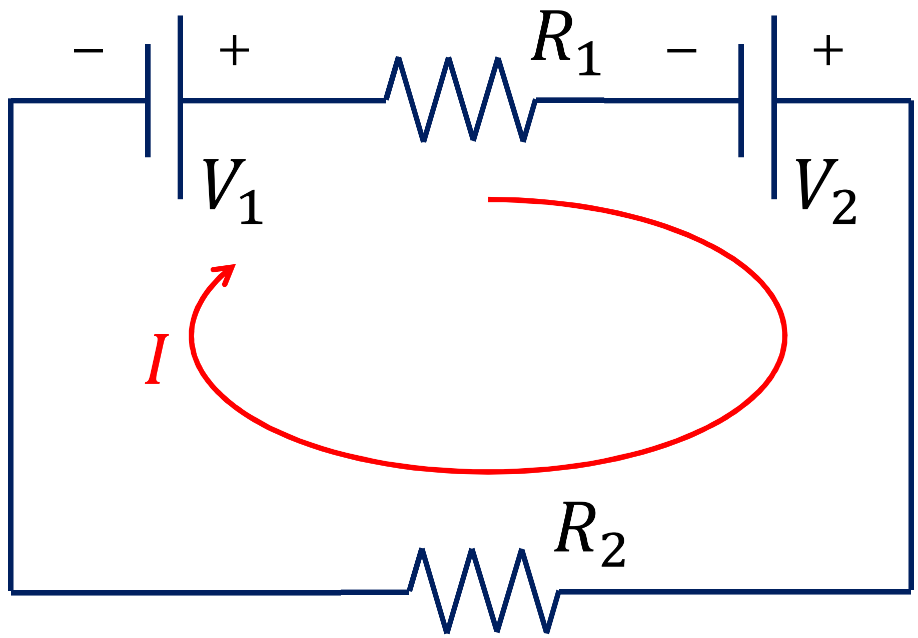
- Step 2: Apply Kirchhoff's Loop Rule…



V_1

Circuit Analysis

- Step 3: Solve for I…

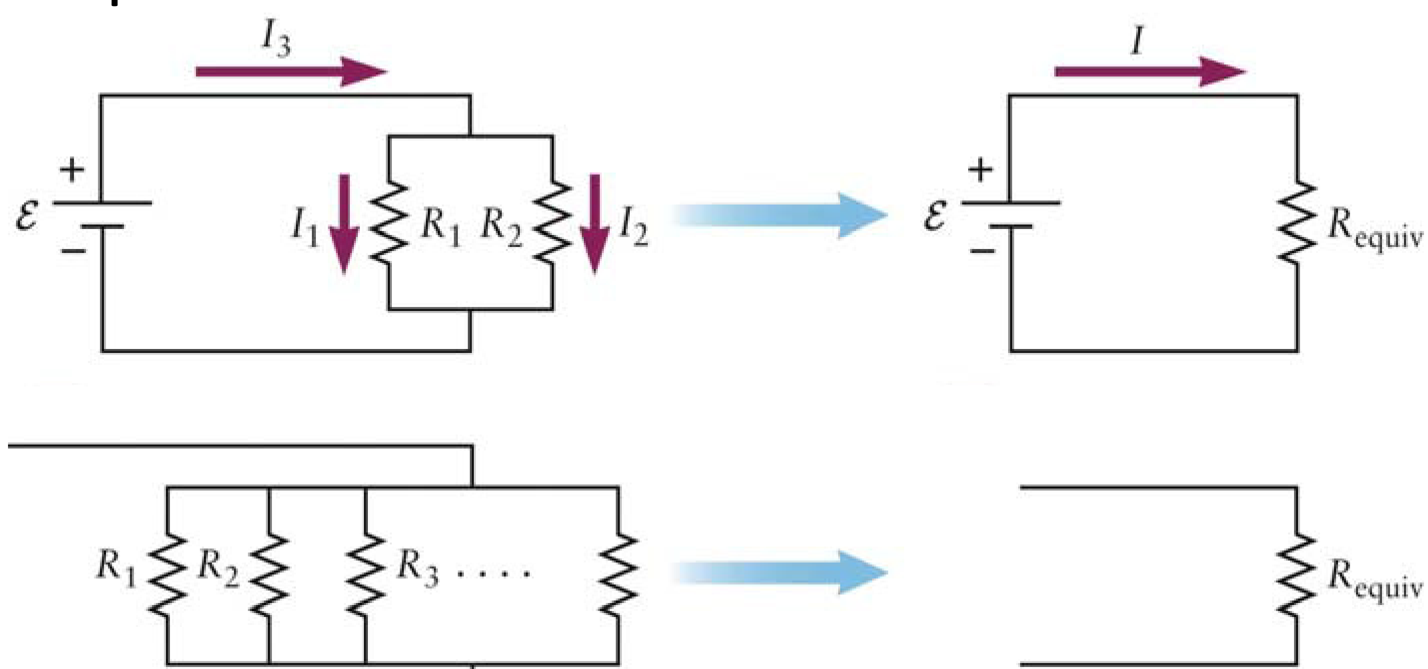


$$I = \frac{V_1 + V_2}{R_1 + R_2}$$

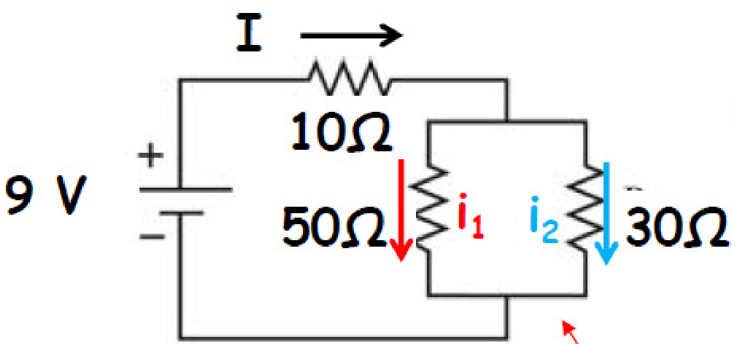
What if I is negative? Then it means the current flows in the opposite direction.

More Complex Circuits

- Sometimes it is easier to first replace combinations of resistors with their series or parallel equivalent.
- Example:



$$\frac{1}{R_{equiv}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

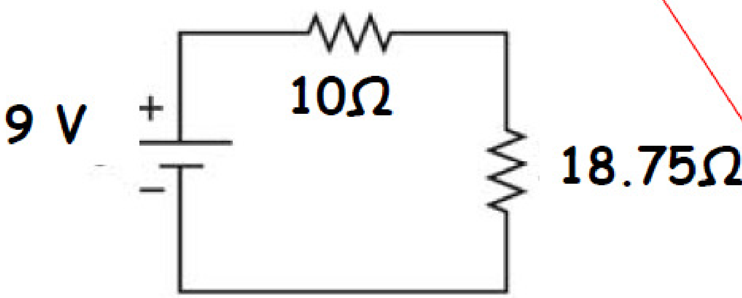


a)

$$\frac{1}{R_{equiv}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$= \frac{1}{50\,\Omega} + \frac{1}{30\,\Omega} = 0.0533\ \Omega^{-1}$$

$$R_{equiv} = 18.75\,\Omega$$



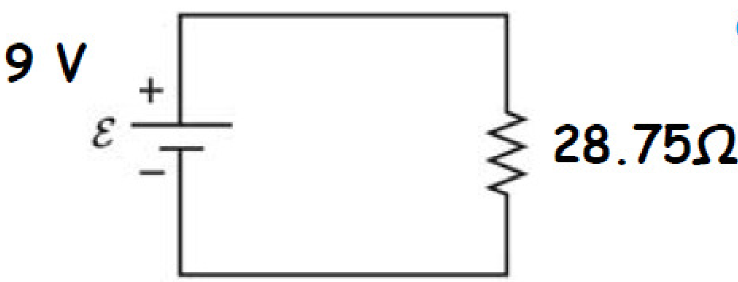
b)

$$R_{equiv} = 18.75\,\Omega + 10\,\Omega$$

$$= 28.75\,\Omega$$

c)

$$I = \frac{V}{R} = \frac{9.0\text{V}}{28.75\,\Omega} = 0.313A$$



d)

$$I = i_1 + i_2 = 0.313A$$

$$50 i_1 = 30 i_2$$

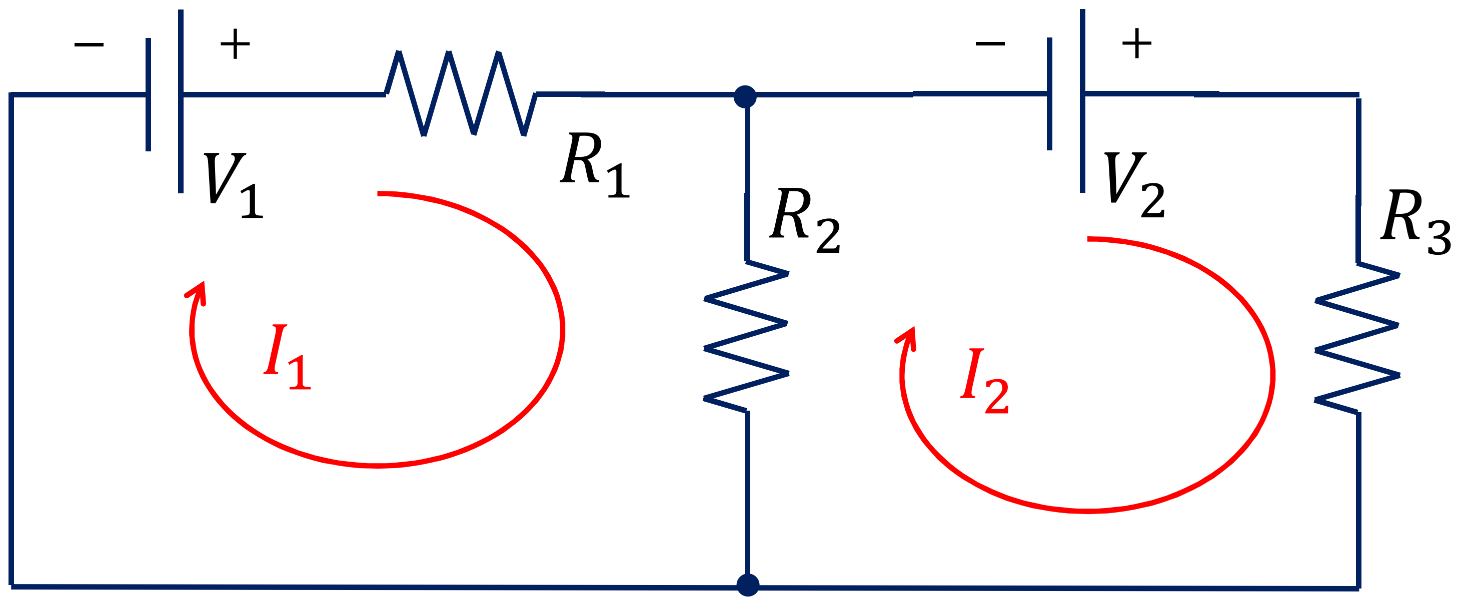
two equations, two unknowns

$$i_1 = 0.117\,A$$

$$i_2 = 0.196\,A$$

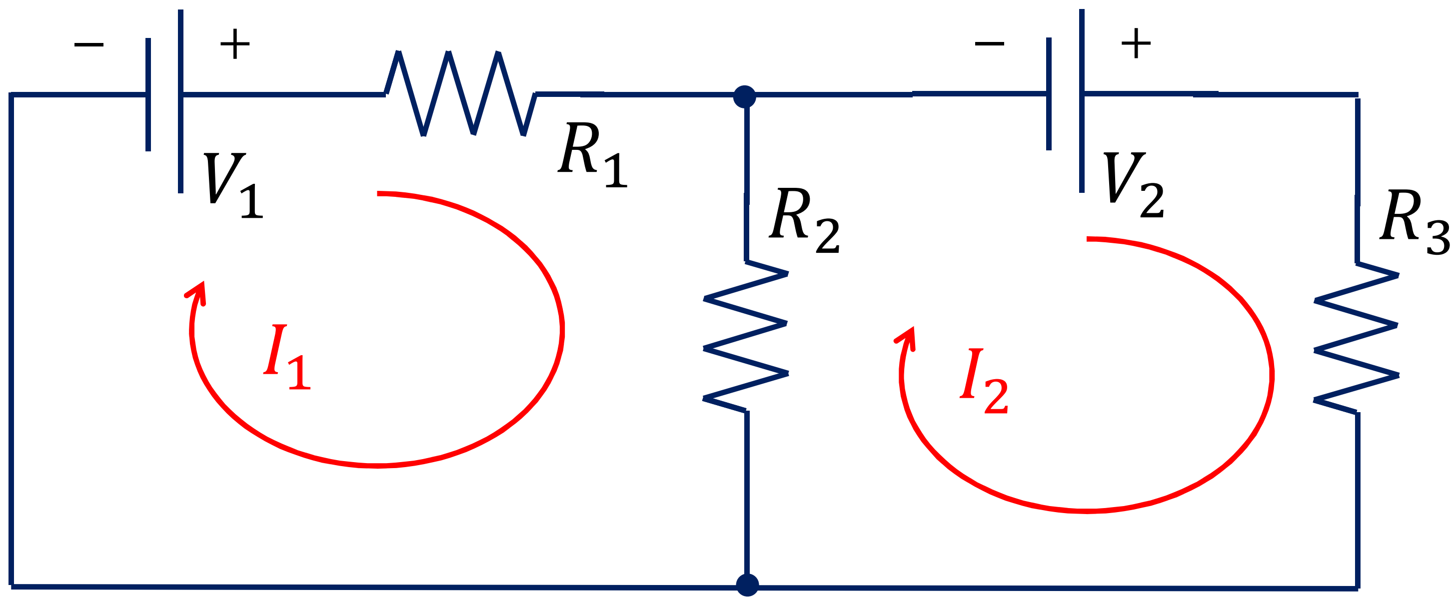
Two Loops

- Step 1: Assign currents to each loop



Two Loops

- Step 2: Apply Kirchhoff's Loop rule



$$V_1 - I_1 R_1 - (I_1 - I_2) R_2 = 0$$
$$V_2 - I_2 R_3 - (I_2 - I_1) R_2 = 0$$

Two Loops

$$V_1 - I_1R_1 - (I_1 - I_2)R_2 = 0$$
$$V_2 - I_2R_3 - (I_2 - I_1)R_2 = 0$$

- This is a system of two equations in two unknowns.
- Use whatever techniques you can think of to solve them.
- There are some general strategies, but these make use of Linear Algebra (matrices and stuff like that) which we won’t discuss.

Two Loops

$$V_1 - I_1R_1 - (I_1 - I_2)R_2 = 0$$
$$V_2 - I_2R_3 - (I_2 - I_1)R_2 = 0$$

- Collect the I_1 and I_2 terms together:

$$V_1 - I_1(R_1 + R_2) + I_2R_2 = 0$$
$$V_2 + I_1R_2 - I_2(R_2 + R_3) = 0$$

- Solve for I_1:

$$I_1 = \frac{V_1 - I_2R_2}{R_1 + R_2} \quad (*)$$

- Substitute it into the second equation:

$$V_2 + \frac{V_1R_2}{R_1 + R_2} - I_2\left(\frac{R_2^2}{R_1 + R_2} + R_2 + R_3\right) = 0$$

- Solove for I_2… then substitute back into (*).

Lots of Loops

- In general, the analysis of complex circuits is done numerically using computers.
- Programs like SPICE were developed to analyze circuits that were to be implemented on silicon chips in the 1970's.
- General idea:
  - Describe the circuit in a form that a computer can interpret (generally not a circuit diagram!)
  - Identify all relevant loops in the circuit
  - Generate the system of equations
  - Solve the system of equations numerically to get all the currents
  - Print out the results
- For an example, look at

http://www.physics.purdue.edu/~mjones/phys536_Spring2008/spice.html