PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 21900

General Physics II

Electricity, Magnetism and Optics

Lecture 23 – Chapter 24.1,5-6

Intensity, energy density and polarization

Fall 2015 Semester

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Properties of Electromagnetic Waves

Basic properties of any wave:

- Frequency: f
- Period: $T = 1/f$
- Wavelength: λ
- Speed (in vacuum): c
- Speed (in a medium): $v = c/n < c$
- Angular frequency, $\omega = 2\pi f$

$$\lambda = \frac{v}{f} = \frac{c}{nf}$$

$$f(x,t) = A\sin\left(2\pi\left(\frac{x}{\lambda} \mp \frac{t}{T}\right)\right)$$

- means the wave propagates in the +x direction.

+ means the wave propagates in the –x direction.

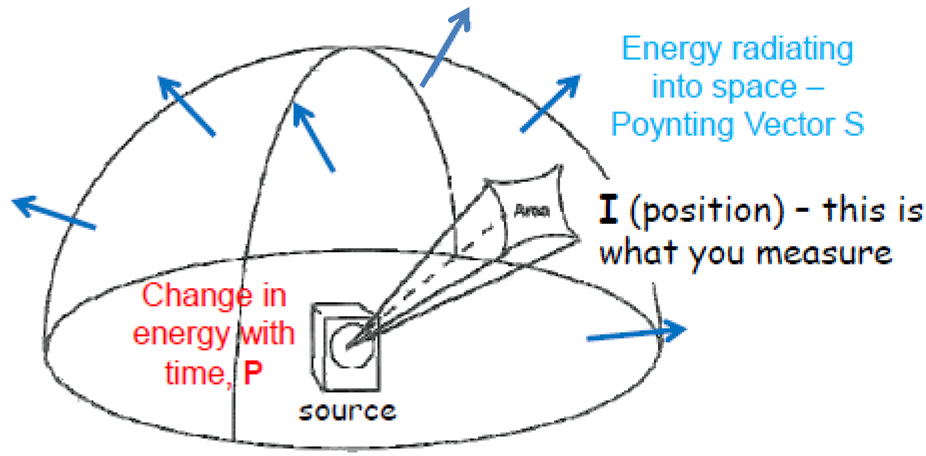
Intensity of an EM Wave

- Electromagnetic waves carry energy
- A source of waves is characterized by its power (the rate at which it radiates energy)
- If the source is isotropic, the energy spreads out uniformly in all directions, over the surface of a sphere
  - Surface area of a sphere of radius R is $A = 4\pi R^2$
- Intensity is defined as the power per unit area
- Intensity at a distance R from a source with power P

$$I = \frac{P}{4\pi R^2}$$

Intensity of an EM Wave

Intensity is defined as the energy that passes through a unit area during a unit time interval.



$$Intensity = I = \frac{Energy\ (J)}{Time\ interval\ (s) \times area\ (m^2)} = \frac{Power}{area}\ [W/m^2]$$

Not all sources are isotropic…

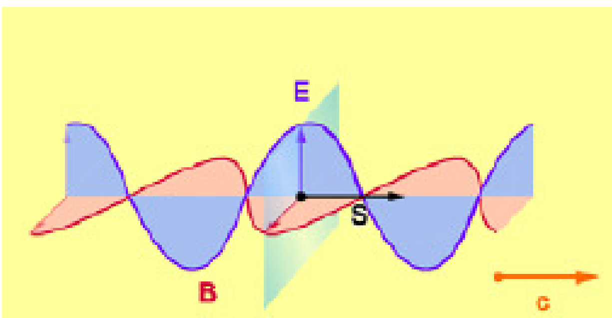
(lasers and flashlights aren't)

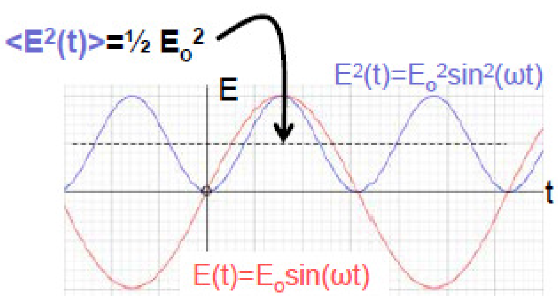
But some are…

(stars, incandescent light bulbs, candles, hot objects)

The Poynting Vector

- Energy moves with the same speed as the wave and in the same direction as the velocity.
- The Poynting vector has a magnitude equal to the instantaneous wave intensity and points in the direction of wave propagation.





$$\vec{S} = \frac{\vec{E}(t)\times\vec{B}(t)}{\mu_0}$$

$$c = \frac{|\vec{E}|}{|\vec{B}|} = \frac{1}{\sqrt{\epsilon_0\mu_0}}$$

$$|\vec{S}| = \frac{E^2(t)}{c\mu_0} = c\epsilon_0 E^2(t)$$

$$S_{avg} = \langle|\vec{S}|\rangle = c\epsilon_0\langle E^2(t)\rangle = c\epsilon_0\frac{E_0^2}{2}$$
$$= c\ \epsilon_0\ E_{rms}^2$$

Units: [W/m^2]

Time Averaged Poynting Vector

- The time-averaged value of the Poynting vector, S, is the same as the wave intensity

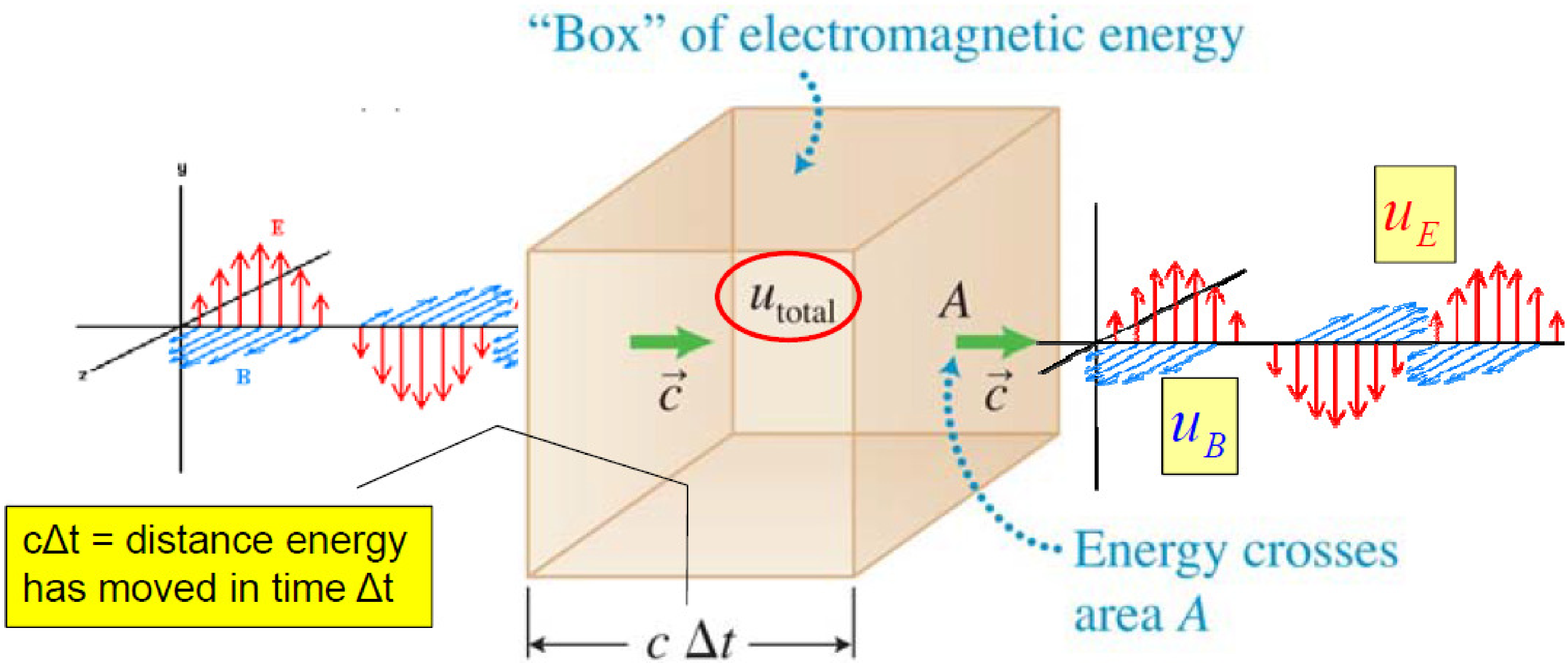
$$I = \left|\vec{S}\right|_{avg} = \frac{1}{2} c\ \epsilon_0\ E_0^2 = c\epsilon_0 E_{rms}^2$$

$$I = \left|\vec{S}\right|_{avg} = \frac{1}{2}\frac{c}{\mu_0} B_0^2 = \frac{c}{\mu_0} B_{rms}^2$$

(Intensity can be expressed either in terms of $E_0,\ B_0,\ E_{rms}$ or B_{rms})

- Intensity specifies the power per unit area (W/m^2) carried by an EM wave, averaged over time.
- Key idea: intensity is proportional to E^2 (or B^2).

Concept of energy density u_{total} (units: J/m^3)



$$u_{total} = u_E + u_B$$

$$because\ E / B = c, \quad u_E = u_B$$

$$u_{total} = 2u_E = 2u_B$$

What is time averaged u_{tot} for an EM wave?

KEY IDEA: Energy is stored in **both** E and B fields

time aver. power thru hole: $P = IA\,[W]$

time aver. energy thru hole: $\Delta U = P\Delta t\,[J]$

Define time aver. energy density $u_{tot}\left[\text{in } \frac{J}{m^3}\right]$:

$$u_{tot} = \frac{\Delta U}{(Ac\Delta t)} = \frac{P\Delta t}{(Ac\Delta t)} = \frac{IA\Delta t}{(Ac\Delta t)} = \frac{I}{c}$$

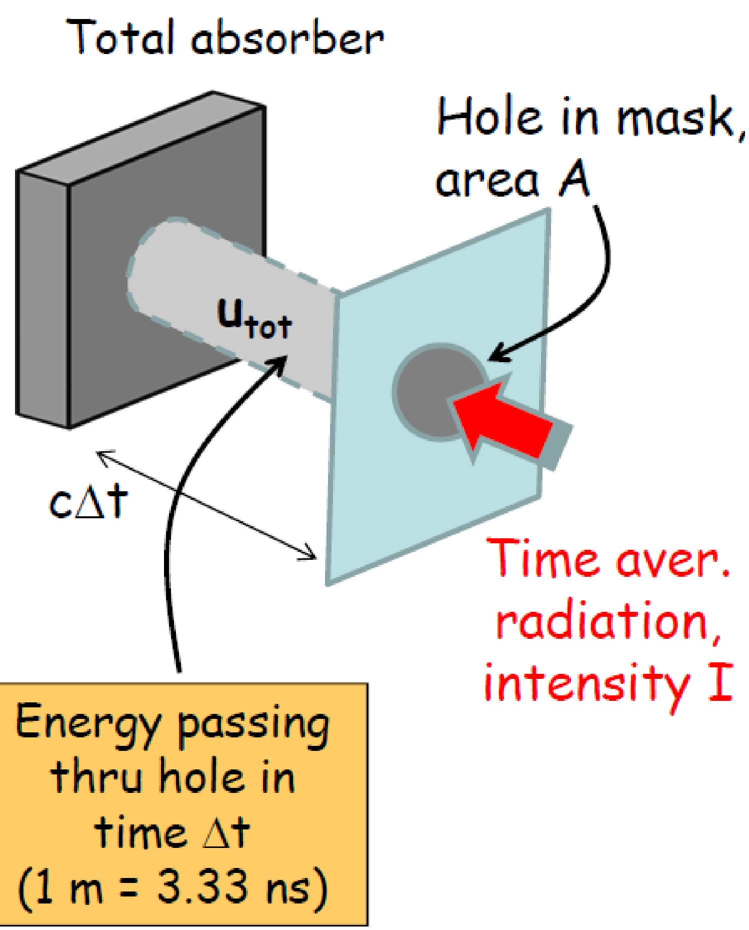
$$u_{tot} = \frac{I}{c} = \frac{1}{c}\left(\frac{1}{2}c\varepsilon_o E_o^2\right) = \frac{1}{c}\left(\frac{c}{2\mu_o}B_o^2\right)$$

$$u_{tot} = \frac{1}{2}\varepsilon_o E_o^2 = \frac{1}{2\mu_o}B_o^2$$

OR

$$u_{tot} = \varepsilon_o E_{rms}^2 = \frac{1}{\mu_o}B_{rms}^2$$

Units: J/m^3



Important to distinguish between what we can measure and what we can calculate.

Closely Related Concepts

- **Intensity** (in W/m^2) of an EM wave: **I**
- **Power** (in W or J/s) carried (or transmitted) by EM wave through an area *A*: $\mathbf{P=I\times A}$
- **Energy** (in J) delivered by an EM wave in a time Δt: $\mathbf{U=P\Delta t=(I\times A)\Delta t}$
- **Energy density** (in J/m^3) of an EM wave: $\mathbf{u_{tot}=I/c}$

SUMMARY

Speed of EM Wave in free space (m/s)	$c = \frac{1}{\sqrt{\varepsilon_o \mu_o}}$
Ratio of Peak Fields	$E_o = cB_o$
Frequency-Wavelength relationship	$\lambda f = c$
Poynting Vector S (W/m²)	$\vec{S} = \frac{1}{\mu_o}\vec{E} \times \vec{B}$
Intensity of EM Wave (W/m²) (time average of Poynting vector for sinusoidal, linearly polarized, plane EM wave)	$I = \langle S \rangle = \lvert \vec{S} \rvert_{AVERAGE} = \langle \lvert \vec{S} \rvert \rangle$ $= c\varepsilon_o \langle E^2(t) \rangle = c\varepsilon_o \frac{E_o^2}{2}$
Energy density (J/m³)	$u_{tot} = u_E + u_B = \frac{1}{2}\left\langle \varepsilon_o E^2(t) + \frac{1}{\mu_o} B^2(t) \right\rangle$ $= \frac{1}{2}\left(\varepsilon_o \frac{E_o^{\ 2}}{2} + \frac{1}{\mu_o}\frac{B_o^{\ 2}}{2} \right)$ $= \varepsilon_o \frac{E_o^{\ 2}}{2} = \frac{1}{\mu_o}\frac{B_o^{\ 2}}{2} = \frac{I}{c}$

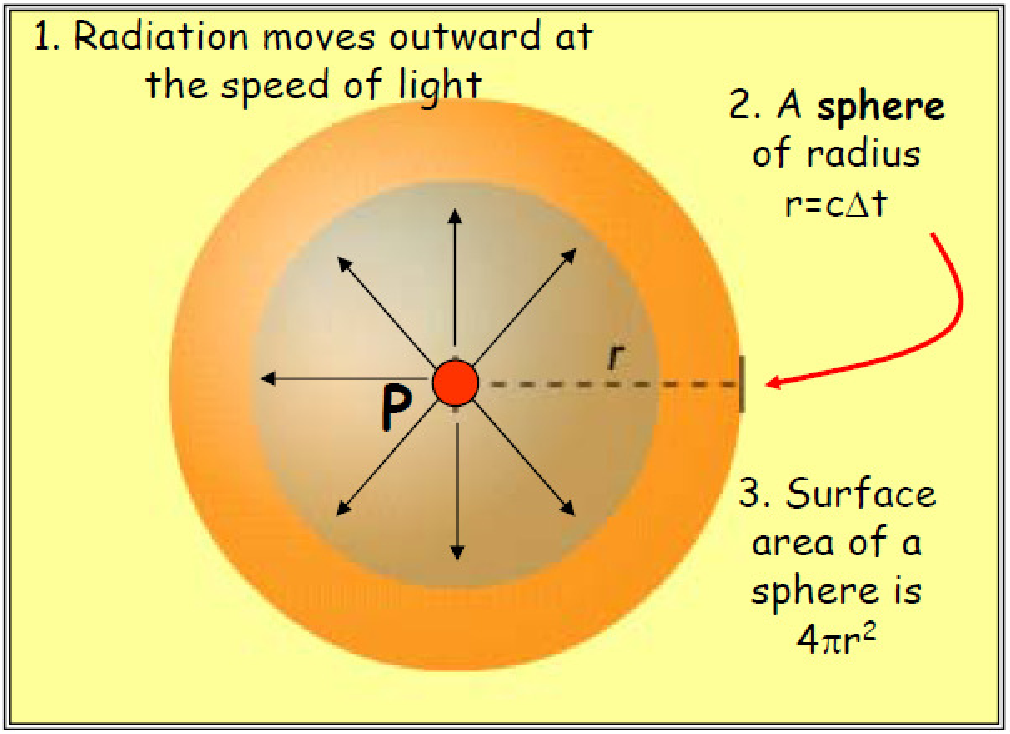
Propagation of Radiation

- In general, radiation can propagate outward in all directions
- The ideal case of a very small source producing spherical wave fronts is called a ***point source***
- The intensity of a spherical wave decreases with distance: $I \propto 1/r^2$
- The intensity must decrease as $1/r^2$ because a "constant" amount of energy spreads out over greater areas
- The intensity relationship ($I \propto 1/r^2$) applies to many situations, including the strength of a radio signal from a distant station

Intensity vs. Distance?

Radiation from Ideal Point Source

Ideal case: Uniform radiation from a **point source** emitting EM radiation at an average power P



$$I = \frac{P}{4\pi r^2}$$

Key Idea: Intensity decreases as r^{-2}

Example 1

Sunlight striking the earth's surface on a cloudless summer day has an intensity of about 1000 W/m^2. If the average **surface area** for a child (9 years old) is 1.07 m^2, estimate the maximum energy absorbed during 1 hour of fun in the summer sun.

$$I = \frac{\text{Energy}}{\text{time interval} \times \text{area}}$$

$$\text{Energy} = I \times \text{time interval} \times \text{area}$$

$$= 1000\ W/m^2 \times 3600\ s \times 1.07\ m^2$$

$$= 3.85 \times 10^6\ J$$

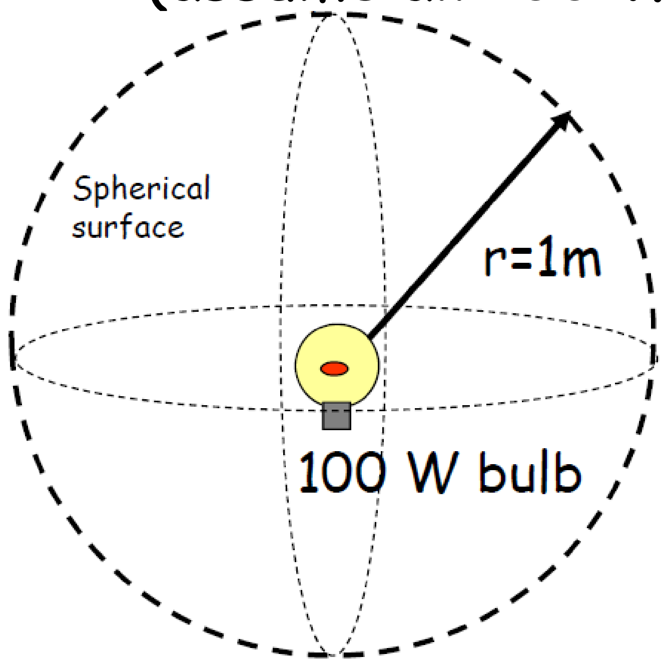
since $\frac{1}{2}$ of body is not illuminated

$$\text{Energy absorbed} \simeq 0.5\left(3.85 \times 10^6\ J\right) = 1.93 \times 10^6\ J$$

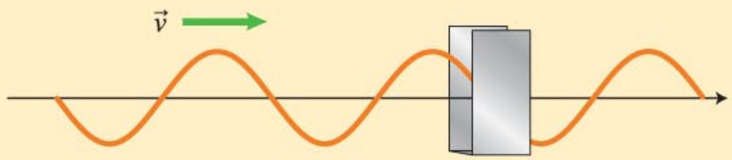
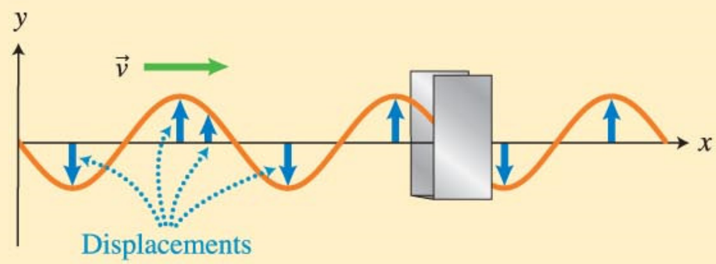
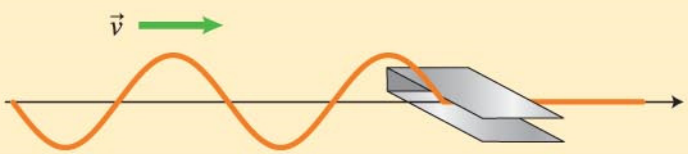
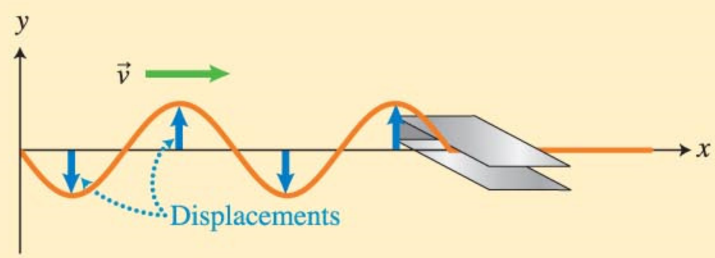
Example 2:

Calculate (a) the intensity, (b) energy density, (c) amplitude of the E-field and (d) amplitude of the B-field, all at 1 m from a 100 W light bulb?

(assume all 100 W goes into EM radiation.)



Polarization

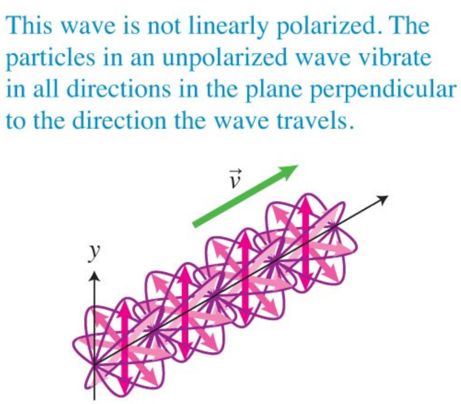
Observational experiment	Analysis
Experiment 1. A rope passes through the open ends of a narrow rectangular box. Shake the rope in a vertical plane, producing a transverse wave. The long sides of the box are parallel to the shaking direction. The rope wave is unaffected by the box. 	Vectors represent the displacement of each section of rope at one instant. They are parallel to the long sides of the box. 
Experiment 2. Rotate the box 90° with respect to the original orientation. Shake the rope the same way as in Experiment 1. The long sides of the box are perpendicular to the shaking direction. The wave does not pass through the box. 	The displacement vectors of the rope sections are perpendicular to the long sides of the box. 

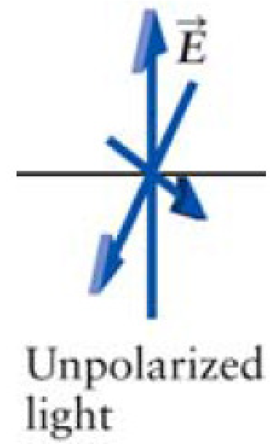
Does this concept apply to light?

Polarization of Light

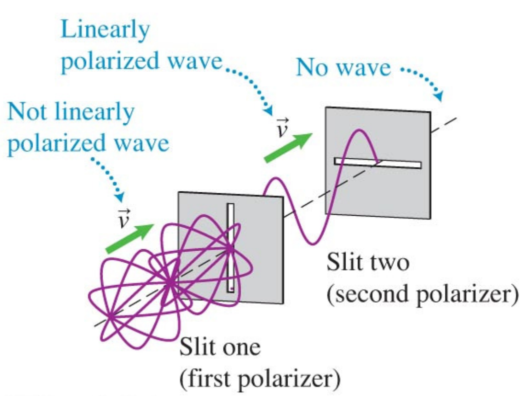
Polarization is a property of light that describes the orientation of the $\vec{E}$-field oscillation in the wave.

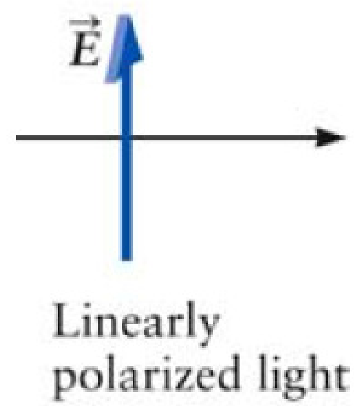
(a)





(b)



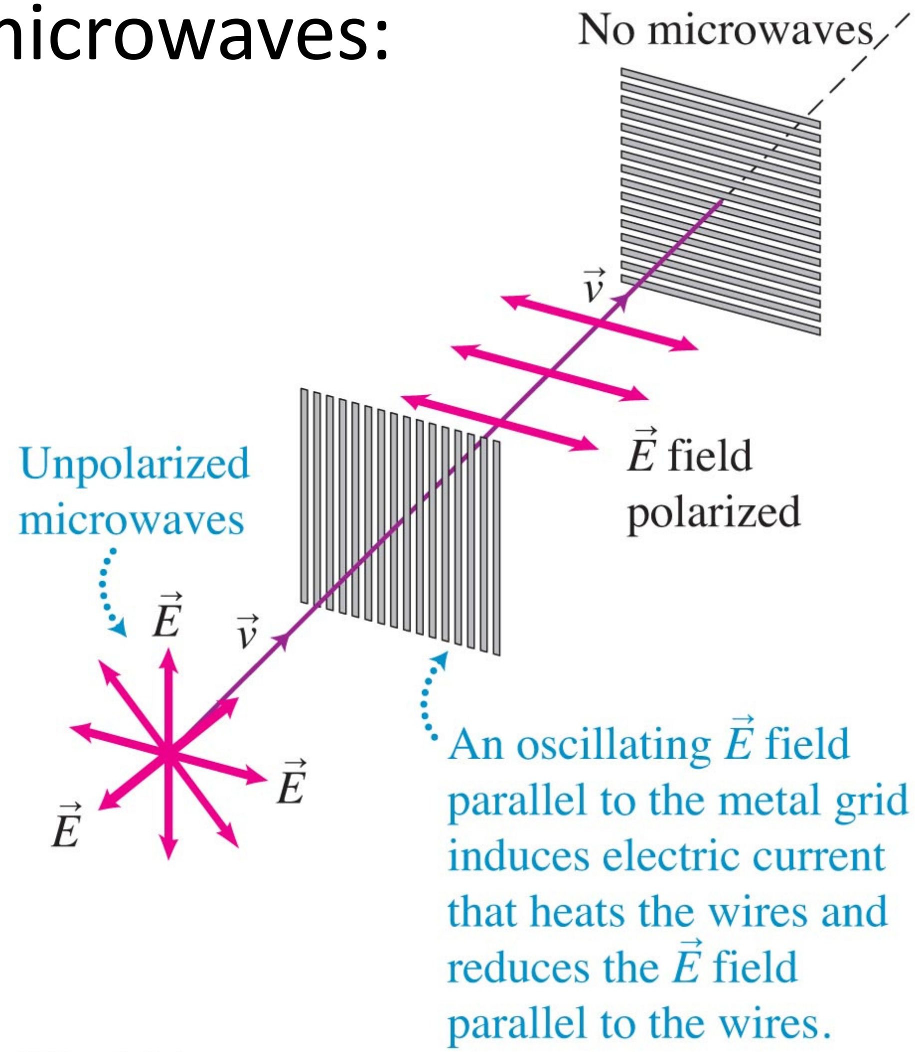


Polarization of EM Radiation

- For any EM wave, the electric field is perpendicular to the direction of propagation
- There are many possible directions of the electric field in an EM wave
- Knowing the direction of the electric field in an EM wave is important for determining how the wave interacts with matter.
- Most light is unpolarized
- Polarized light can be produced from unpolarized light by using a **polarizer**
- A polarizer often consists of a thin, plastic film that allows only the **component** of the electric field parallel to a particular direction to pass through. This direction is called the **axis of the polarizer**.
- A polarizer absorbs the component of the electric field that is perpendicular to the polarizer's axis.

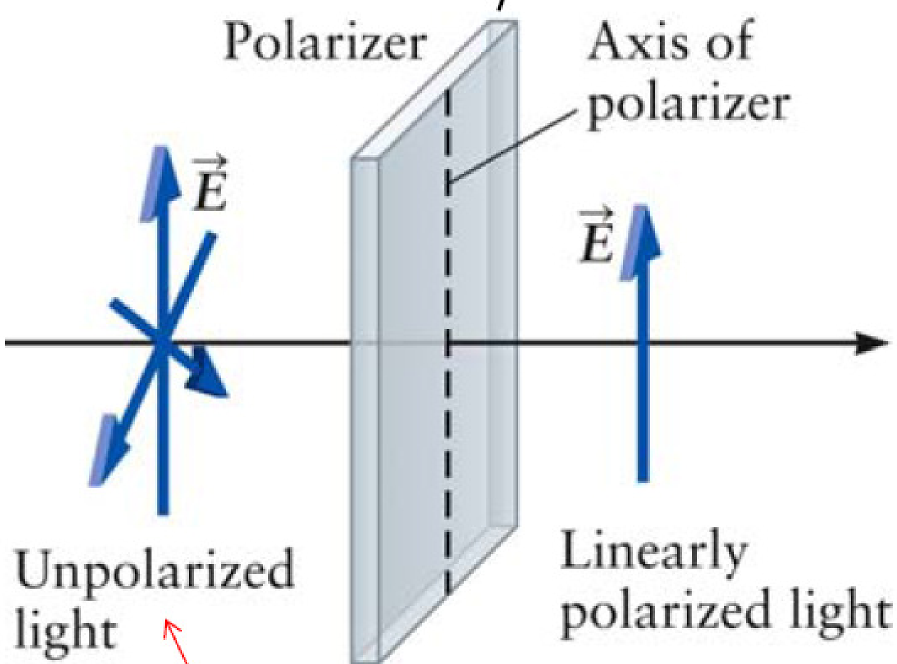
The Effect of a Polarizer

- Example using microwaves:

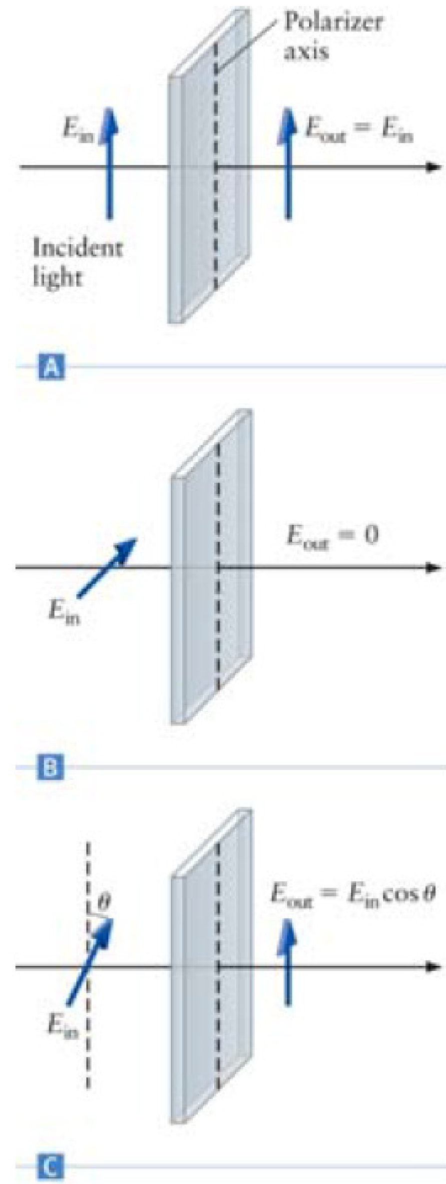


The Effect of a Polarizer

In 1928, Edwin Herbert Land invents a sheet-type dichroic linear polarizer (as an undergrad at Harvard University)

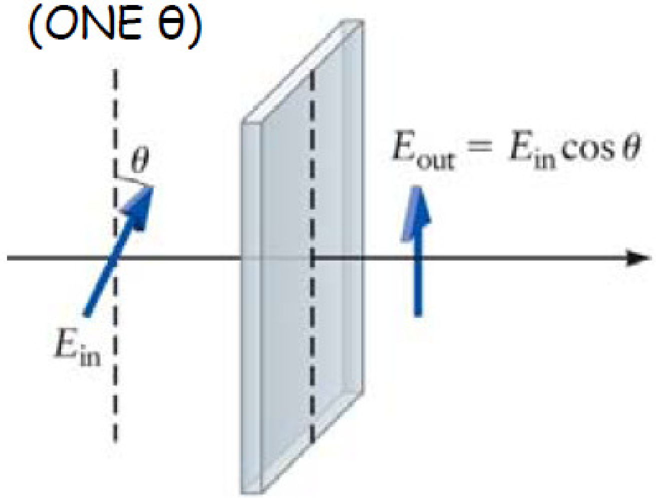


Light emitted by a lightbulb consists of many waves that originate at random times with random polarizations.



The Law of Malus' (~1810)

Polarized light (ONE θ)



Recall......... $I = \langle S \rangle = \frac{cB_o^2}{2\mu_o} = \frac{\varepsilon_o c}{2}E_o^2$

$$I_{in} = \frac{\varepsilon_o c}{2}E_{in}^2$$

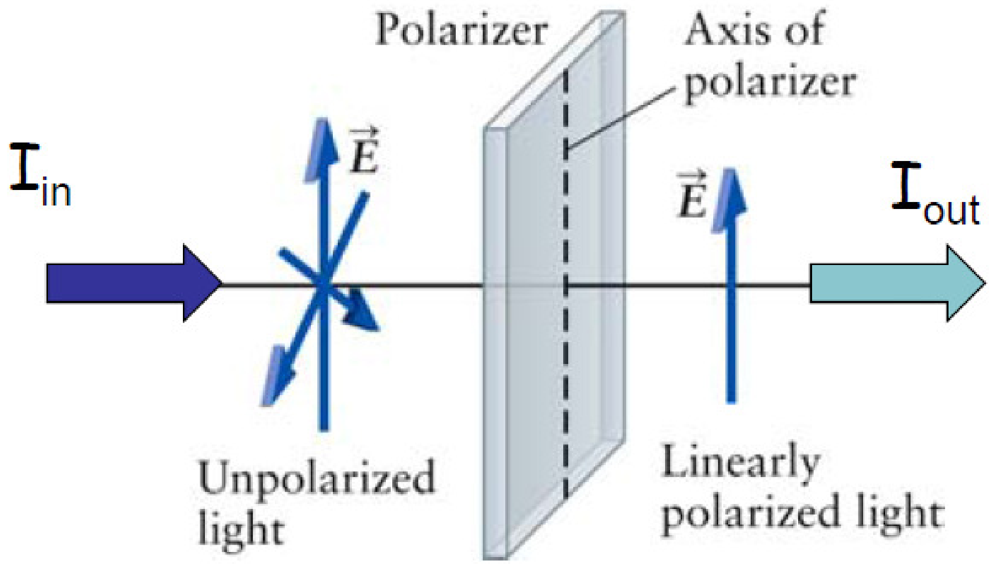
$$I_{out} = \frac{\varepsilon_o c}{2}E_{out}^2 = \left(\frac{\varepsilon_o c}{2}E_{in}^2\right)\cos^2\theta$$

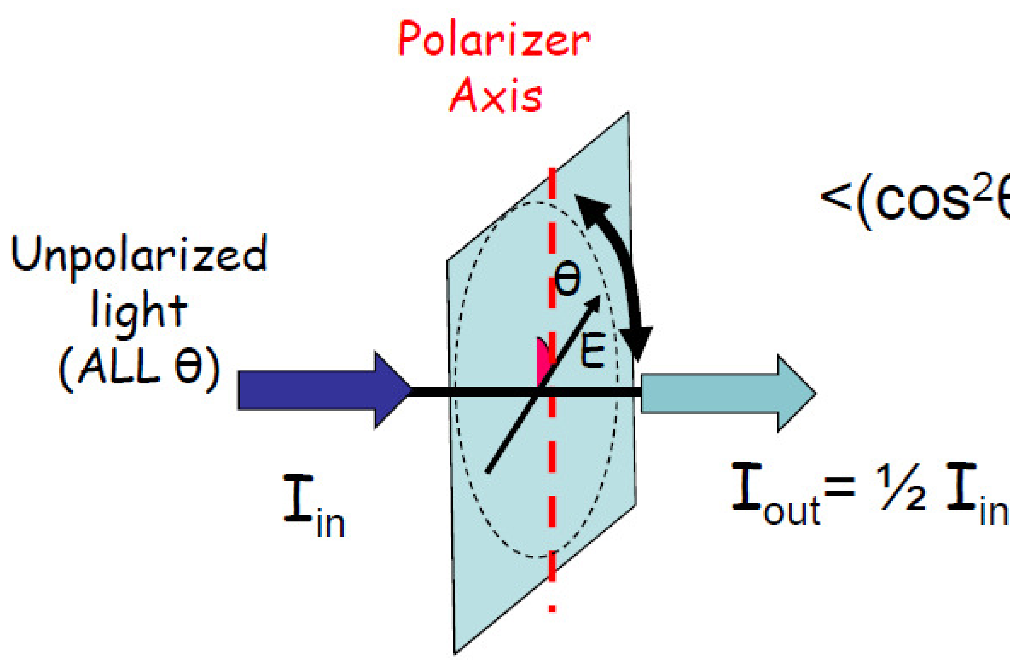
$$= I_{in}\cos^2\theta$$

$$I_{out} = I_{in}\cos^2\theta$$

Note: E_{in}, E_{out} represents amplitudes or maximum values

Unpolarized Light Through a Polarizer

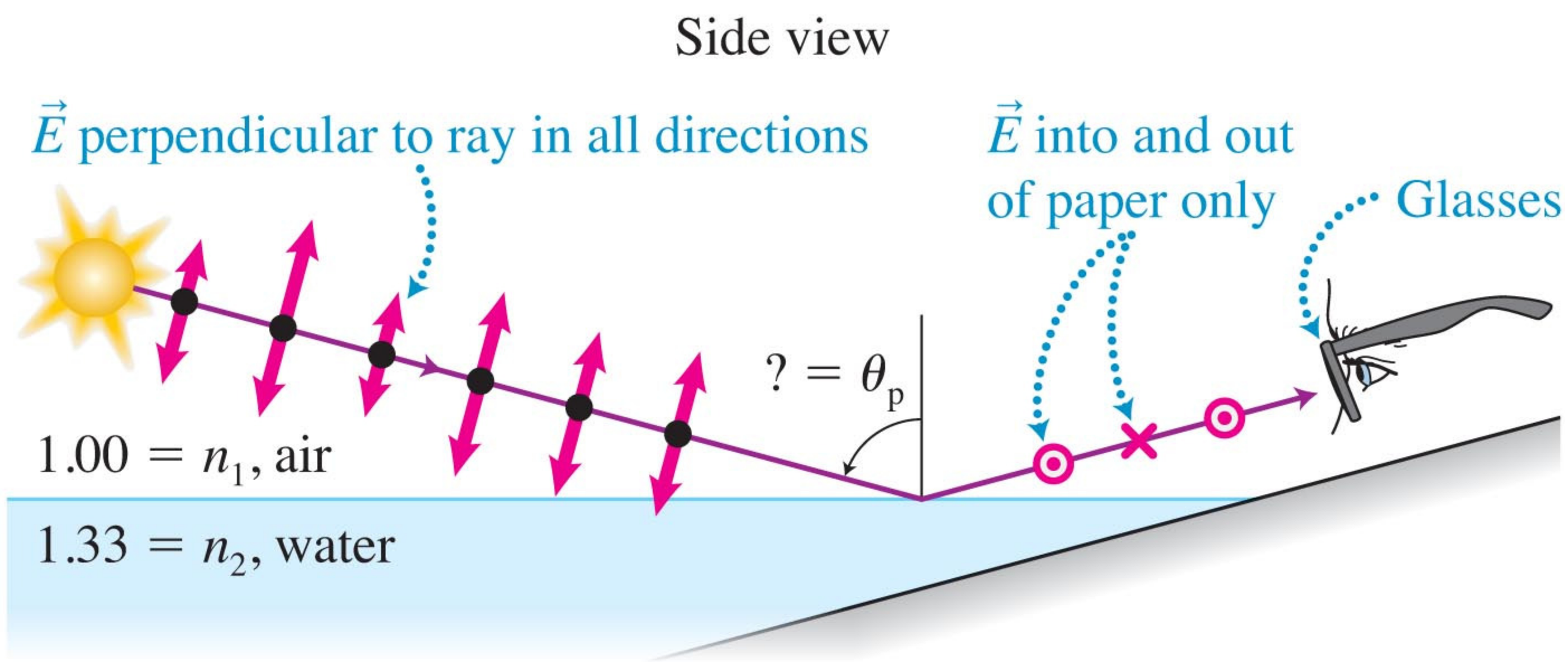




$<(\cos^2\theta)> = ½$

Polarization by Reflection

- Reflected light has its electric field preferentially oriented parallel to the surface.
- The other component is preferentially transmitted into the medium.



Polarized Sun Glasses