Purdue University Department of Physics

Physics 21900

General Physics II

Electricity, Magnetism and Optics

Lecture 25 – Chapter 27

The Hydrogen Atom

Fall 2015 Semester

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Review

- Photons are *quanta* of electromagnetic radiation
- The energy of a photon depends on its frequency:

$$E = hf$$

$$h = 6.626 \times 10^{-34}\, J \cdot s \quad \text{(Planck's constant)}$$

- Wavelength is related to frequency:

$$\lambda = \frac{c}{f}$$

- Energy is related to wavelength:

$$E = \frac{hc}{\lambda}$$

Visible EM Radiation

From Maxwell in the 1860s, we know that light is an EM wave.

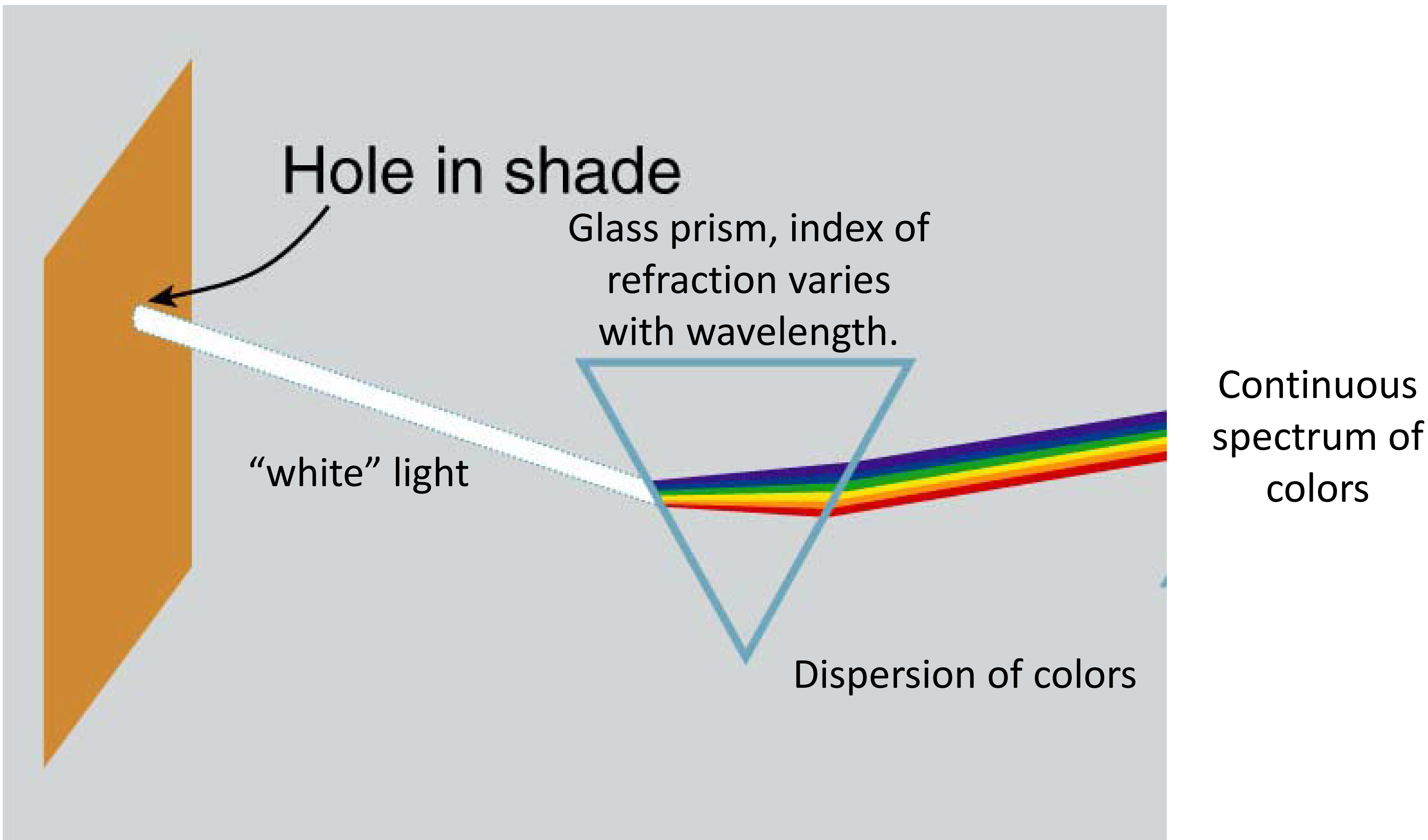
- In 1888, Hertz showed that EM waves with **long wavelengths** could be launched and detected using transmitters and receivers (the forerunners of today's communication industry).

But what about visible light? How is it generated?

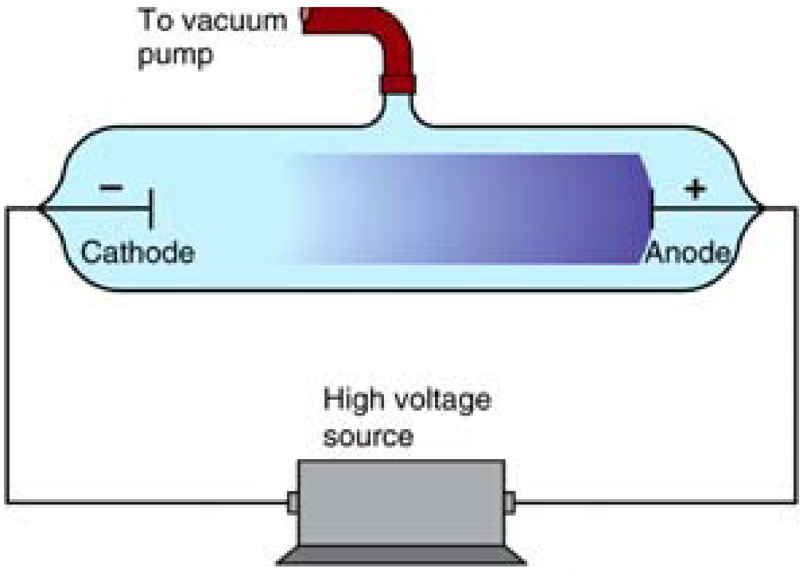
- Consider a few sources of visible light:
- Sun
- Fire
- Oil lamp
- Heat radiation (blackbody)
- Gas discharge tube

Consider sunlight – "white" light containing a continuous spectrum of colors.

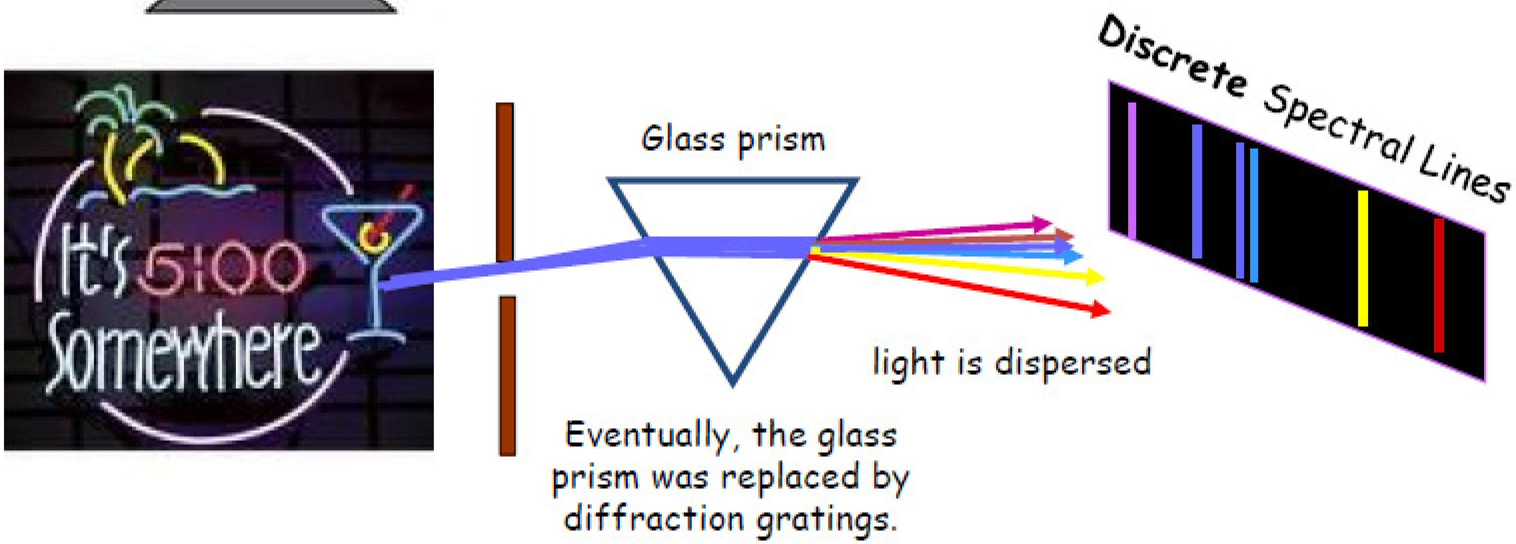
Newton, 1666



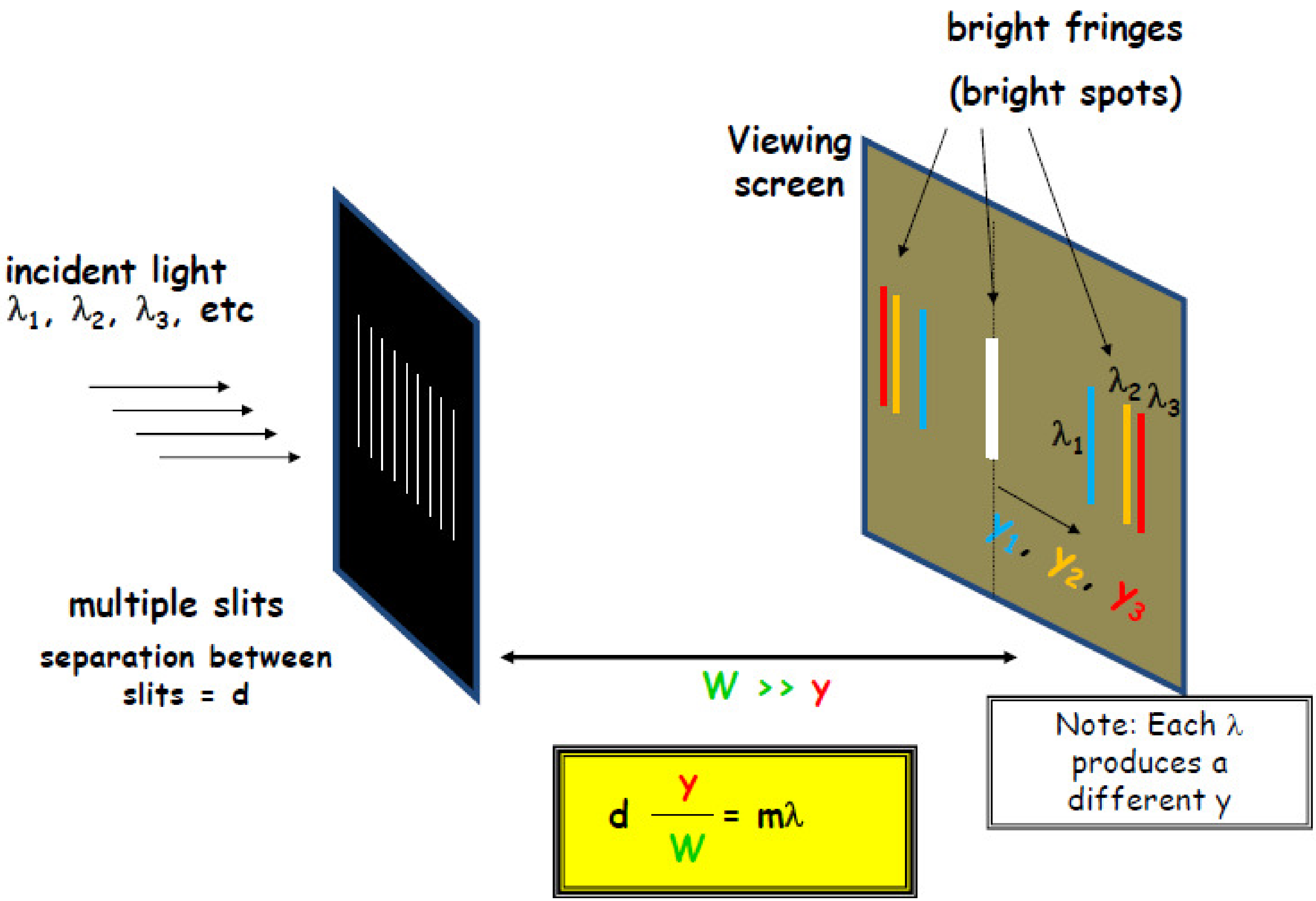
Newton's result is inconsistent with light emitted from a gas discharge tube.



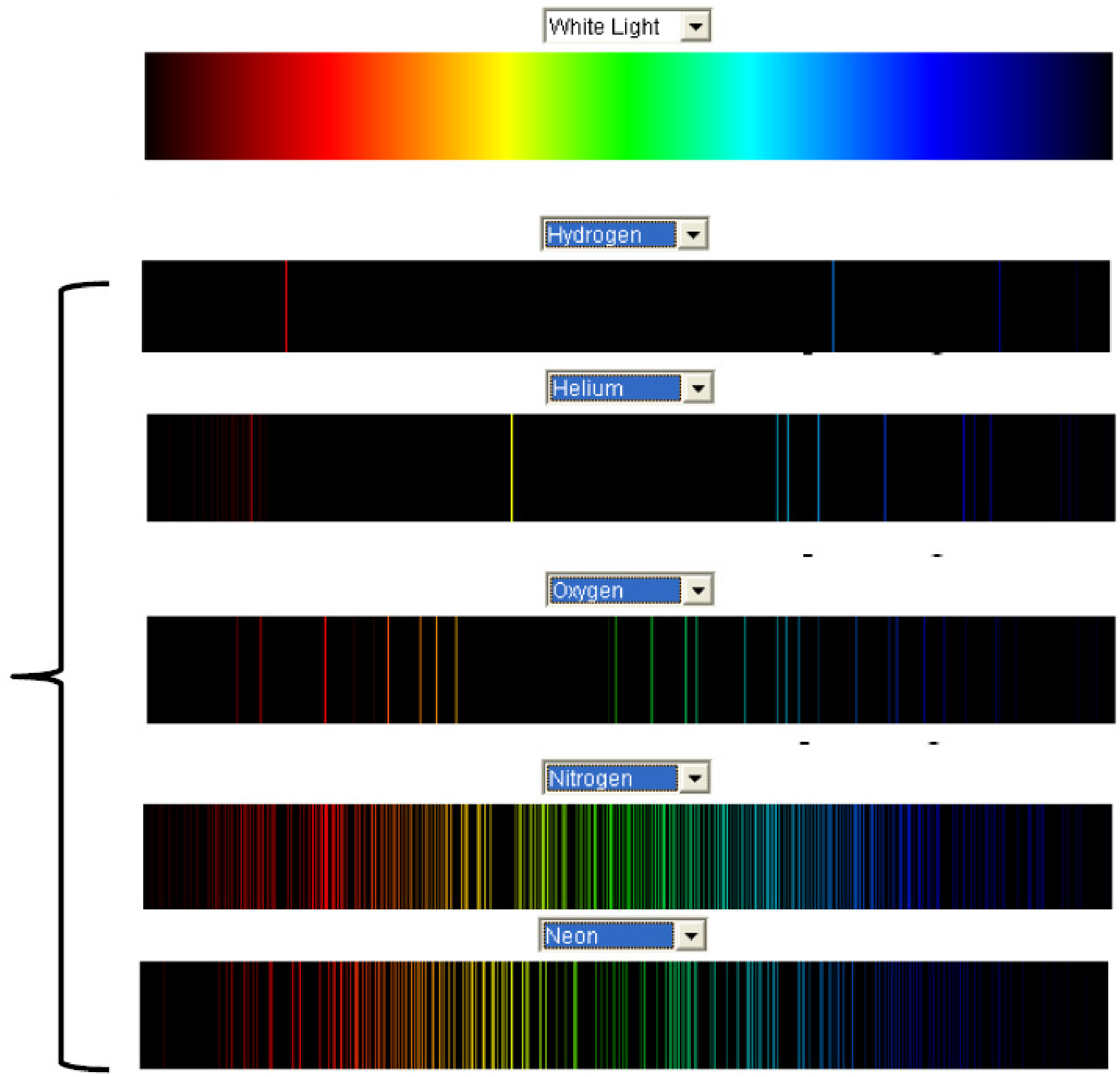
Glass tube, evacuated and then filled with a low pressure gas, like hydrogen, helium, neon, etc…



Wavelengths can be measured more accurately using a diffraction grating



Spectral Lines for Common Gases

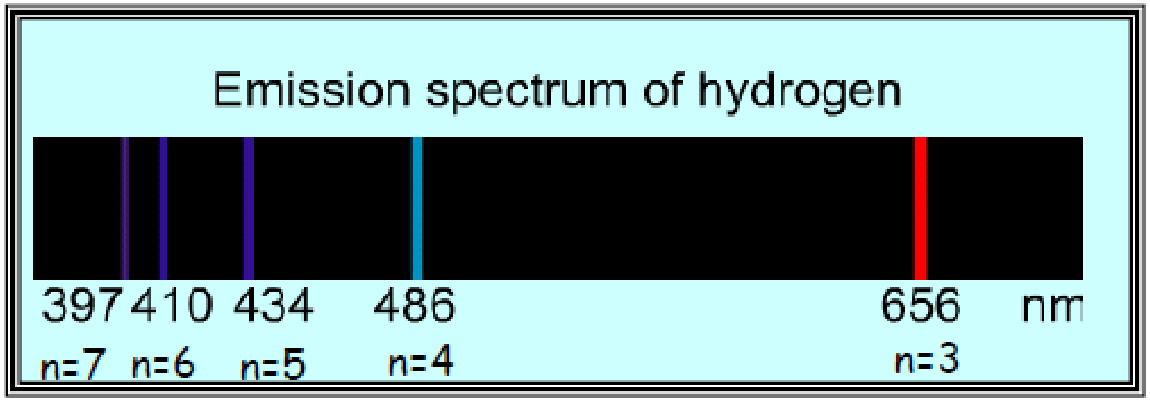


Wavelengths accurately measured

Why are there so many DISCRETE lines?

Hydrogen discharge tube

Balmer's **empirical** formula (1884) explains the observed **visible** wavelengths from hydrogen gas with *high accuracy*



$$\frac{1}{\lambda} = R_H \left[\frac{1}{2^2} - \frac{1}{n^2} \right]; \; n = 3, 4, 5...$$

R_H adjusted to fit experiment

$$R_H = 1.097 \times 10^7 \ m^{-1}$$

Why does this work?

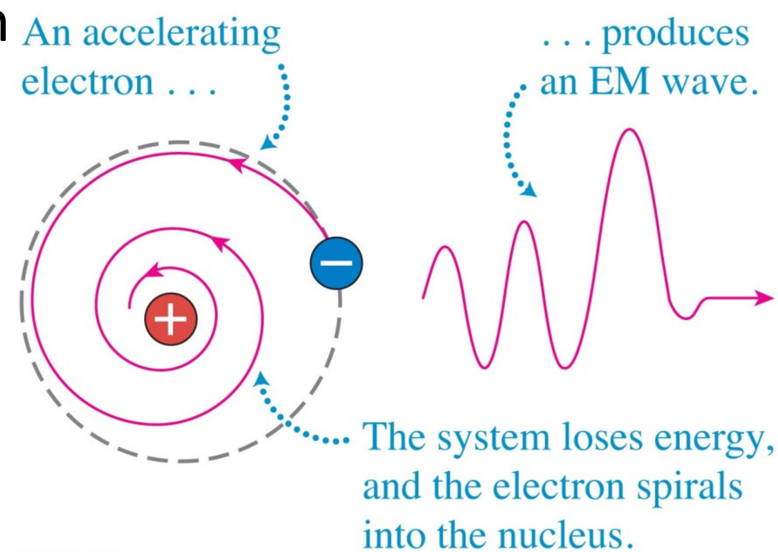
The Hydrogen Atom

- One proton (charge +e) and one electron (charge –e)
- Attractive force via Coulomb's law:

$$\vec{F} = -k\frac{e^2}{r^2}\hat{r}$$

- Classically, an accelerated charge radiates electromagnetic energy.

- Circular motion requires acceleration to change the direction of the velocity
- Classically, a hydrogen atom should be unstable because the electron would spiral inwards as it radiates energy



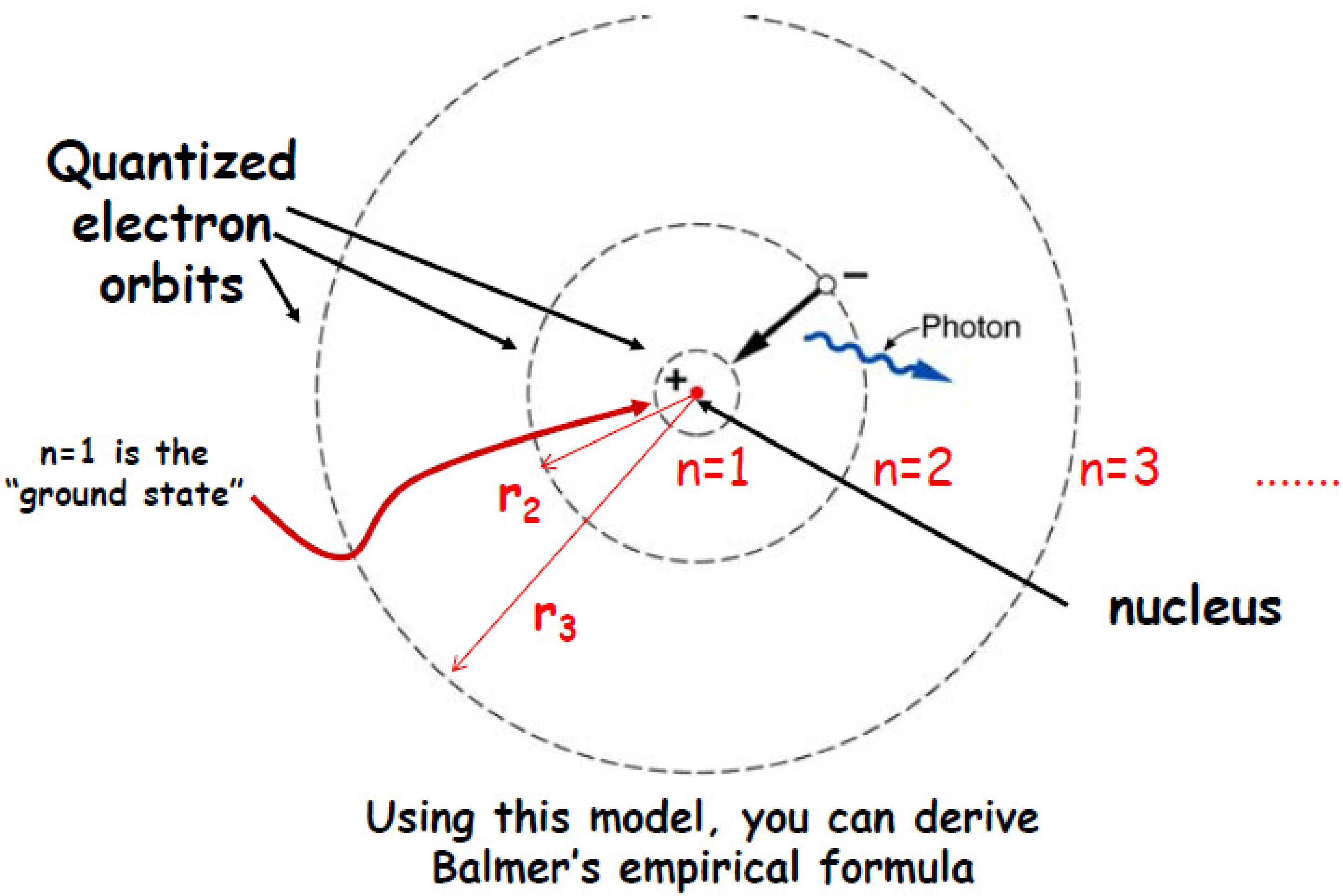
Bohr Model (1913) Assumptions

- Electron moves in special circular orbits – stationary states
- Only certain orbits are allowed; quantization of angular momentum determines the radius of the orbit:

$$r = n^2 a_0 \quad (n \text{ is a positive integer})$$

- Electron gives off no radiation when in a stationary orbit
- Radiation is emitted only when an electron makes a transition from one stationary orbit to another.

Bohr's model for light emission from H

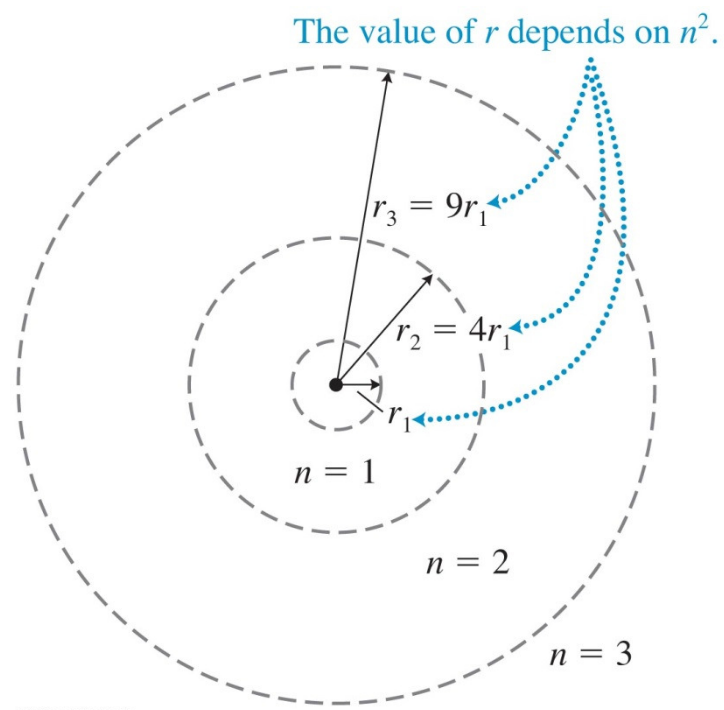


Size of the hydrogen atom

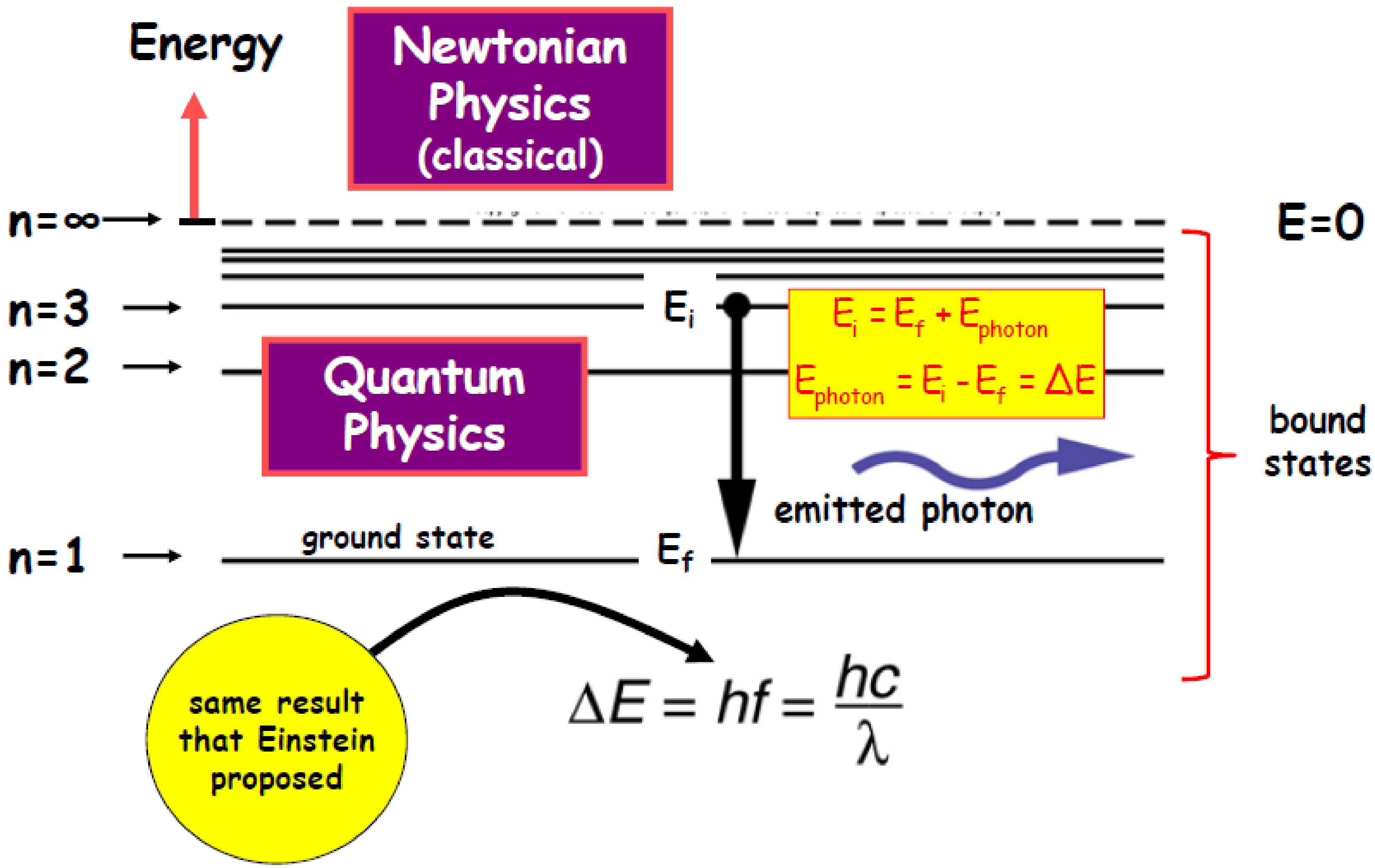
$r_n = (0.53 \times 10^{-10}\ \text{m})n^2$, for $n = 1,2,3, \ldots$

- n is called the principal quantum number and must be a positive integer
- Only certain radii represent stable electron orbits.

$$r_n = n^2 a_0$$
$$a_0 = 0.0529\ \text{nm}$$



Each orbit gives rise to a discrete energy level



Allowed energy levels for the H atom

Prediction of Bohr's Model

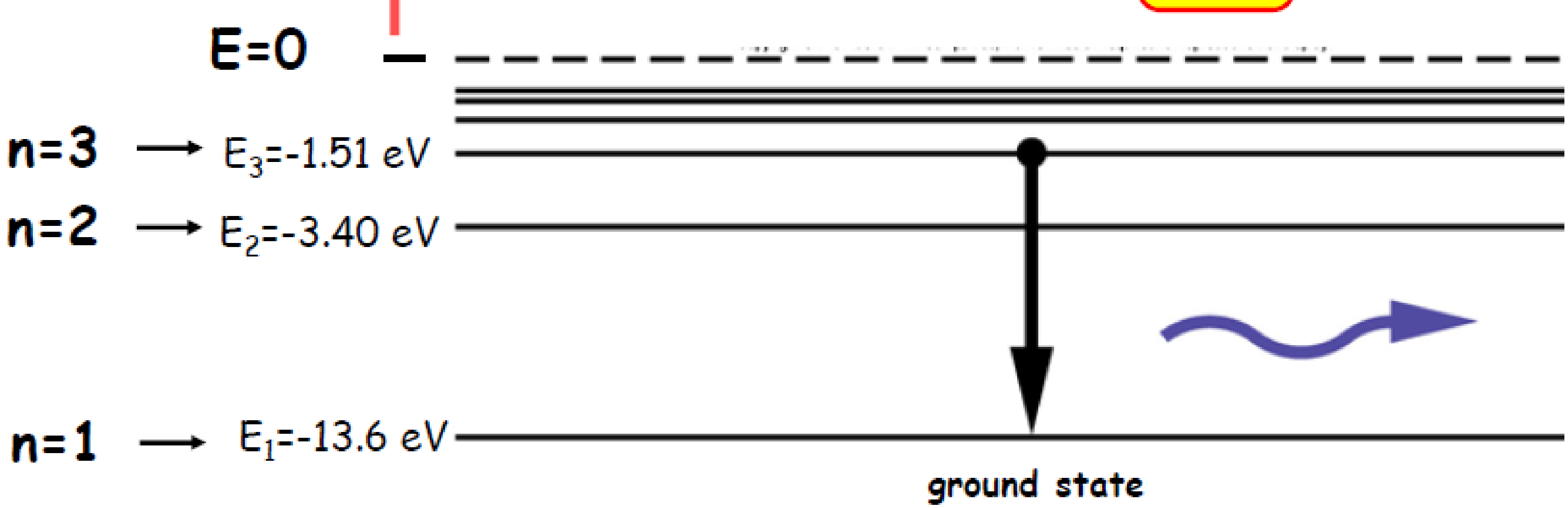
$$E_n = -\left(\frac{1}{4\pi\varepsilon_o}\frac{2\pi e^2}{h}\right)^2 \frac{m_e}{2}\frac{1}{n^2}; \quad n = \text{positive integer}$$

$$= -\left(\frac{1}{4\pi(8.85\times10^{-12})}\frac{2\pi(1.602\times10^{-19})^2}{6.626\times10^{-34}}\right)^2 \frac{9.11\times10^{-31}}{2}\frac{1}{n^2}$$

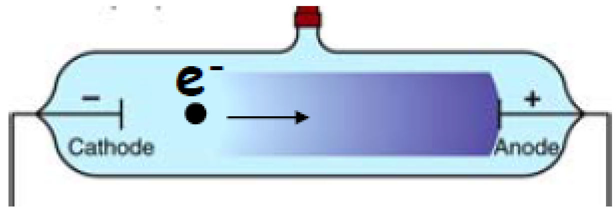
$$= -(2.188\times10^{6})^2(4.555\times10^{-31})\frac{1}{n^2}$$

$$= -\frac{2.181\times10^{-18}\,J}{n^2}\cdot\left(\frac{1\,eV}{1.602\times10^{-19}\,J}\right) = -\frac{13.6\,eV}{n^2}$$

Energy



The electron-volt (eV) is a convenient unit of energy



1 electron moved through a potential difference of -1 Volt acquires 1 eV of energy

$$W = q\,\Delta V$$
$$= (-1.6 \times 10^{-19}\ C)\,(-1V)$$
$$= (-1.6 \times 10^{-19}\ C)\,(-1J/C)$$
$$= +1.6 \times 10^{-19}\ J$$
$$\equiv 1\ eV$$

$$h = 6.63 \times 10^{-34}\ Js = 4.14 \times 10^{-15}\ eVs$$

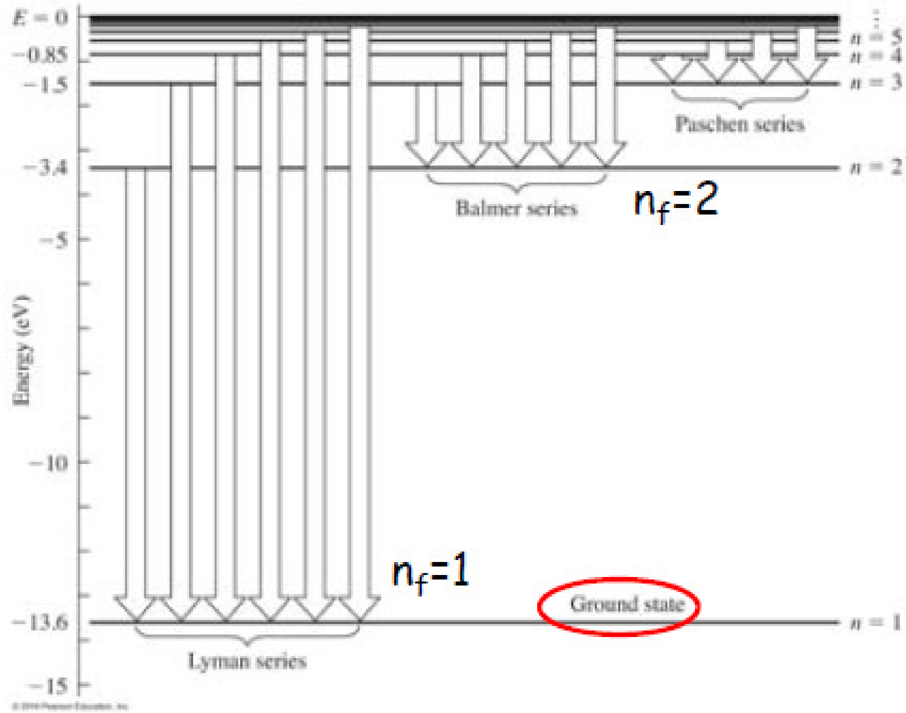
Example - Atomic transitions

$$\Delta E = E_i - E_f$$

where $E_i = -\frac{E_o}{n_i^2}$

and $E_f = -\frac{E_o}{n_f^2}$ with $E_o = 13.6\ eV$

$$\therefore \quad \Delta E = -\frac{E_o}{n_i^2} - \left(-\frac{E_o}{n_f^2}\right) = hf = h\frac{c}{\lambda}$$



$$\frac{1}{\lambda} = \frac{E_o}{hc}\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$= R_H\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

$$R_H = \frac{E_o}{hc} = \frac{(13.6eV)\left(\frac{1.6\times10^{-19}J}{1eV}\right)}{(6.626\times10^{-34}Js)(2.998\times10^{8}m/s)}$$

$$= 1.097\times10^{7}m^{-1}$$ (compare to Balmer's formula)

Example

- An electron in the $n = 3$ orbit has an energy of

$$E_3 = \frac{-13.6\ eV}{3^2} = -1.51\ eV$$

- An electron in the $n = 2$ orbit has an energy of

$$E_2 = \frac{-13.6\ eV}{2^2} = -3.40\ eV$$

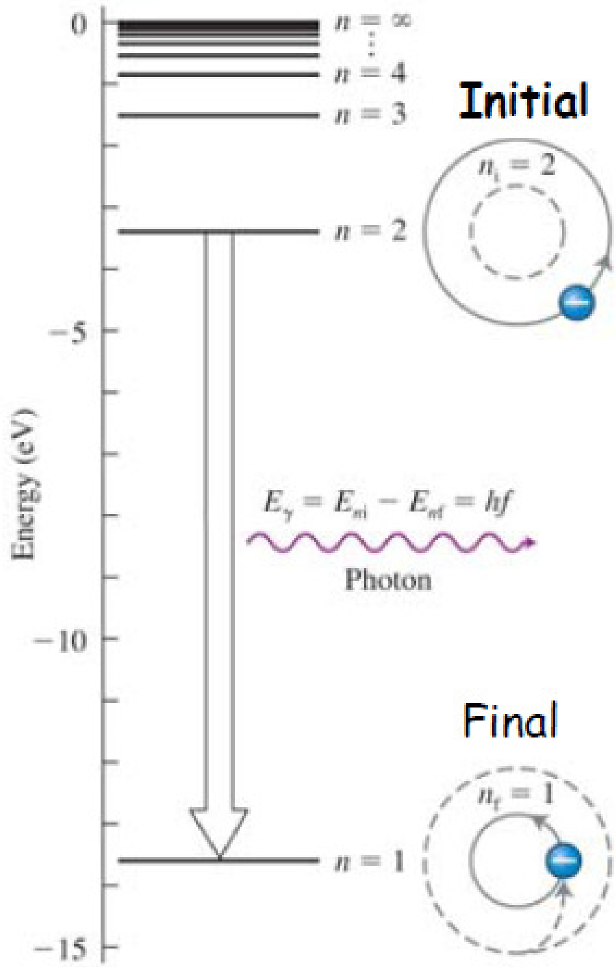
- If the electron "drops" from the *n=3* to the *n=2* orbit, how much energy is released?

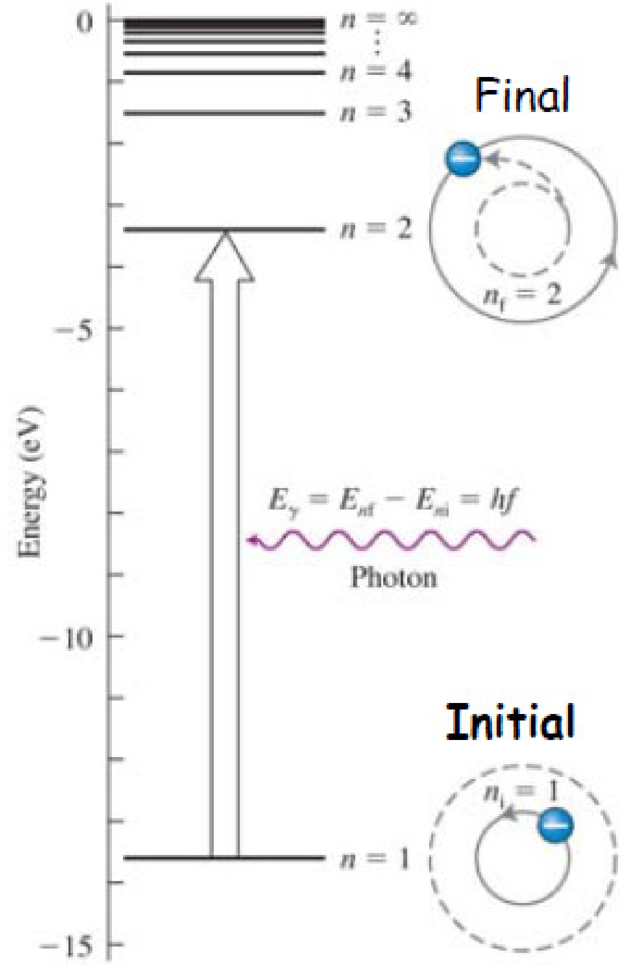
$$\Delta E = E_i - E_f = (-1.51\ eV) - (-3.40\ eV) = \mathbf{1.89\ eV}$$

- From experiment, the 656 nm emission line from H has a frequency of 4.57×10^{14} Hz.
- A photon with this wavelength has an energy of

$$hf = 3.03 \times 10^{-19}\ J = \mathbf{1.89\ eV}$$

Photon Emission and Photon Absorption

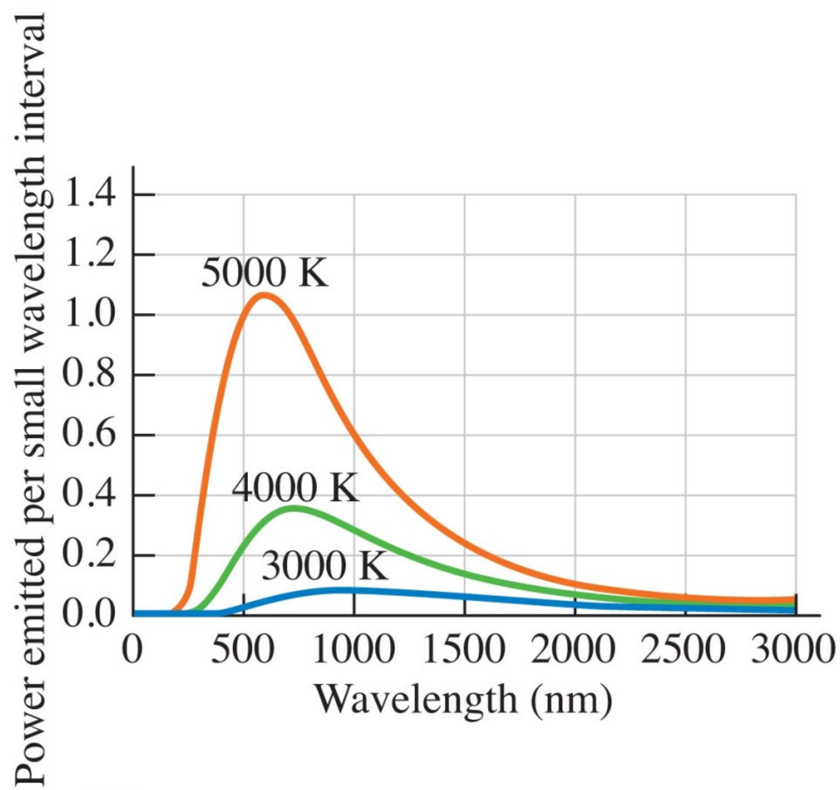




Key process in lasers

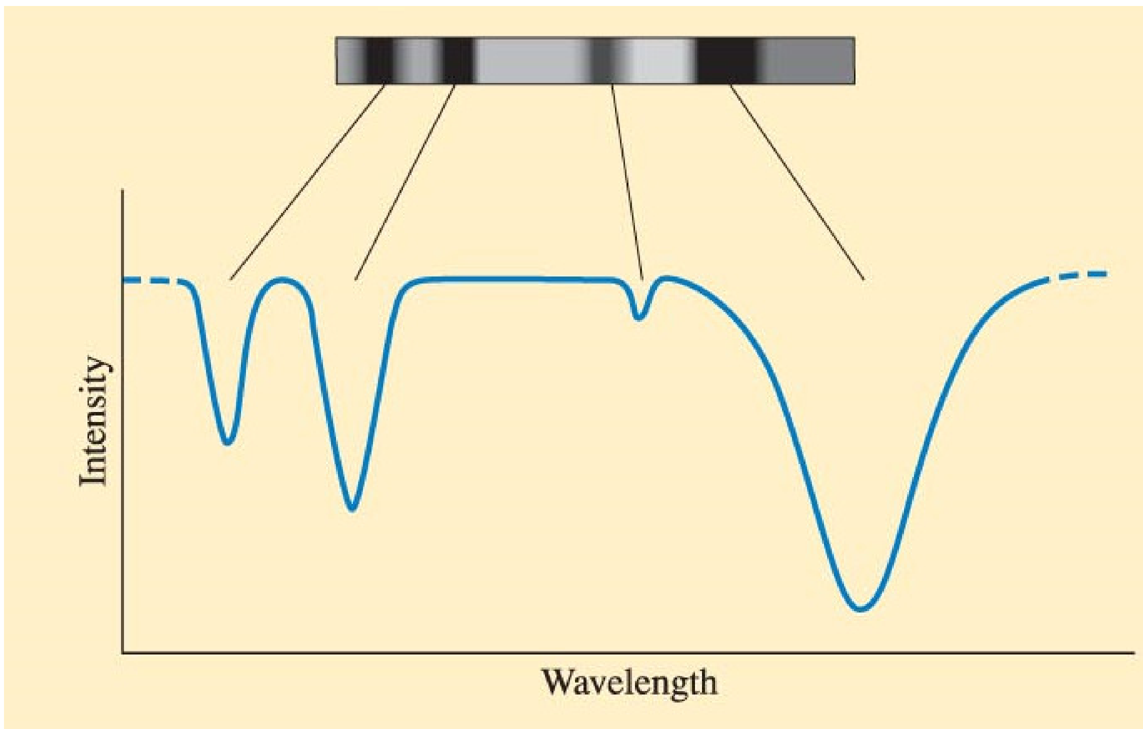
Solar Absorption Spectra

- The sun is composed mostly of hot gas (hydrogen and helium)
- Light from the sun is close to a continuous blackbody spectrum:



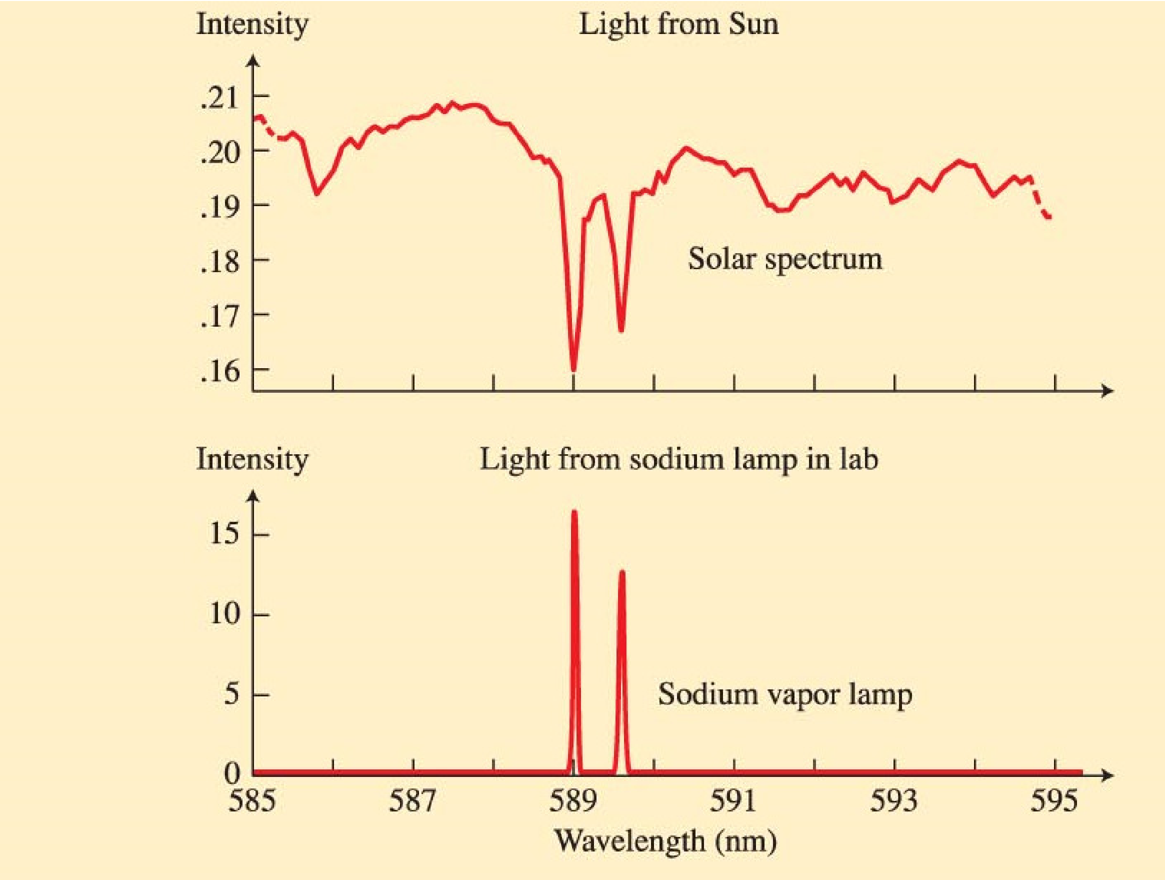
Solar Absorption Spectra

In the visible region, we see a continuous spectrum but with several dark lines.



Solar Absorption Spectra

- Lines in the absorption spectrum correspond to emission lines from a gas discharge lamp



Solar Absorption Spectra