PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 21900

General Physics II

Electricity, Magnetism and Optics
Lecture 6 – Chapter 15.6-7
Capacitors

Fall 2015 Semester

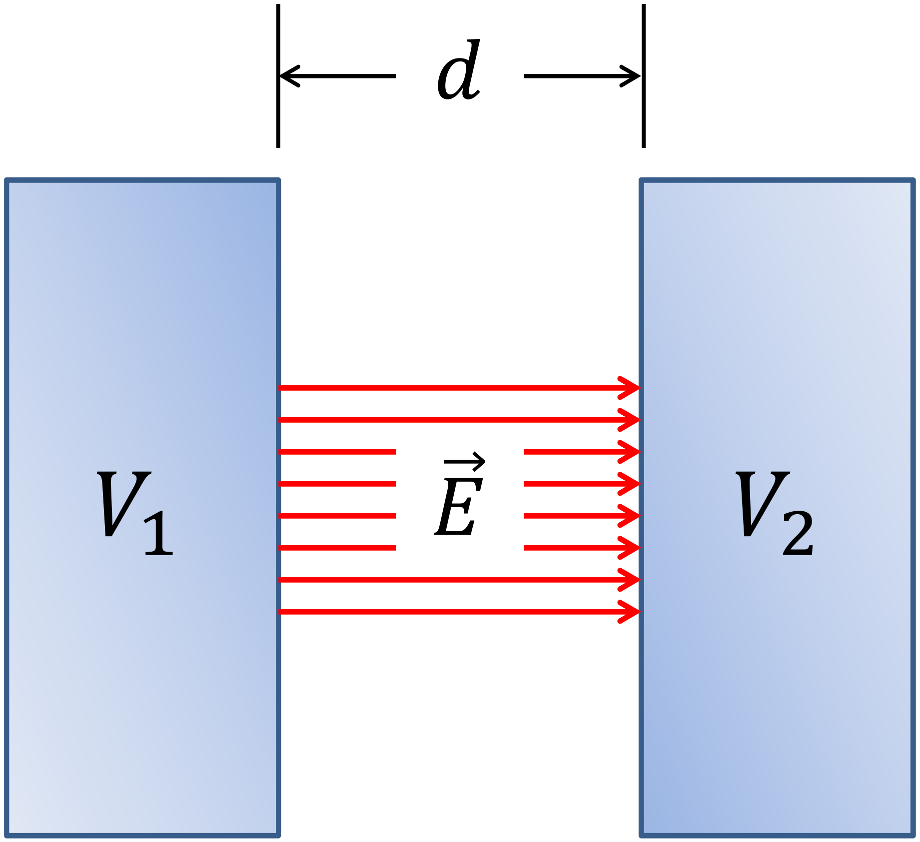
Prof. Matthew Jones

Reminder

- The first mid-term exam will be on Thursday, September 24th.
- Material to be covered is chapters 14 and 15
  - Coulomb's law
  - Electric potential energy
  - Electric field
  - Electric potential
  - Capacitors

Electric Potential

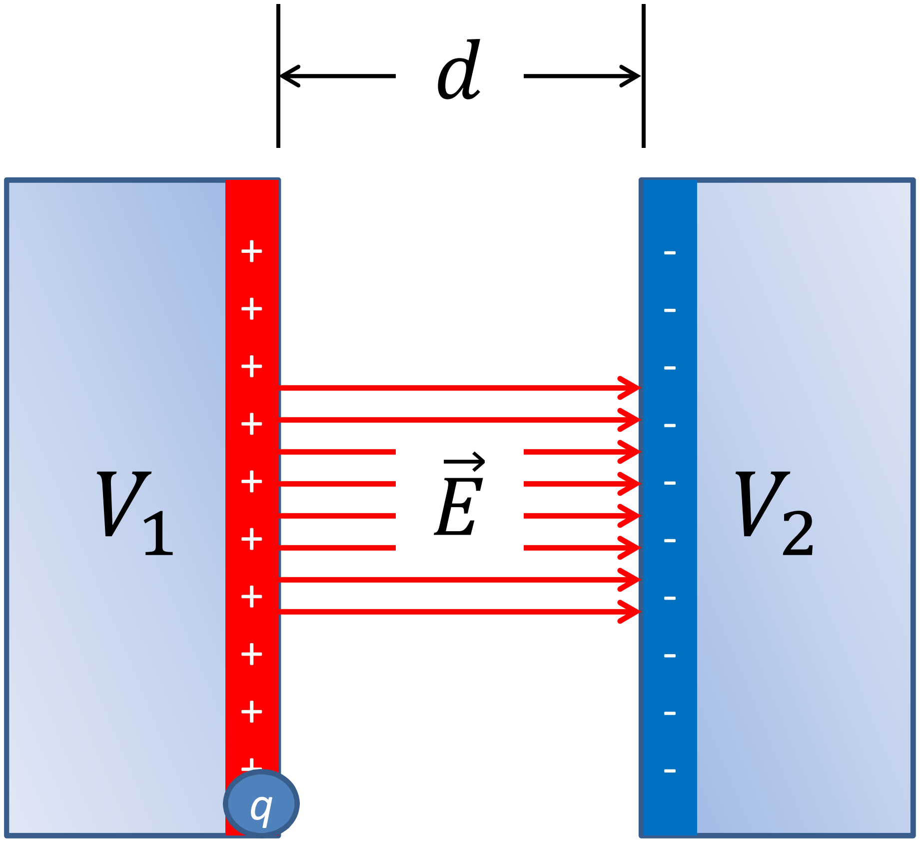
Consider the situation we discussed last week where there were two parallel conductors at different electric potentials.



Electric Potential

The electric field is created by an accumulation of static charge at the surfaces.

How does the potential energy change when a charge q is (somehow) moved from the right to the left?



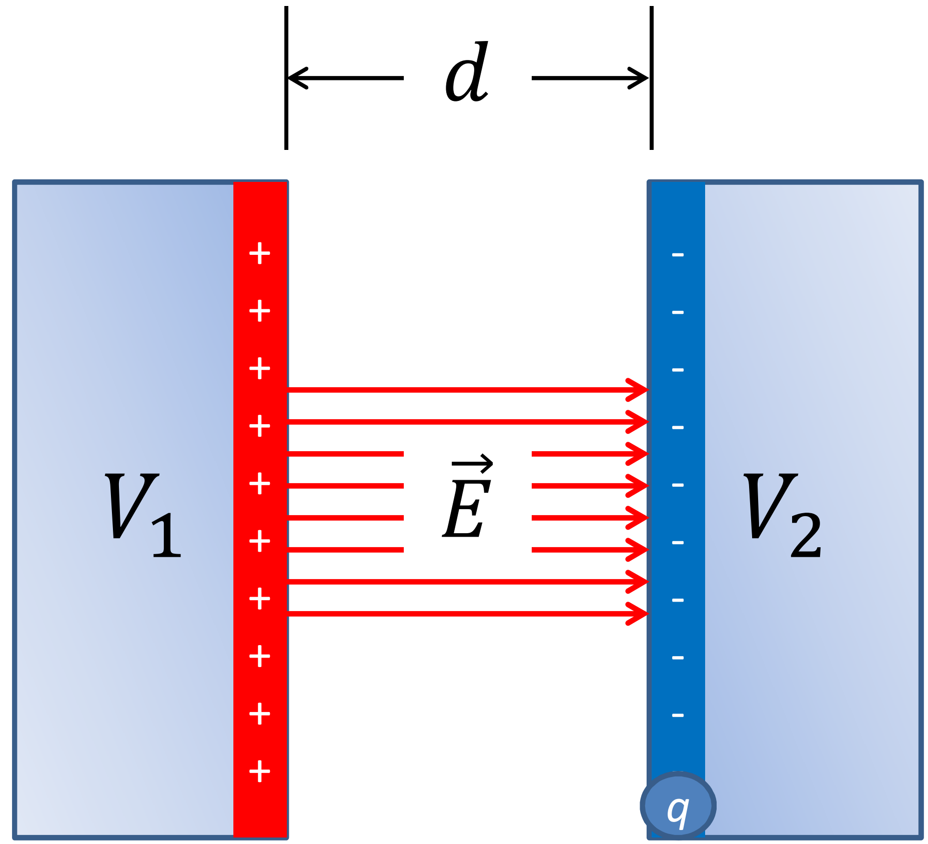
$$\Delta U = q\,(V_2 - V_1)$$

Electric Potential

Did the energy of the system increase or decrease?

Remember that $V_2 < V_1$ (look at the direction of the field lines.)

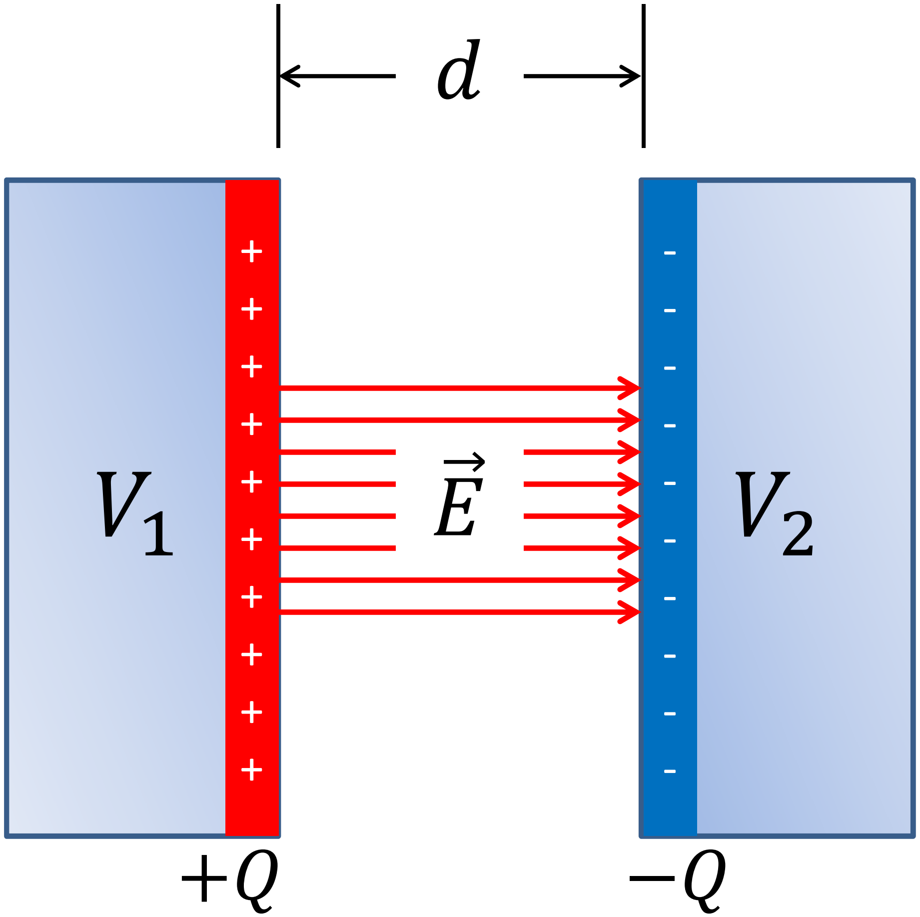
- If q is positive, then the electric potential energy decreases.
- If q is negative then the potential energy increases.



We can *store energy* in this system by moving a bunch of charge from one side to the other.

Electric Potential

How does the potential difference ΔV depend on the total charge, Q?



In general, $Q \propto \Delta V$.

The constant of proportionality is called the capacitance, $\boldsymbol{C}$:

$$Q = \boldsymbol{C}\,\Delta V$$

Note: Q is the charge on either side, not the difference in charge (which would be $2Q$).

Capacitance

- In general, any pair of conductors that stores energy in the form of an electric field is a capacitor.
- Units for capacitance:

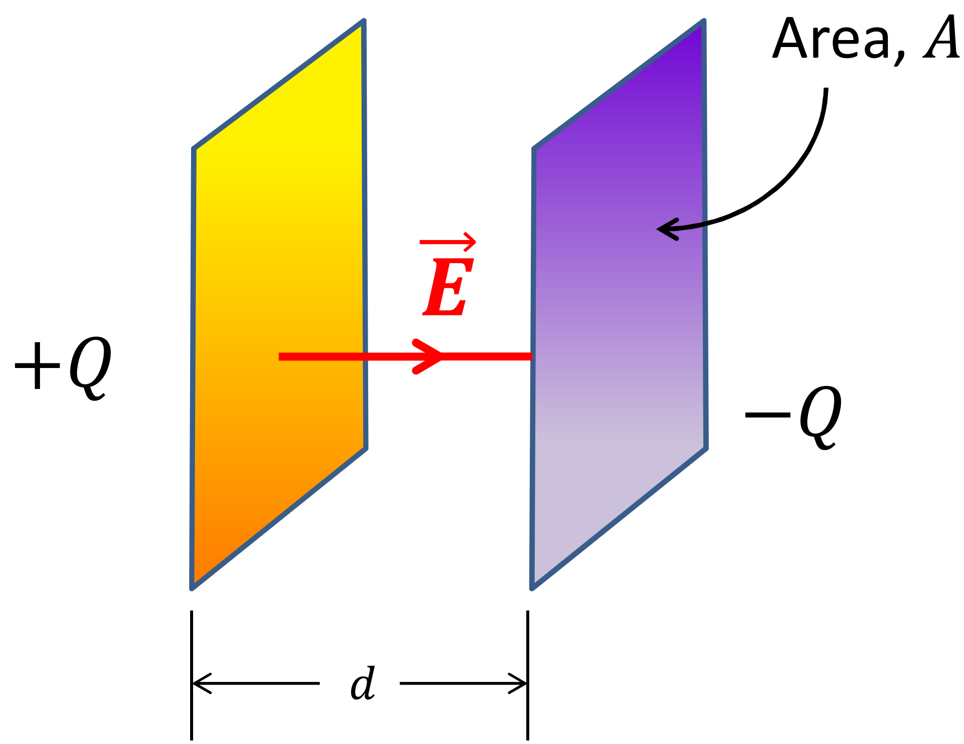
$$C = \frac{Q}{\Delta V}$$

$$\frac{Coulombs}{Volt} = Farad$$

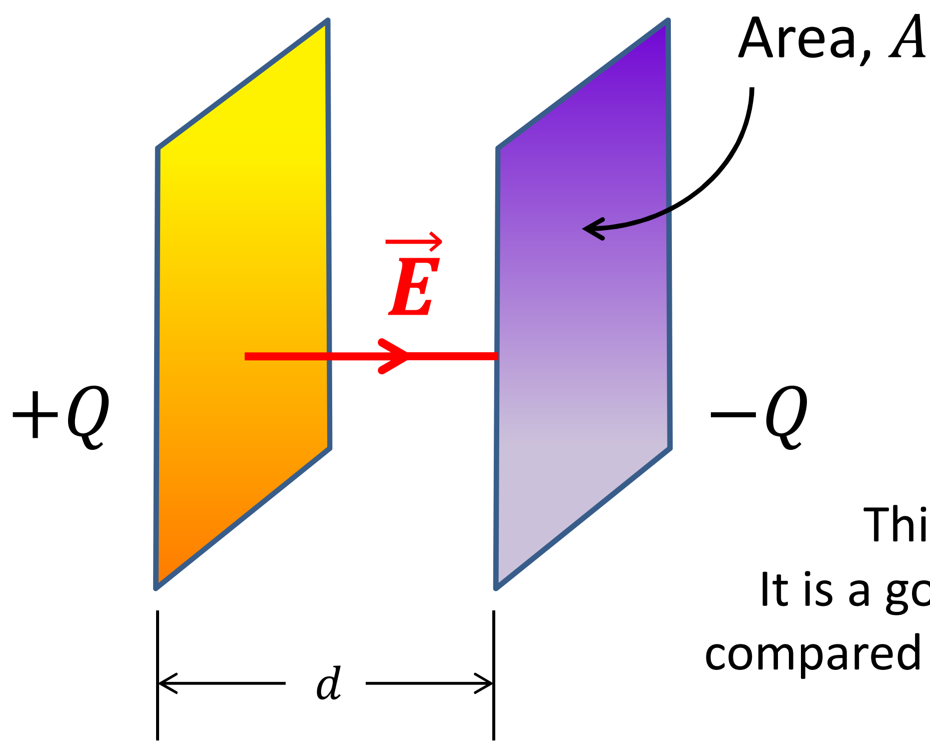
- Also, $F = C^2/J$ but this is rarely used in practice.

Parallel Plate Capacitor

- The capacitance depends on the geometry of the two conductors – not Q or ΔV.
- A simple, but very useful example:



Parallel Plate Capacitor



$$C \propto A$$

$$C \propto 1/d$$

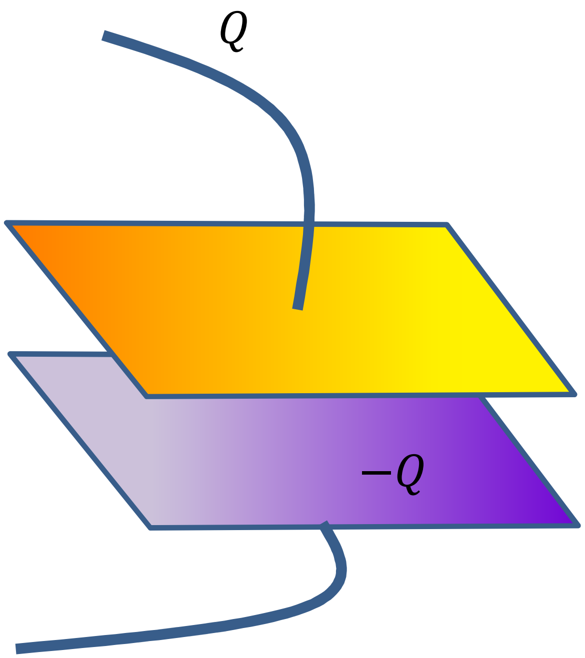
$$C = \frac{\epsilon_0 A}{d}$$

This ignores any "edge effects"…
It is a good approximation when d is small compared with the length and width of a plate.

ϵ_0 is called the "permittivity of free space"

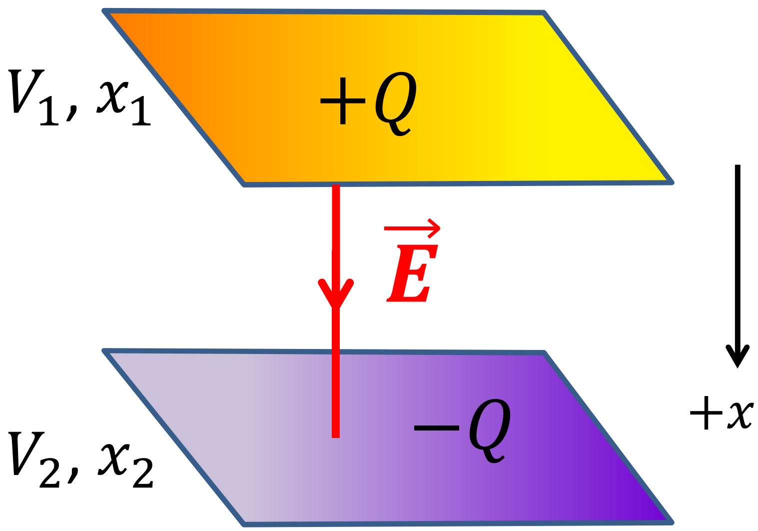
$$\boldsymbol{\epsilon_0 = 8.85 \times 10^{-12}\, F/m} = 8.85 \times 10^{-12}\, C^2/(N \cdot m)^2$$

Capacitors



- Suppose charge Q flows onto the top plate.
- The bottom plate is a conductor so it attracts a charge $-Q$.
- This is the same as a charge Q flowing away from the bottom plate.
- What is the resulting electric field and potential difference?

Capacitors



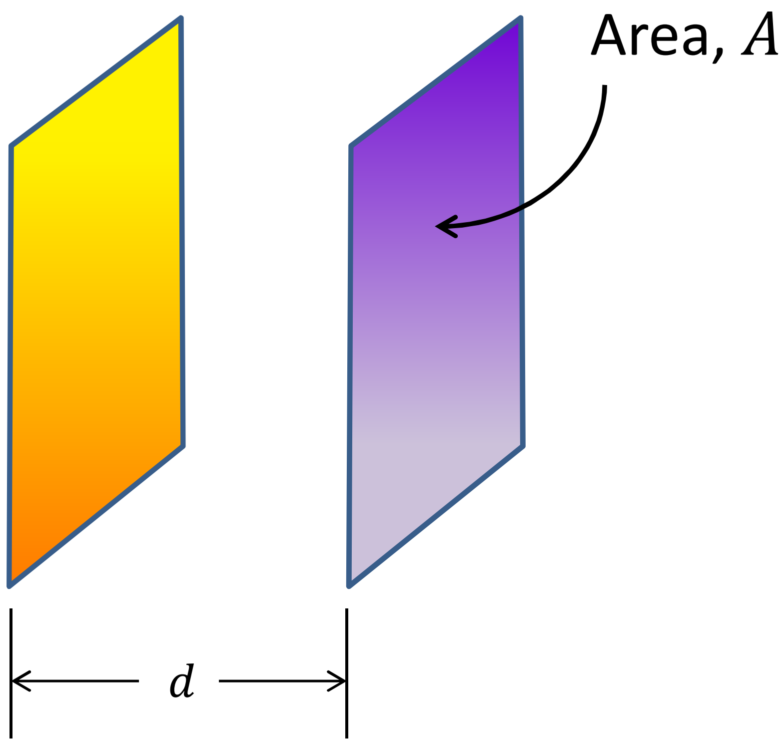
- Electric field lines point away from sources of positive charge.
- Relation between $\vec{E}$ and ΔV:

$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{V_2 - V_1}{x_2 - x_1}$$

- If $\vec{E}$ points in the $+x$ direction, then $V_2 < V_1$.

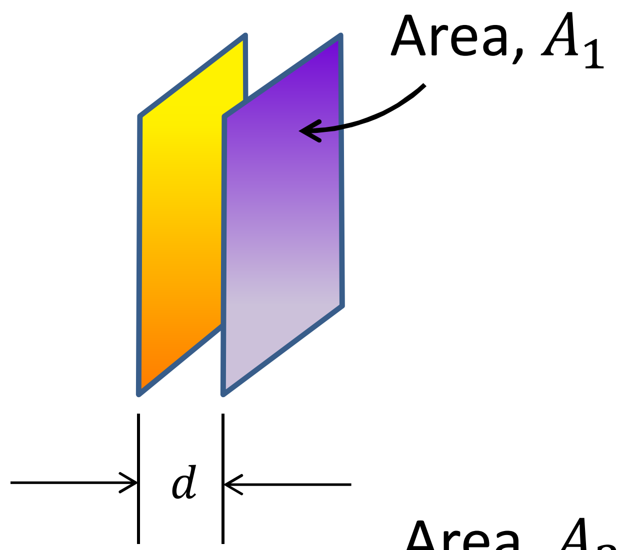
$$V_1 - V_2 = \frac{\mathrm{Q}}{\mathrm{C}}$$

Parallel Plate Capacitor

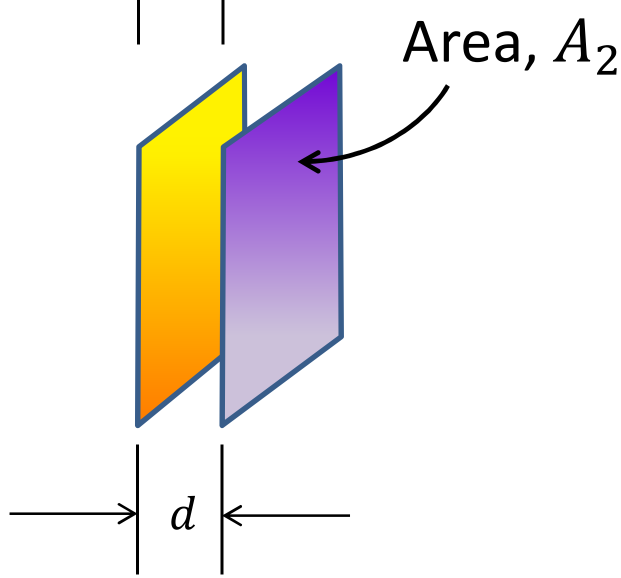


$$C = \frac{\epsilon_0 A}{d}$$

Two Parallel Plate Capacitors

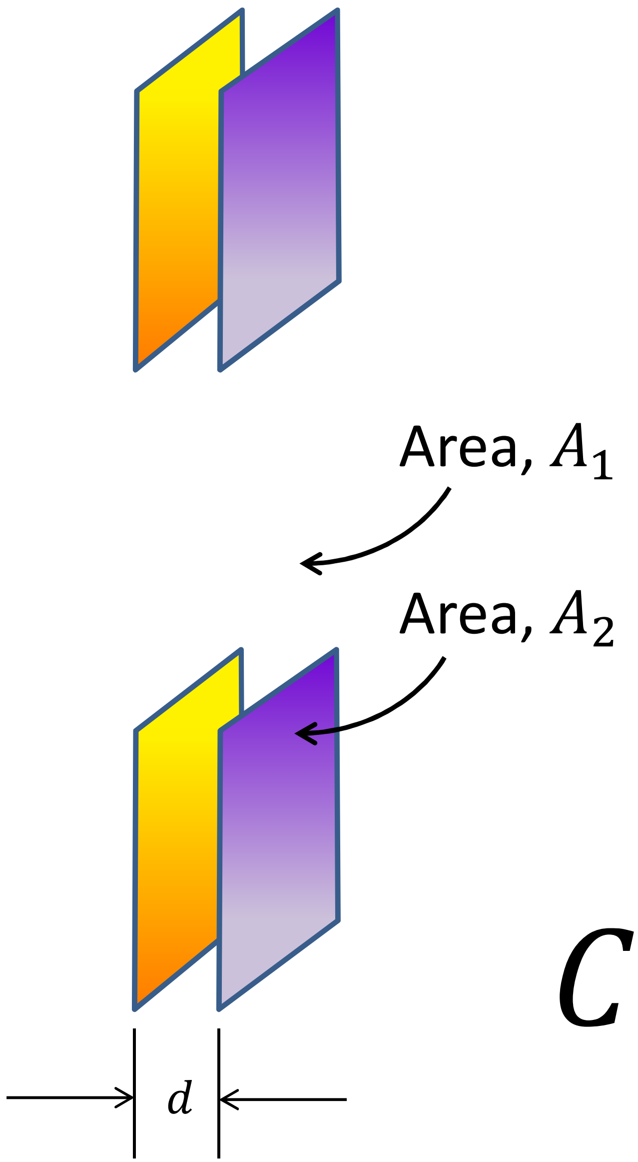


$$C_1 = \frac{\epsilon_0 A_1}{d}$$



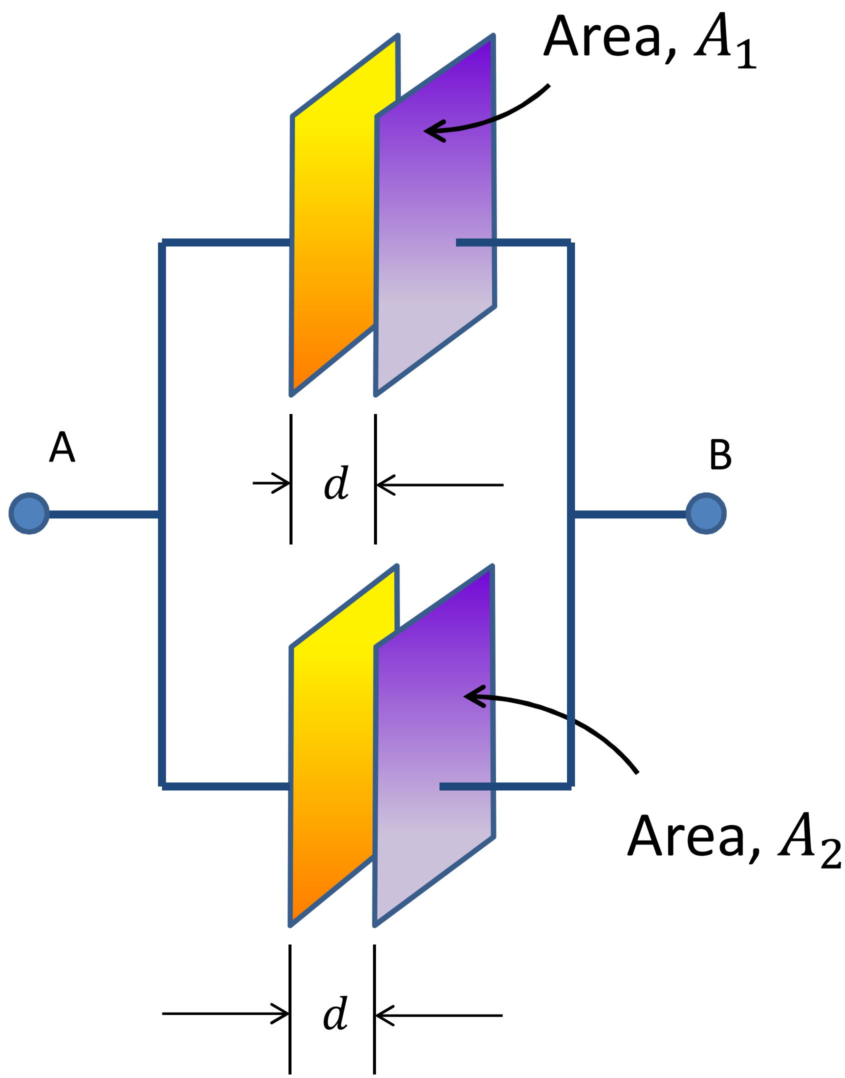
$$C_2 = \frac{\epsilon_0 A_2}{d}$$

One Big Parallel Plate Capacitor



$$C = \frac{\epsilon_0 (A_1 + A_2)}{d}$$

Two Parallel Plate Capacitors



- Remember: *conductors in electrical contact have the same electric potential.*
- We could measure the potential difference between points A and B if they were connected to the plates with wires.

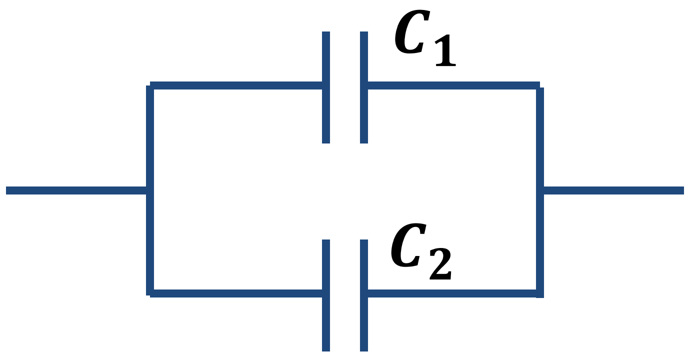
$$Q = C\,V$$

$$C = \frac{\epsilon_0(A_1 + A_2)}{d}$$

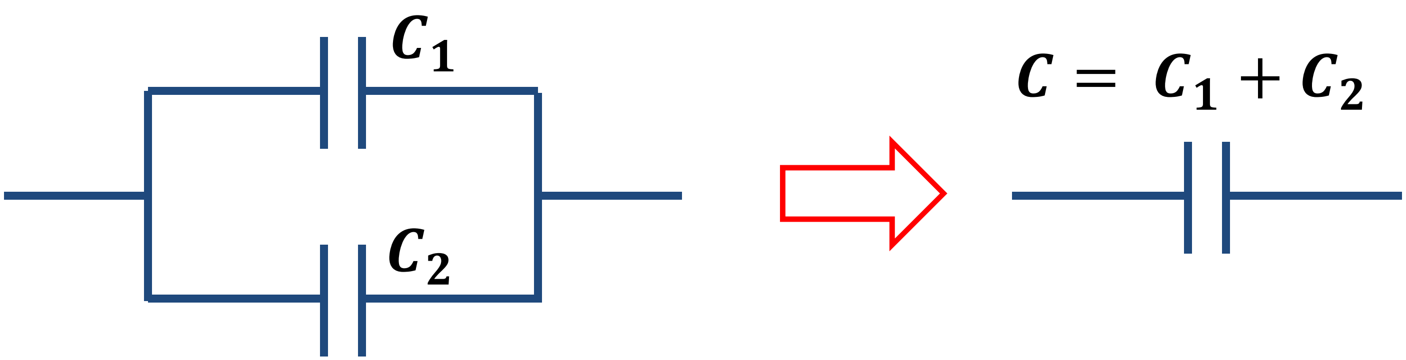
Capacitors In Parallel

- Symbol for a capacitor:

- Capacitors in parallel:



- Equivalent capacitance:



$$C = C_1 + C_2$$

Capacitors in Parallel

- Consider two capacitors, C_1 and C_2, with the same potential difference, V, across them.
- The charge on each one is

$$Q_1 = C_1 V$$
$$Q_2 = C_2 V$$

- If they are connected in parallel then the potential difference doesn't change but now we can write:

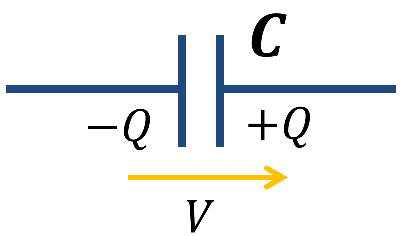
$$Q = Q_1 + Q_2 = (C_1 + C_2)V = C\ V$$

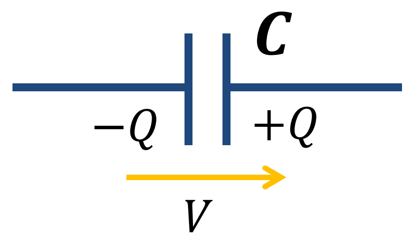
where

$$\boldsymbol{C = C_1 + C_2}$$

Capacitors in Series

- Consider two capacitors, C, each with potential difference, V, and charge $Q = CV$:





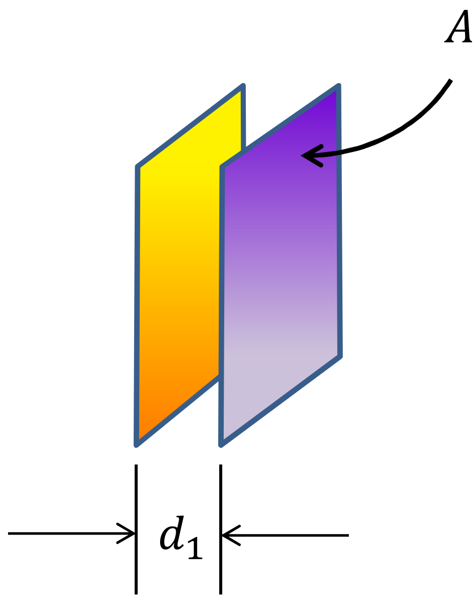
- If they are brought into electrical contact the potential difference is $2V$ but the charge is still Q.
- The effective capacitance is C' where

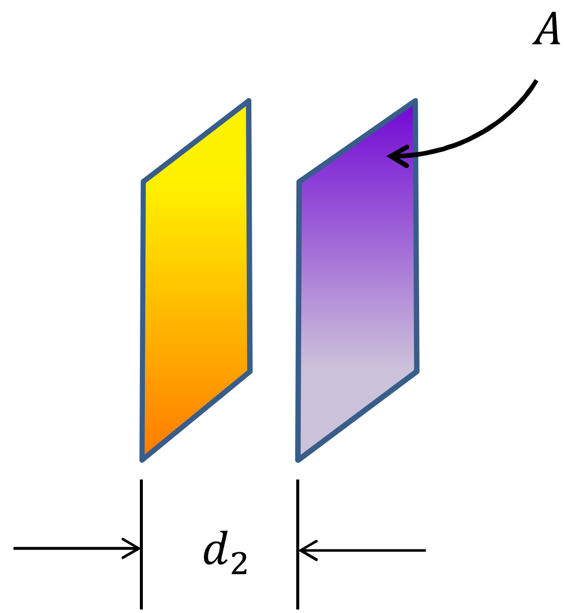
$$C' = \frac{Q}{2V} = \frac{C}{2}$$

- *Equivalent series capacitance is reduced.*

Capacitors in Series

- Consider two parallel plate capacitors:

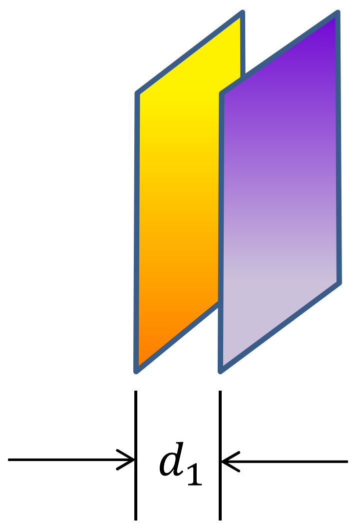


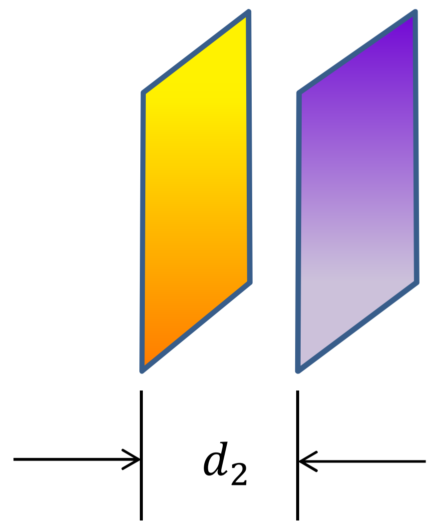


$$C_1 = \frac{\epsilon_0 A}{d_1} \qquad C_2 = \frac{\epsilon_0 A}{d_2}$$

Capacitors in Series

- Consider two parallel plate capacitors:



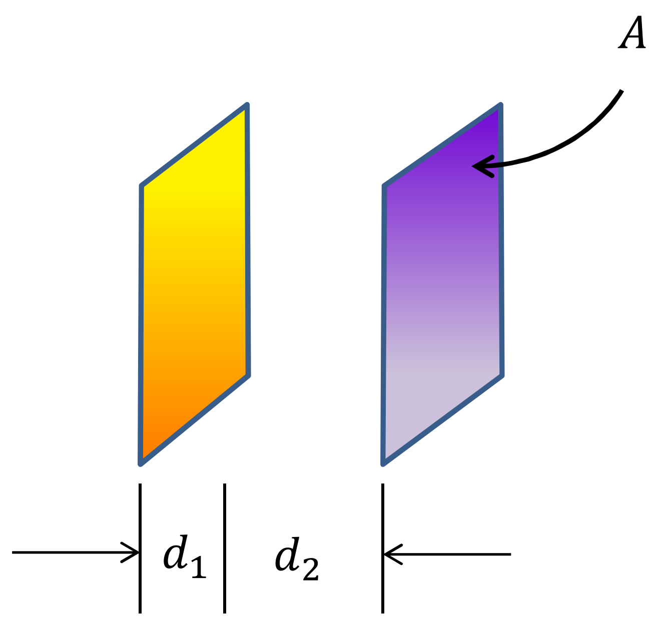


$$C_1 = \frac{\epsilon_0 A}{d_1}$$

$$C_2 = \frac{\epsilon_0 A}{d_2}$$

Capacitors in Series

- Consider two parallel plate capacitors:



$$C = \frac{\epsilon_0 A}{d_1 + d_2}$$

Capacitance in Series

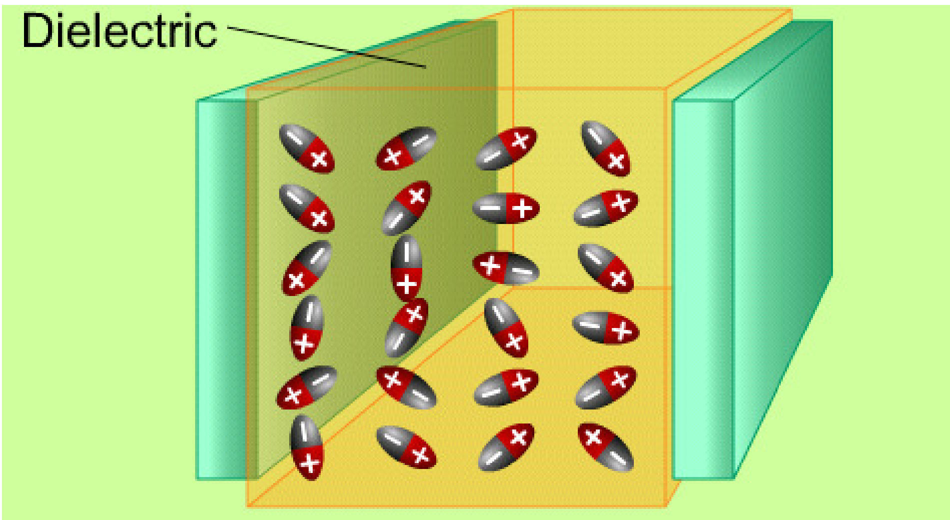
$$C = \frac{\epsilon_0 A}{d_1 + d_2}$$

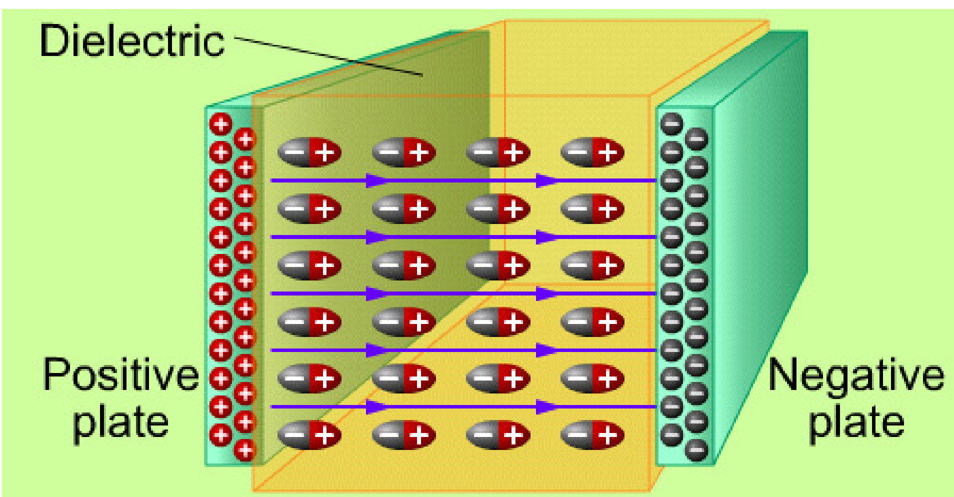
$$\frac{1}{C} = \frac{d_1 + d_2}{\epsilon_0 A} = \frac{1}{C_1} + \frac{1}{C_2}$$

- Equivalent series capacitance:

$$\boldsymbol{C = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1}}$$

Dielectrics





- I dielectric medium is polarized by the electric field.
- The effective charge at the surfaces is reduced.
- The resulting potential difference is also reduced.

$Q = C\,\Delta V$ (no dielectric)

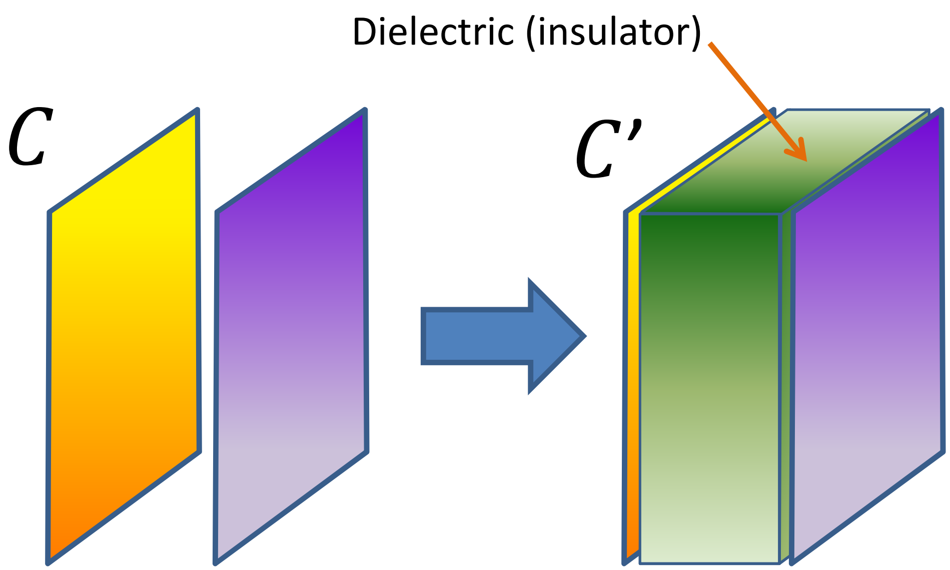
$Q = C'\Delta V'$ (with dielectric)

$\Delta V' < \Delta V$ so $C' > C$

- *A dielectric material will increase the capacitance.*

Adding an Insulating Material (Dielectric)

- Empirical observation: inserting a dielectric material *reduces* the electric potential difference.
- If Q is constant, the capacitance must increase.
- The "dielectric constant", κ, of the material is the ratio of the capacitance when the space between the conductors is filled with the dielectric material, to the original capacitance.



$$\kappa = \frac{C'}{C} > 1$$

$$C = \frac{\kappa \epsilon_0 A}{d}$$

Permittivity

- The ratio κ is called the dielectric constant.
- We can define

$$\epsilon = \kappa \, \epsilon_0$$

which is called the permittivity of the material with dielectric constant κ.

- We can replace ϵ_0 with ϵ in formulas when the space between conductors is not vacuum.
- For example, the parallel plate capacitor…

$$C = \frac{\epsilon_0 A}{d} \quad \Longrightarrow \quad C = \frac{\epsilon \, A}{d}$$

Dielectric Breakdown

- If the electric field is too strong, it can ionize the dielectric (rip electrons off its molecules).
- Free electrons = conductor... (sparks).
- Can result in damage to the dielectric.
- ***DIELECTRIC STRENGTH***: maximum *electric field* a dielectric can tolerate before breaking down.
- ***BREAKDOWN POTENTIAL***: maximum *electric potential* a device can tolerate before breaking down.

Dielectric Constants

material	κ	dielectric strength (kV/mm)
air (1 atm)	1.00054	3
paraffin	2.1–2.5	10
glass (Pyrex)	5.6	14
mica	5.4	10–100
polystyrene	2.55	24
H_2O (20^0 C)	80	(depends on what's dissolved in it)
Strontium titanate	240	8
Sulfur hexafluoride	1.002026	9

Preventing Electrical Discharge

- The PRIME lab in the basement uses a tandem Van de Graaff generator to accelerate atomic fragments.
- It is full of Sulfur Hexafluoride to increase the breakdown potential.

- For more fun with Sulfur Hexafluoride, check out http://www.youtube.com/watch?v=u19QfJWI1oQ