PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 21900
General Physics II

Electricity, Magnetism and Optics

Lecture 21 – Chapter 23.3-4

Thin Films and Diffraction Gratings

Fall 2015 Semester

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Wave Optics

- Light has wave-like properties that are more easily observed in certain situations than in others
- Waves from two sources arriving at a common point will interfere constructively or destructively
- This depends on the phase difference of the waves when they arrive at that point

Interference

- Coherent light:
  - The phase is correlated over time and space
  - The phase is not random
- Two sources of waves:

$$f_1(t) = A\cos(\omega t + \theta_1)$$
$$f_2(t) = A\cos(\omega t + \theta_2)$$
$$f_1(t) + f_2(t) = 2\text{A}\ \cos(\omega t)\cos(\delta)$$
$$\delta = \frac{\theta_1 + \theta_2}{2}$$

- Constructive interference: $\delta = 0$
- Destructive interference: $\delta = \pi$

Interference

- What contributes to the phase difference?
  - Different initial phase
  - Different propagation distances

$$\theta_1 = \frac{2\pi x_1}{\lambda} \qquad \theta_2 = \frac{2\pi x_2}{\lambda}$$

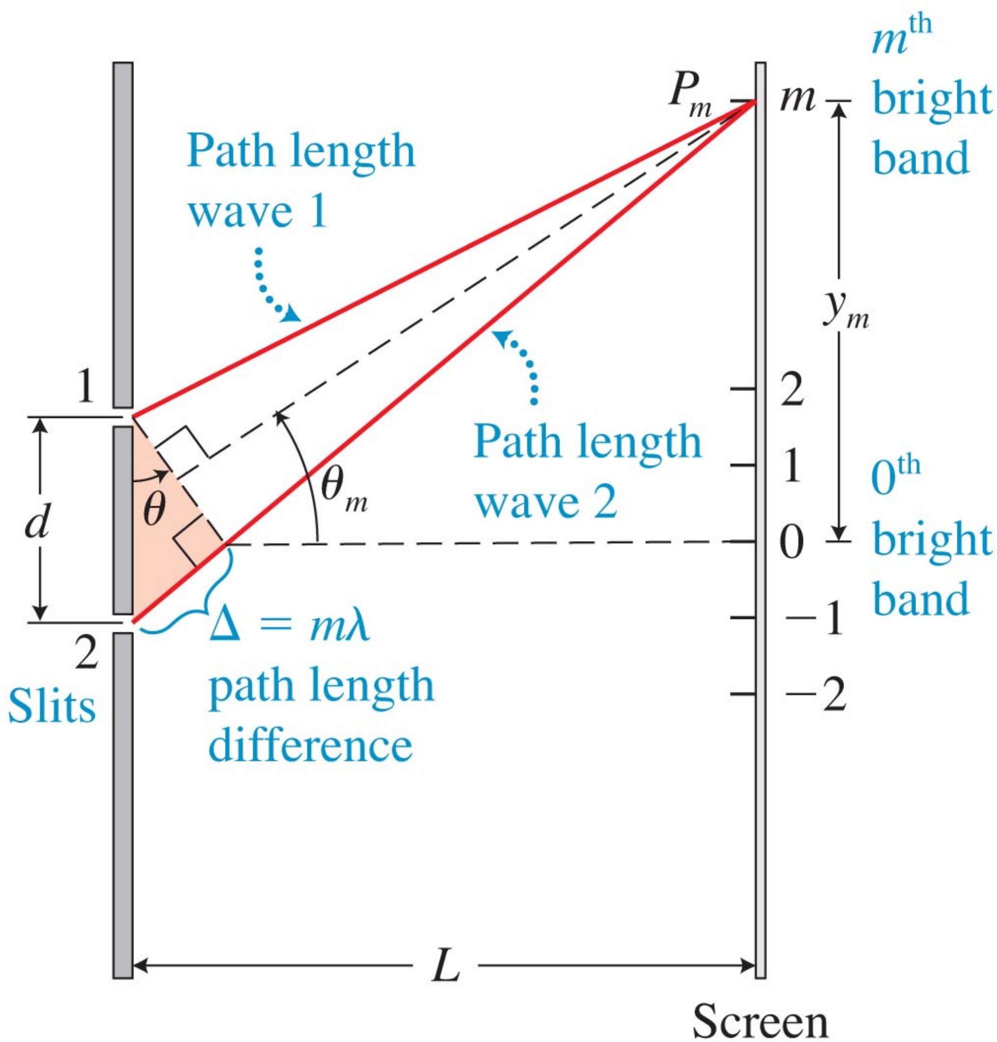
  - Propagation in different media ($\lambda' = \lambda/n$)

$$\theta_1 = 2\pi x \frac{n_1}{\lambda} \qquad \theta_2 = 2\pi x \frac{n_2}{\lambda}$$

  - Reflections (sometimes)

$$\Delta\theta = \pi$$

Double Slit Experiment



When the initial phase is the same, the path length difference is:

$$\Delta = d \sin\theta$$

$$\approx \frac{d\,y}{L}$$

Constructive interference when

$$\Delta = m\,\lambda$$

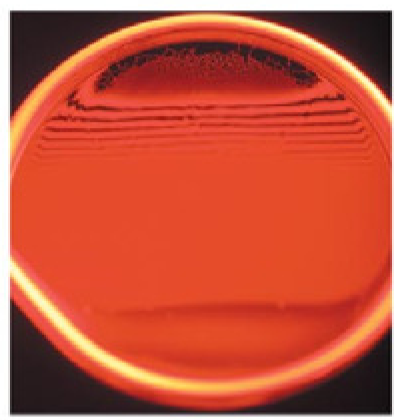
Destructive interference when

$$\Delta = \left(m + \frac{1}{2}\right)\lambda$$

Thin Film Interference

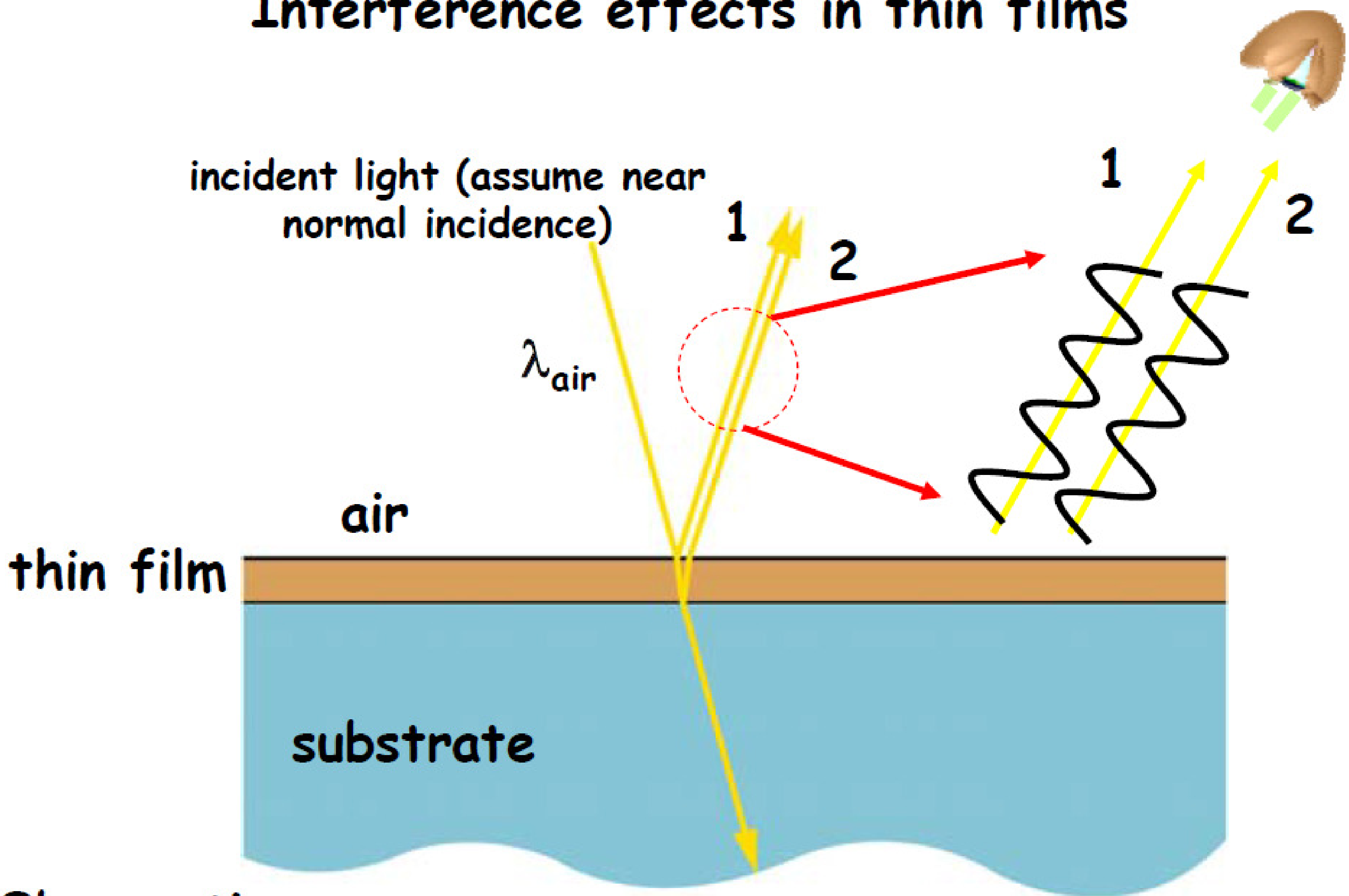
These are examples of interference from thin films:

Iridescence, structural coloration



White light *vs* red light

Interference effects in thin films



Observations:

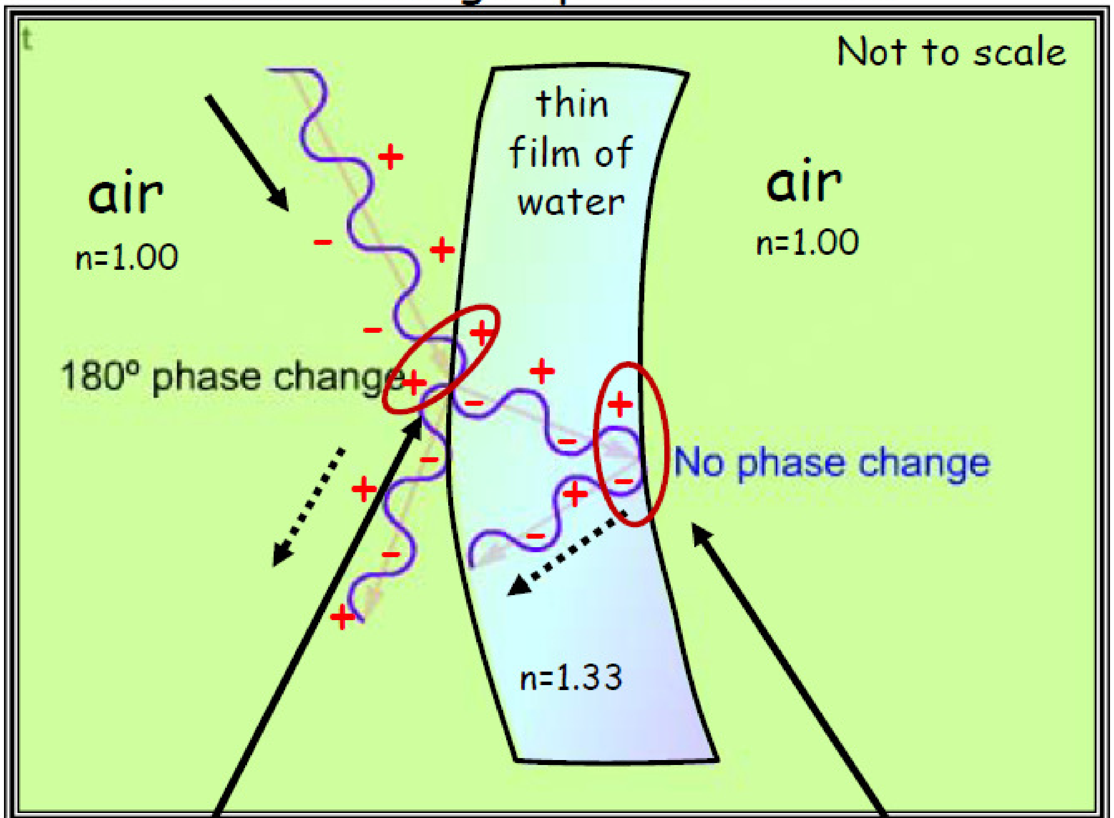
- Ray 2 travels through a longer path than ray 1
- If path difference is integer number of wavelengths, then constructive interference results

Tracking the Phase Upon Reflection

Must Include Phase Change upon Reflection

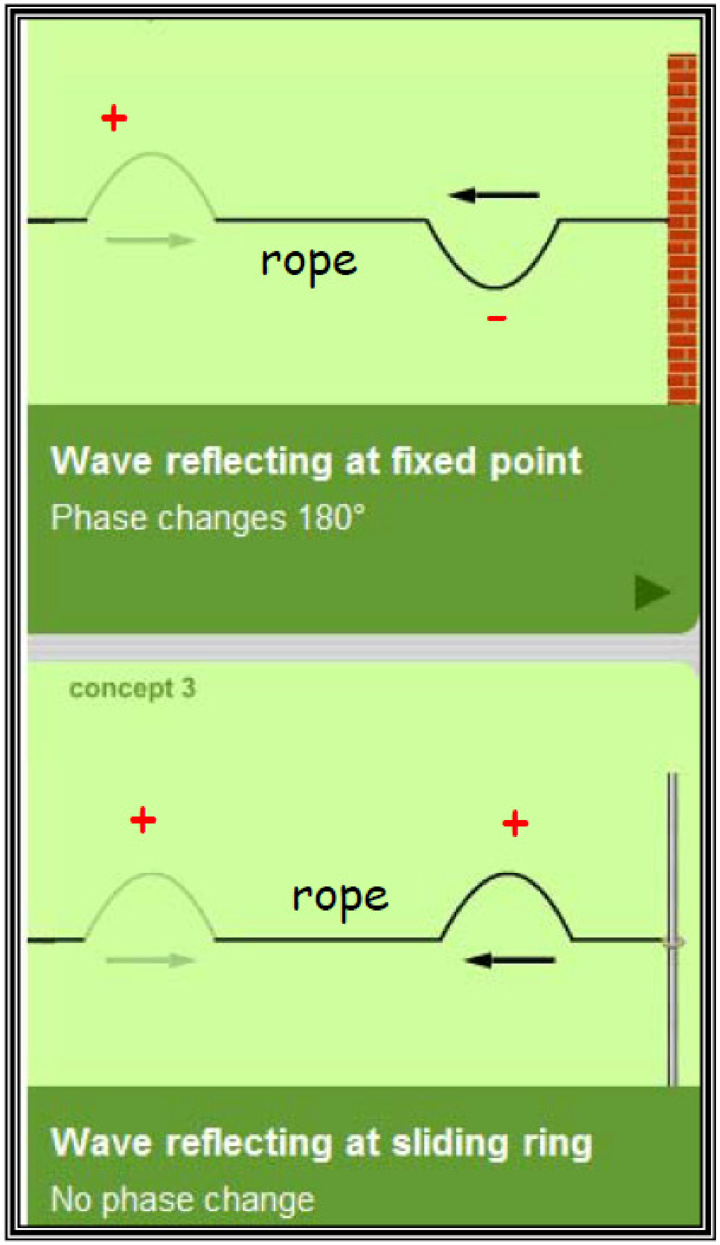
KEY IDEA

Learn to keep track of the phase!!



Reflection off interface from **low** **n** **to** **high** **n:** 180° (π) phase change

Reflection off interface from **high** **n** **to** **low** **n:** no phase change



Phase change upon reflection? Simple Analogy

Take home lesson: phase change upon reflection depends on boundary conditions

Summary so far…

TWO KEY IDEAS

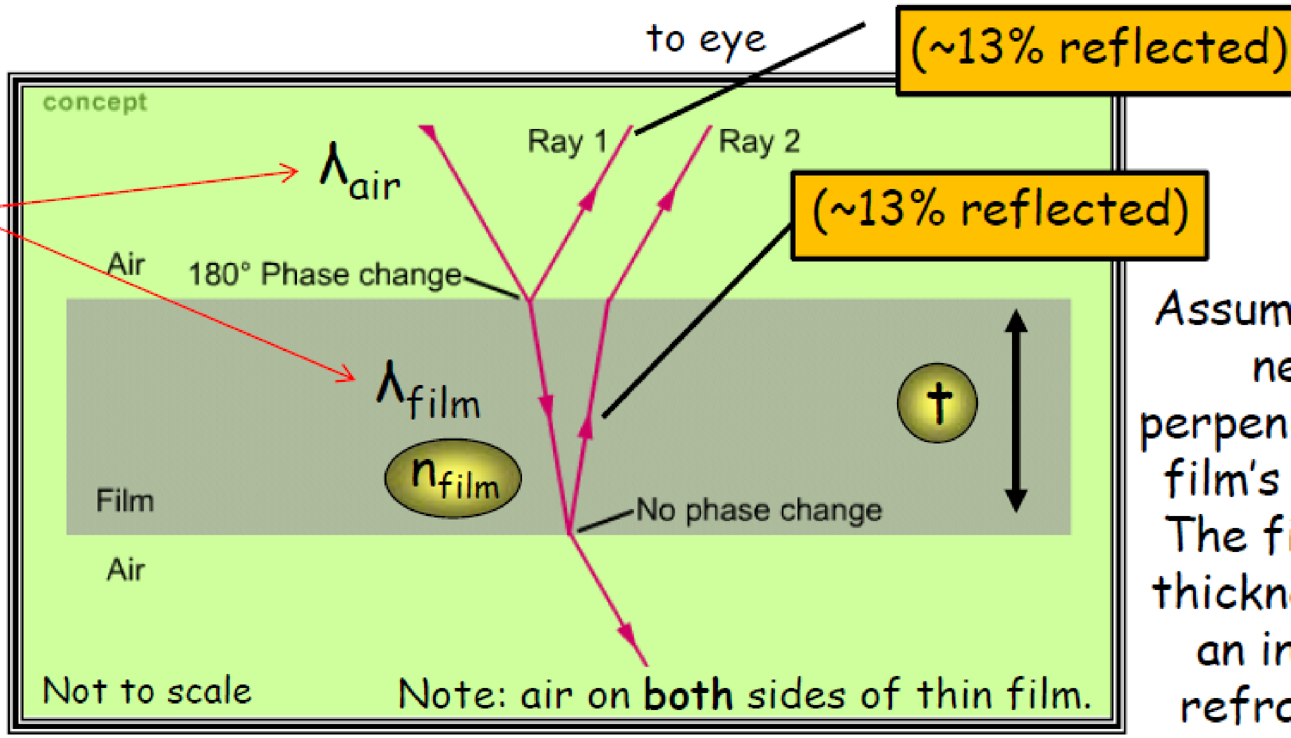
Two ways to produce a phase difference between two waves:

1. One wave travels an extra distance
2. A reflection from an optically dense material produces a phase change of π upon reflection.

   (A phase change of π is the same as a path length difference of $\lambda/2$)

Analyzing the situation (qualitative)

You must keep track of λ which is different depending on the medium through which the light travels.



Assume light is nearly perpendicular to film's surface. The film has a thickness t and an index of refraction n.

Two contributions to **phase difference** between Ray 1 and Ray 2:

i) Difference in path length between ❶ and ❷: $\Delta x = 2t$

ii) Any phase shifts due to reflection: Yes

Analyzing constructive interference (quantitative)

See Appendix from Lecture 18:

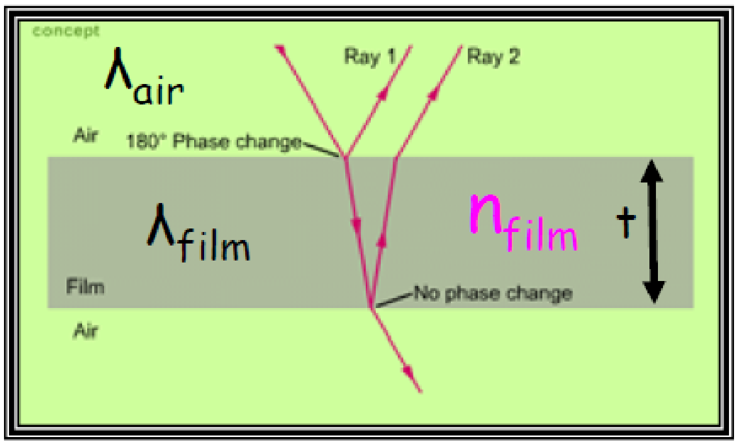
$$\lambda_{film} = \frac{\lambda_{air}}{n_{film}}$$

KEY IDEA

A net 360° (or 2π) phase change between two waves produces an **in phase** condition.

Condition on thickness t for complete constructive interference (maximum light reflected):

recall $\Delta\varphi = 2\pi \frac{\Delta x}{\lambda_{film}}$



here, $\Delta x = 2t$

Phase change between ray 2 and ray 1

constructive if $\left(2\pi \frac{2t}{\lambda_{film}} - \pi\right) = 0, 2\pi, 4\pi, 6\pi.....$

$$2\pi \frac{2t}{\lambda_{film}} = 1\pi, 3\pi, 5\pi, 7\pi.....$$

$$\frac{2t}{\lambda_{film}} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, ...\frac{m}{2} \text{ where } m = \text{odd integer}$$

$$t = \frac{m}{4}\lambda_{film} = \frac{m}{4}\left(\frac{\lambda_{air}}{n_{film}}\right); \text{ where } m = 1,3,5...$$

Note: Valid for air on **both** sides of thin film.

Analyzing destructive interference (quantitative)

See Appendix from Lecture 18:

$$\lambda_{film} = \frac{\lambda_{air}}{n_{film}}$$

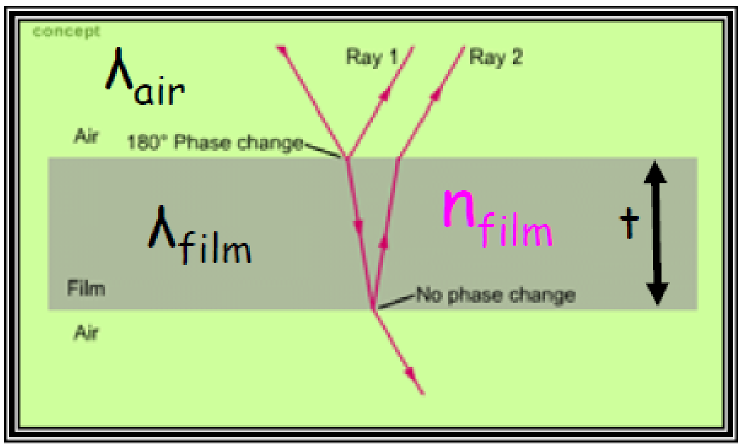
KEY IDEA

A net 180° (or π) phase change between two waves produces an **out of phase** condition.

Condition on t for complete destructive interference (no light reflected):

recall $\Delta\varphi = 2\pi \frac{\Delta x}{\lambda_{film}}$

Phase change between ray 2 and ray 1



here, $\Delta x = 2t$

$$\text{destructive if } \left(2\pi \frac{2t}{\lambda_{film}} - \pi\right) = \pi, 3\pi, 5\pi, \ldots$$

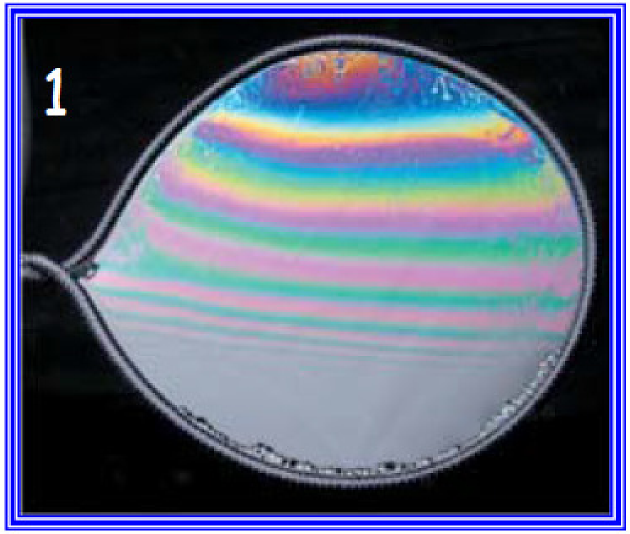
$$2\pi \frac{2t}{\lambda_{film}} = 2\pi, 4\pi, 6\pi, \ldots$$

$$\frac{2t}{\lambda_{film}} = 1, 2, 3 \ldots m \quad \text{where } m \text{ is positive integer}$$

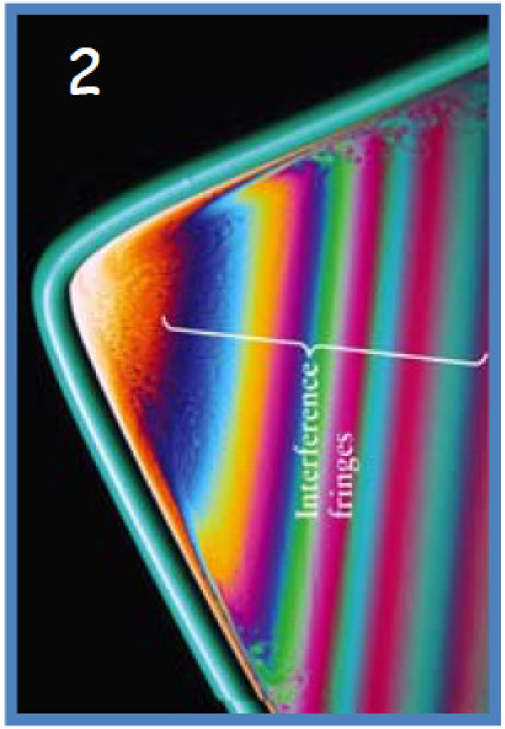
$$t = \frac{m}{2}\lambda_{film} = \frac{m}{2}\left(\frac{\lambda_{air}}{n_{film}}\right); \; m = 1, 2, 3, \ldots$$

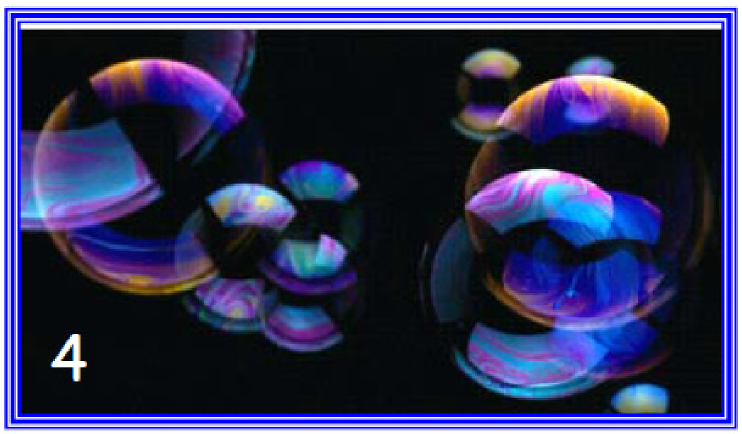
Note: Valid for air on **both** sides of thin film.

Example I: Thin Film Interference is in Soap Films



The color indicates where the wavelength and <u>local</u> film thickness satisfy constructive interference criteria.

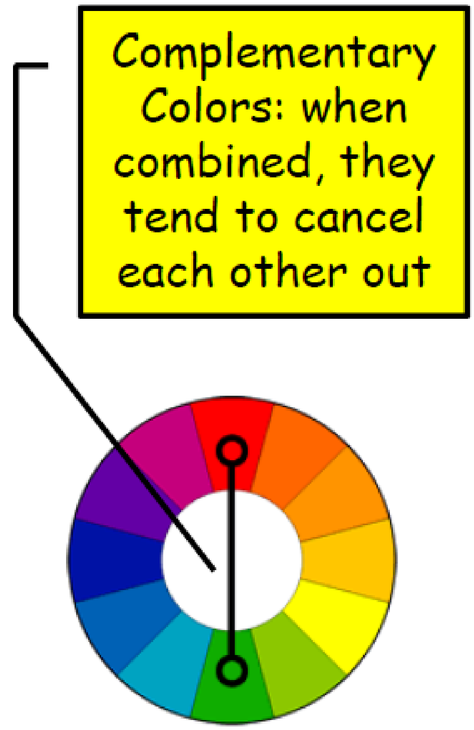




Note: First four illustrations involve thin film of soapy water with air on both sides

Caution: Understanding What Is Observed in Thin Film Interference

- The human brain tends to reject what it cannot organize.
- How to organize colors so they are "pleasing" to the eye? – Color Theory
- Due to different wavelengths, different thicknesses, and different angles, light of only a small wavelength range is destructively reduced in intensity at a particular location on a soap bubble.
- In thin film interference you often see **complementary** colors which are the white light colors left after light from a small wavelength range is **subtracted**.
- A black region on a soap bubble usually indicates a region where the soap bubble is very thin.



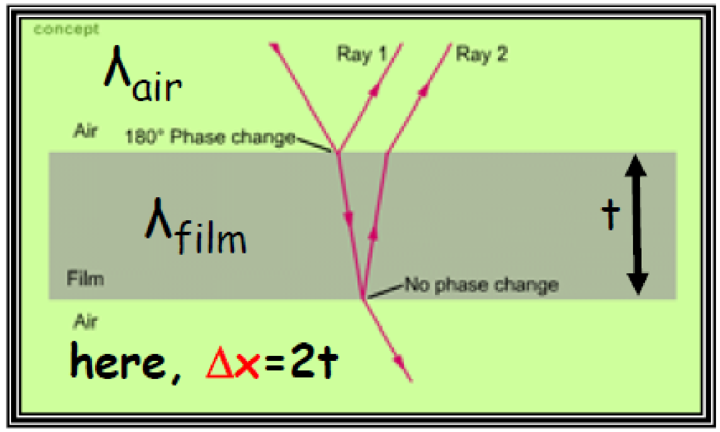
White light contains all the colors above

Example I: A region of a soap bubble looks red. What must be its thickness?

Implication: thickness of film must produce constructive interference for red light ($\lambda_{air} \approx 650$ nm).

Physical model:

Note: air on **both** sides of thin film.



constructive **if** phase change between ray 1 and ray 2 is $0, 2\pi, 4\pi$

$$\left(2\pi\frac{2t}{\lambda_{film}} - \pi\right) = 0, 2\pi, 4\pi, 6\pi.....$$

Calculation:

$$2\pi\frac{2t}{\lambda_{film}} = 1\pi, 3\pi, 5\pi, 7\pi.....$$

$$\frac{2t}{\lambda_{film}} = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, ...\frac{m}{2} \quad m = \text{odd integer}$$

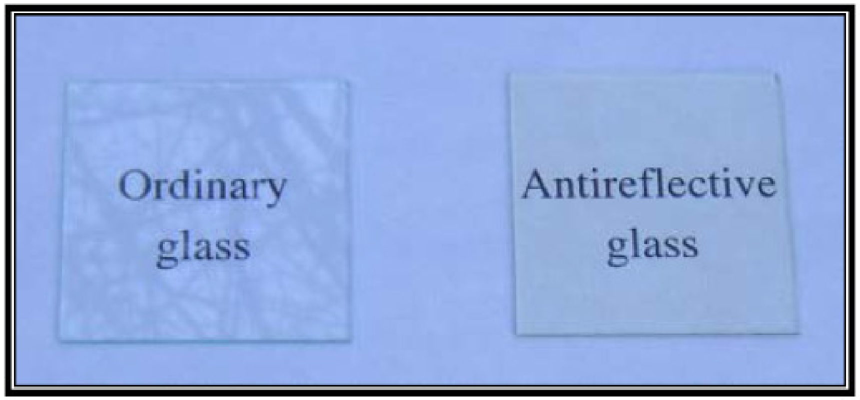
$$t = \frac{m}{4}\lambda_{film} = \frac{m}{4}\left(\frac{\lambda_{air}}{n_{film}}\right); \quad m = 1,3,5...$$

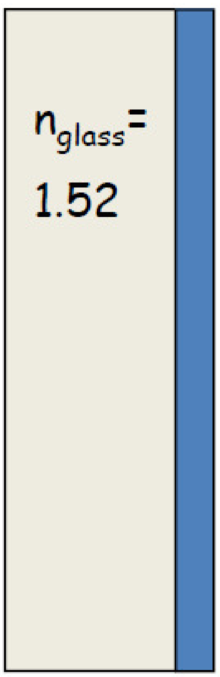
$$t = \frac{m}{4}\lambda_{film} = \frac{m}{4}\left(\frac{\lambda_{air}}{n_{film}}\right); \quad m = 1,3,5...$$

$$= \frac{1}{4}, \frac{3}{4}, \frac{5}{4}, ... \times \left(\frac{650\text{ nm}}{1.33}\right)$$

$$= 122\text{ nm}, 366\text{ nm}, 611\text{ nm}, ...$$

Example II: Antireflective Coatings





$n_{coating}$= 1.40

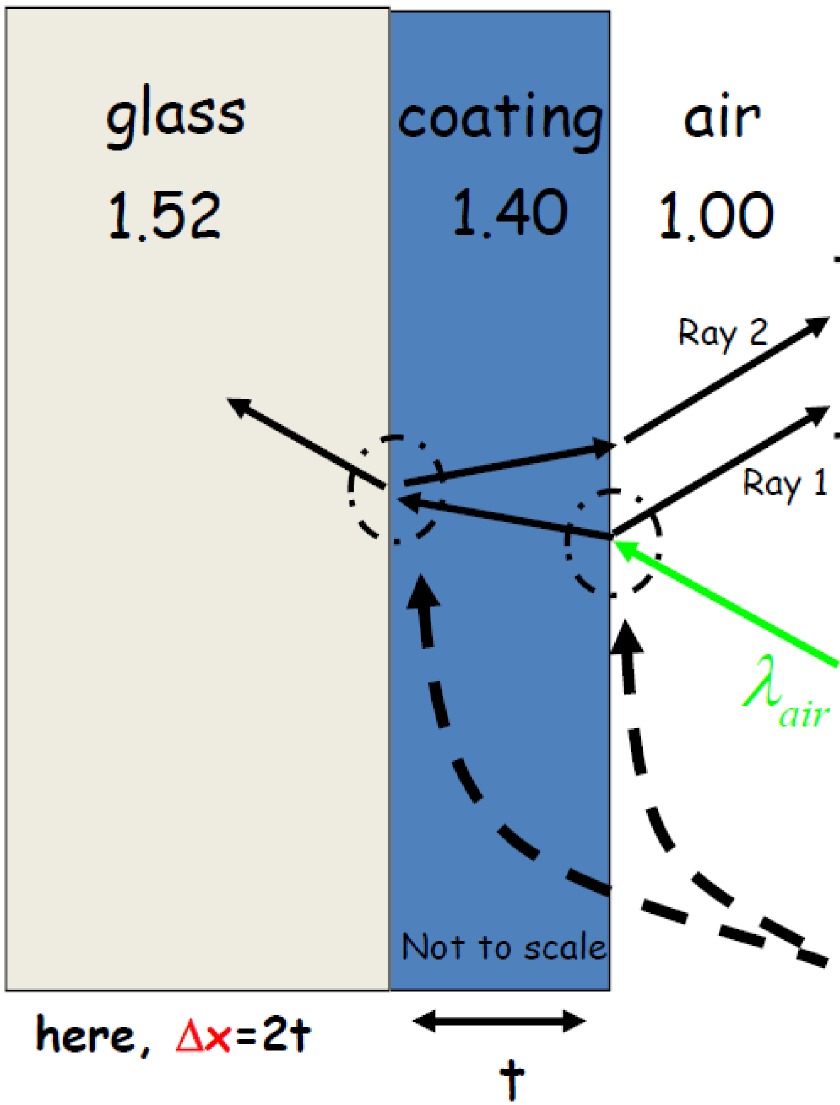
White Light from Sun

(peak intensity for sunlight is near λ=565 nm)

How to <u>minimize</u> reflected light at λ=565 nm?

n_{air}= 1.00

What are possible values for t to <u>minimize</u> reflected light?



these two rays should **destructively** interfere to reduce reflection

$\lambda_{air} = 565\ nm$ (incident light)

In this case, both reflected rays suffer a 180° (π) phase change upon reflection. Why?? What is implication?

here, Δx=2t

Working it out

Phase change between ray 2 and ray 1 — Destructive Condition

$$\text{destructive if}\left(\left[2\pi\left(\frac{2t}{\lambda_{coating}}\right)+\pi\right]-\pi\right)=(1,3,5....)\pi$$

$$2\pi\left(\frac{2t}{\lambda_{coating}}\right)=(1,3,5....)\pi$$

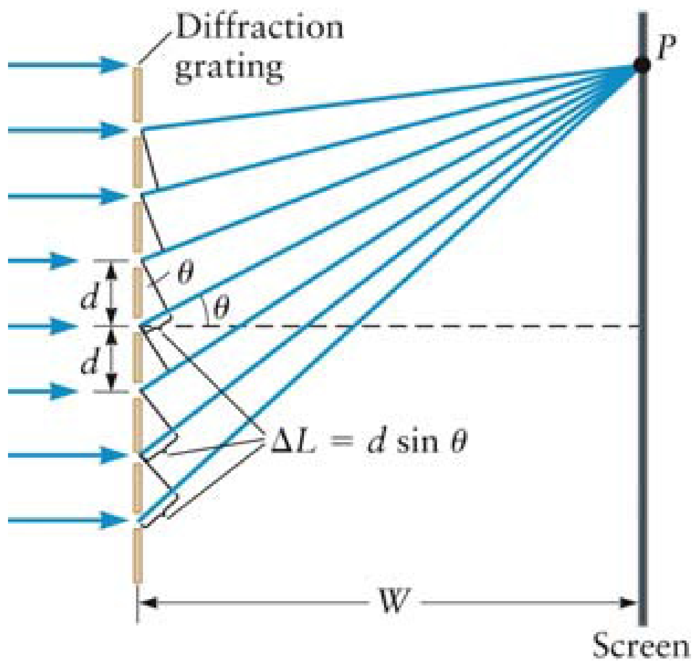
$$\left(\frac{2t}{\lambda_{coating}}\right)=\left(\frac{1}{2},\frac{3}{2},\frac{5}{2}....\right)=\left(m+\frac{1}{2}\right);\quad m=0,1,2,...$$

$$2t=\left(m+\frac{1}{2}\right)\lambda_{coating}=\left(m+\frac{1}{2}\right)\frac{\lambda_{air}}{n_{coating}};\quad m=0,1,2,...$$

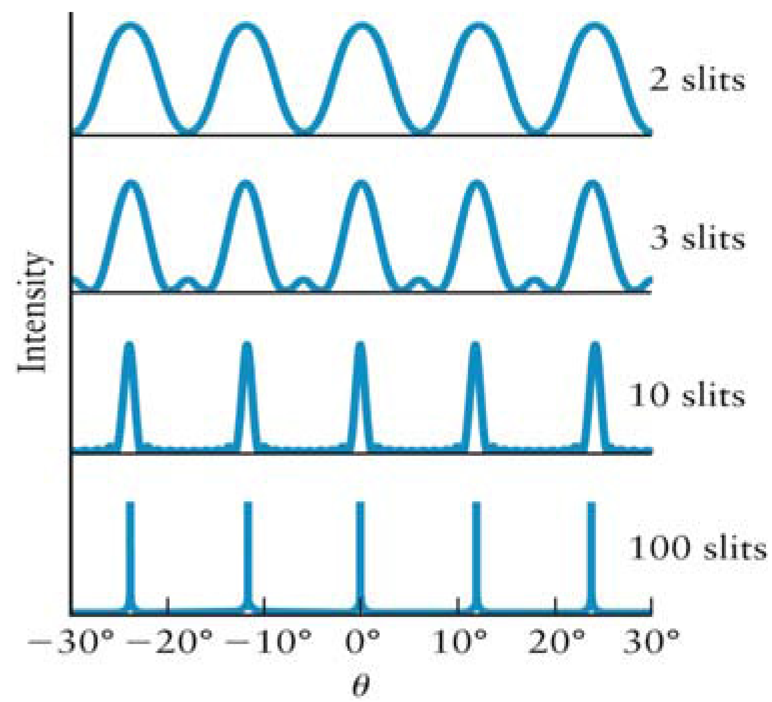
$$t=\frac{1}{2}\left(m+\frac{1}{2}\right)\frac{565\text{ nm}}{1.40}=\frac{1}{2}\left(m+\frac{1}{2}\right)403.6\text{ nm}=\left(m+\frac{1}{2}\right)201.8\text{ nm};\quad m=0,1,2,...$$

$$t=100.9\text{ nm}, 302.7\text{ nm}, 504.5\text{ nm}, ...$$

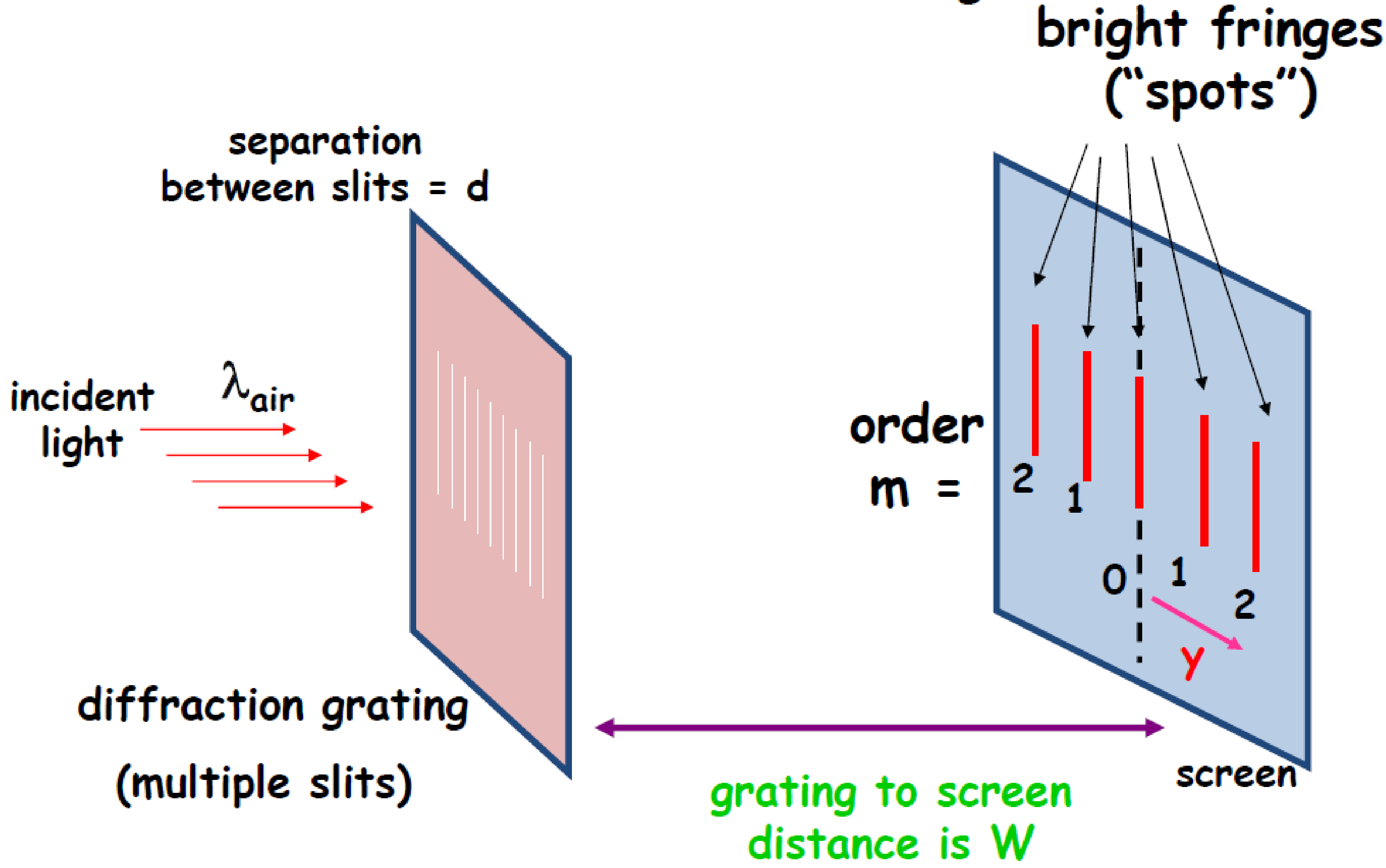
Diffraction Gratings: More than Two Slits



Intensity pattern observed on the screen:

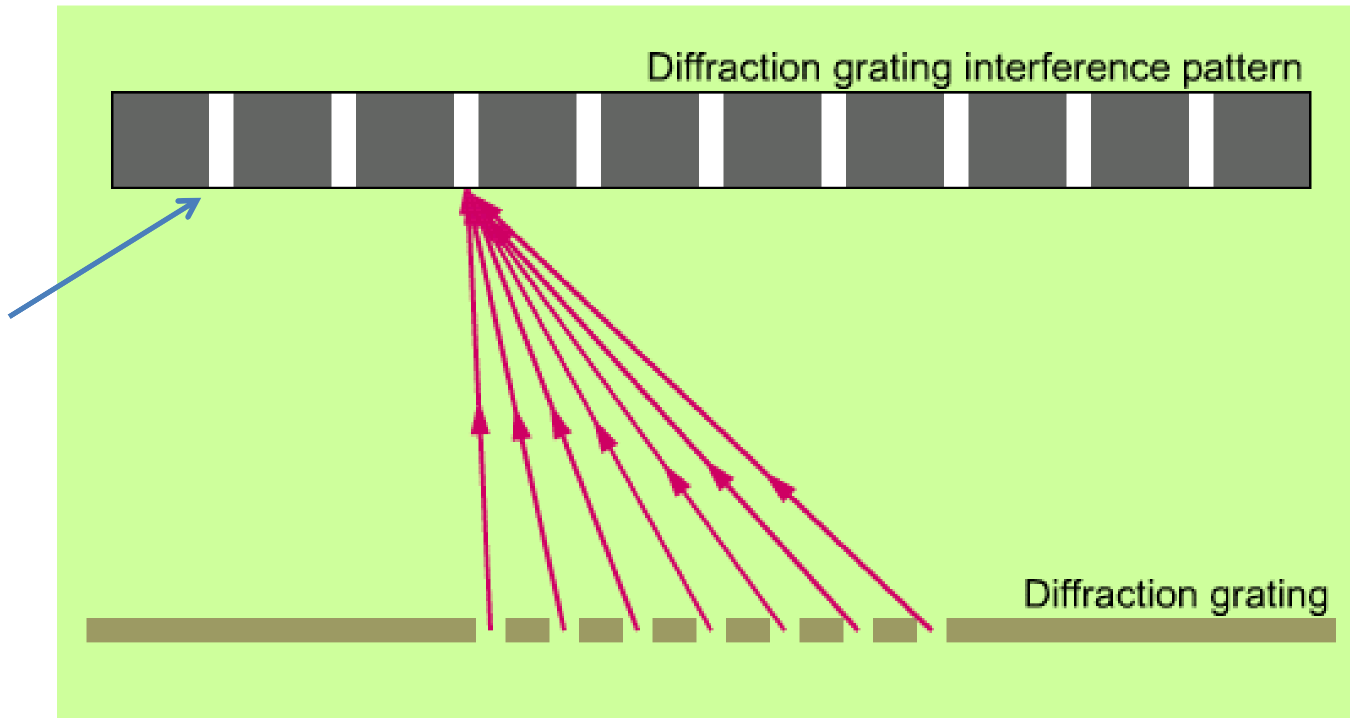


Accurately Measuring the Wavelength of Light
- Diffraction Gratings -



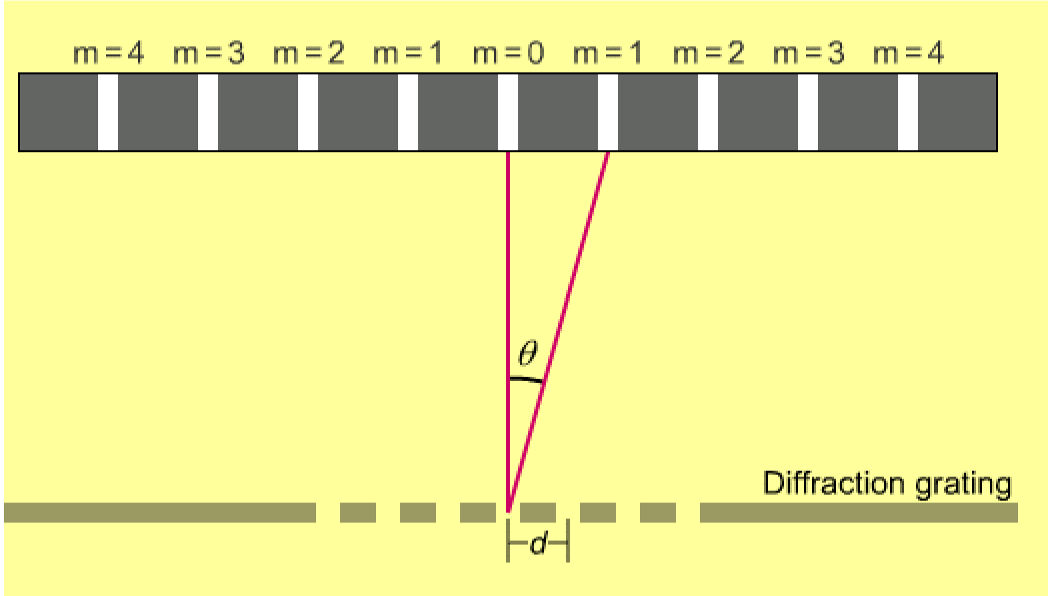
Not to scale

The underlying physics of the problem is the same as before



Bright spots on a viewing screen are produced by the constructive interference of light from many, many slits.

Where are the bright spots located?



Spacing between slits is d.

Bright spots when

$$\sin\theta = \frac{m\lambda}{d}$$

Example

Light of unknown wavelength is directed onto a diffraction grating, forming a third order bright fringe which is located on a screen 18.7 mm from the center bright line. If the distance between the screen and the diffraction grating is 1 m, what is the wavelength of the light?

The diffraction grating has 10 slits/mm.

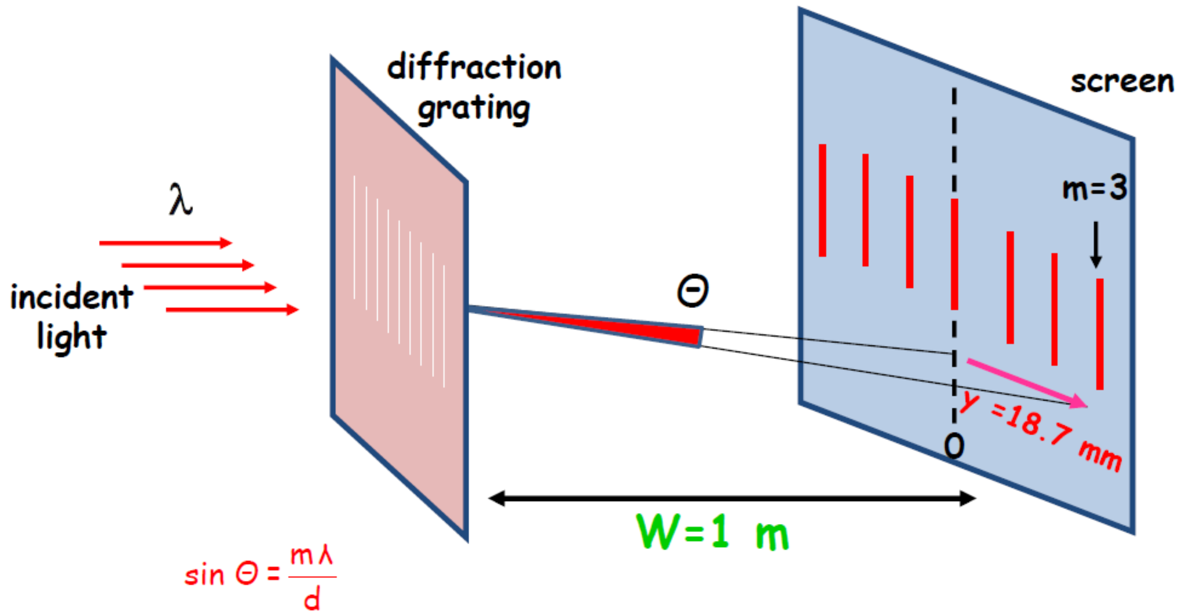
1. What is d?

10 slits/mm = 0.1 mm/slit

$$d = 0.1\ mm$$

Example

2. What is λ?



$$\frac{y}{W} \approx \sin\theta$$

$$= \frac{m\lambda}{d}$$

$$\lambda = \frac{yd}{mW}$$

$$= \frac{(18.7\ mm)(0.1\ mm)}{3\ (1000\ mm)} = 623 \times 10^{-6}\ mm$$

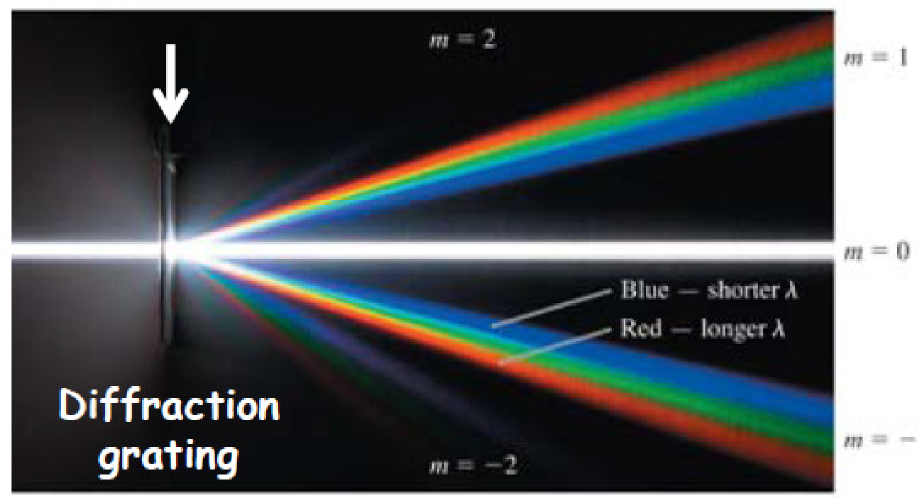
$$= 623\ nm$$

CDs and DVDs: Reflection gratings

The grooves in a CD disc are "effective" slits: the reflected white light forms interference maxima for different colors at different angles.

White light incident on grating

A spectrum produced by a grating is a result of the light of different wavelengths interfering constructively at different locations.



APPENDIX: Flow Chart for Thin Films