PURDUE UNIVERSITY DEPARTMENT OF PHYSICS

Physics 21900

General Physics II

Electricity, Magnetism and Optics

Lecture 19 – Chapter 22.4-6

Thin Lenses

Fall 2015 Semester

Prof. Matthew Jones

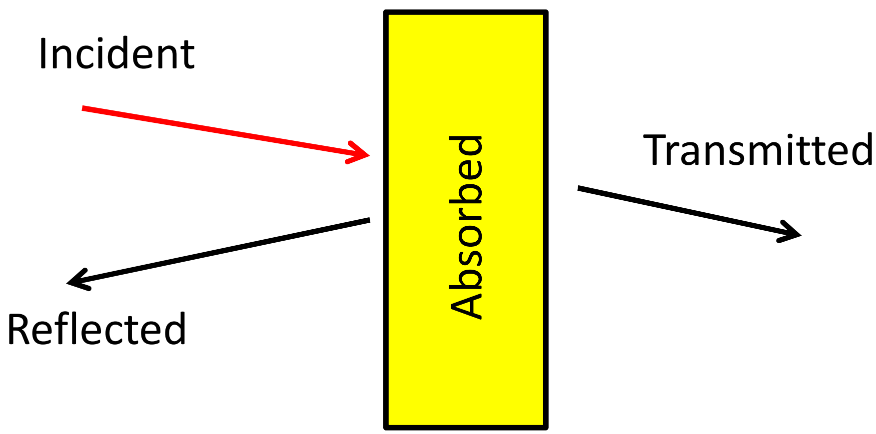
Announcement

Exam #2 will be on November 5th in Phys 112 at 8:00 pm

Electric current, DC circuits, Kirchhoff's Rules
Magnetic Fields, Lorentz Force, Forces on Currents
Ampere's Law, Magnetic Induction, Lenz's Law
Induced EMF, AC Voltage, Transformers

To keep the number of questions reasonable, some will require that you combine knowledge from multiple areas. For example, you might need to know Kirchhoff's rules and how to analyze series and parallel resistors in the same problem.

Interaction of Light with Matter



Electromagnetic radiation (ie, light) travels slower in a material:

$$n = \frac{c}{v}$$

c = speed of light in vacuum (or air)
v = speed of light in the material

Material	n
Glass	1.5
Water	1.33
Diamond	2.4

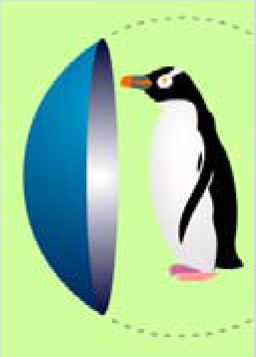
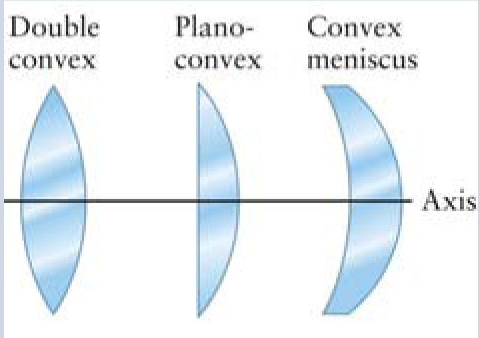
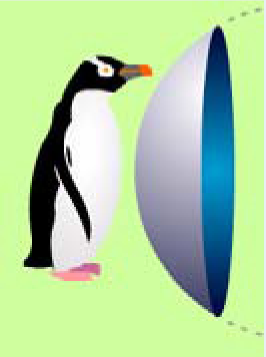
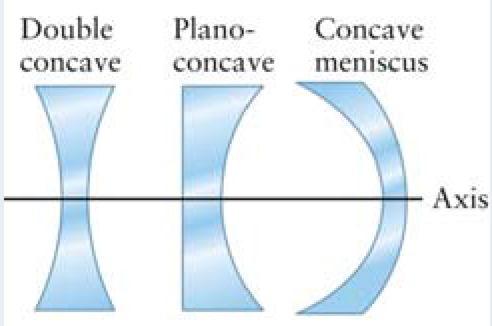
$$I\ (W/m^2) = R + T + A$$

(Energy conservation)

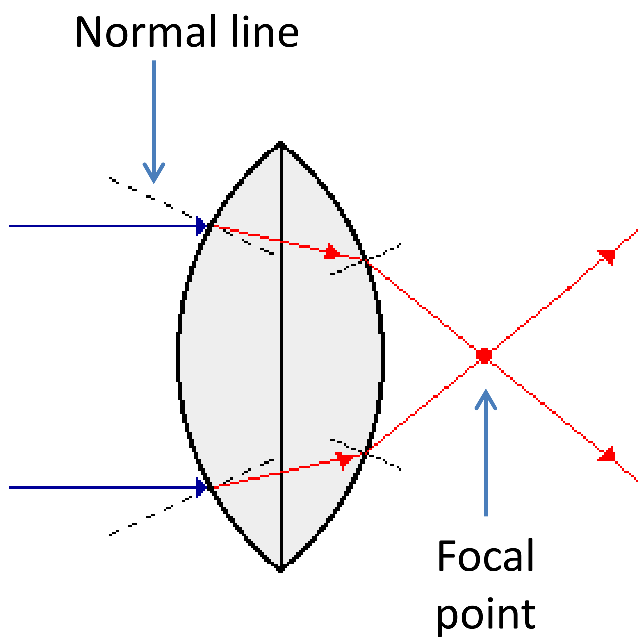
Three situations:

- If $R \approx I$, then it's a mirror
- If $A \approx I$, then it's a filter or absorber
- **If $T \approx I$, then it's a lens**

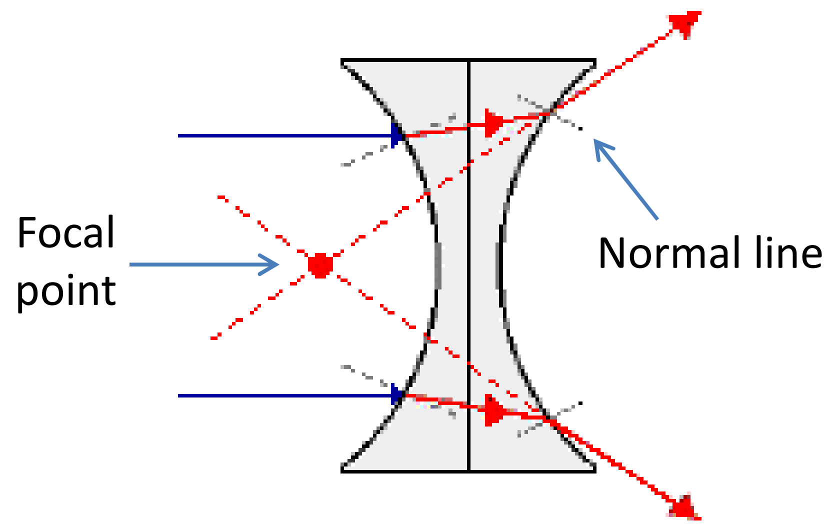
Types of Lenses

Mirrors		Lenses	
Concave mirror		Converging lens	
Convex mirror		Diverging lens	

Why Lenses Work

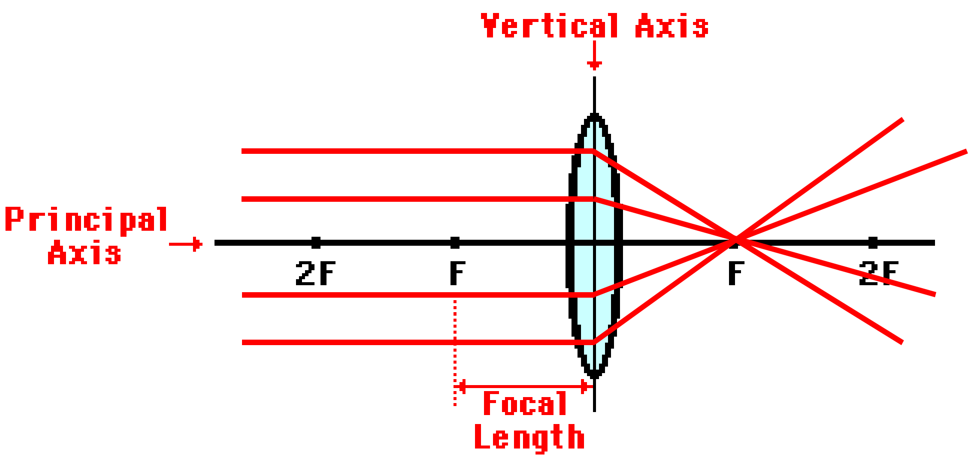


Refraction of light rays by a converging lens.



Refraction of light rays by a diverging lens.

Terminology



An imaginary line passing through the exact center of the lens is referred to as the ***principal axis***.

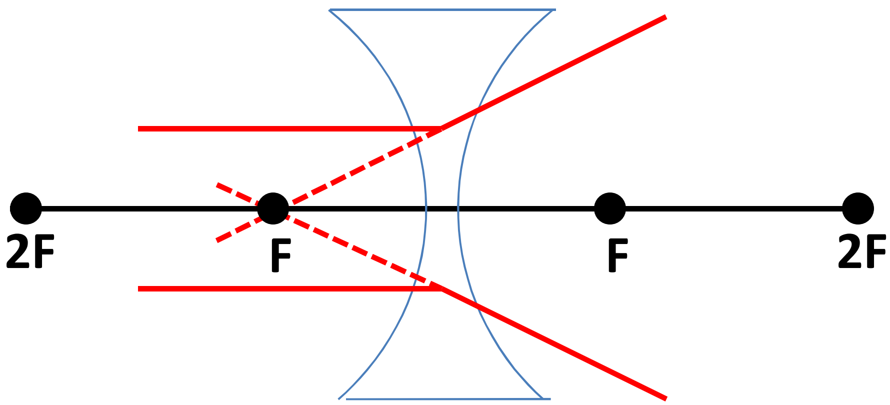
A lens also has an imaginary ***vertical axis*** that bisects the lens.

Light rays on either side of the lens **parallel** to the principal axis will either converge or diverge.

For a converging lens, parallel light rays will converge to a point. This is the ***focal point*** (**F**) of the converging lens.

A point that is twice the distance from the lens as the focal point is labeled **2F**.

Terminology



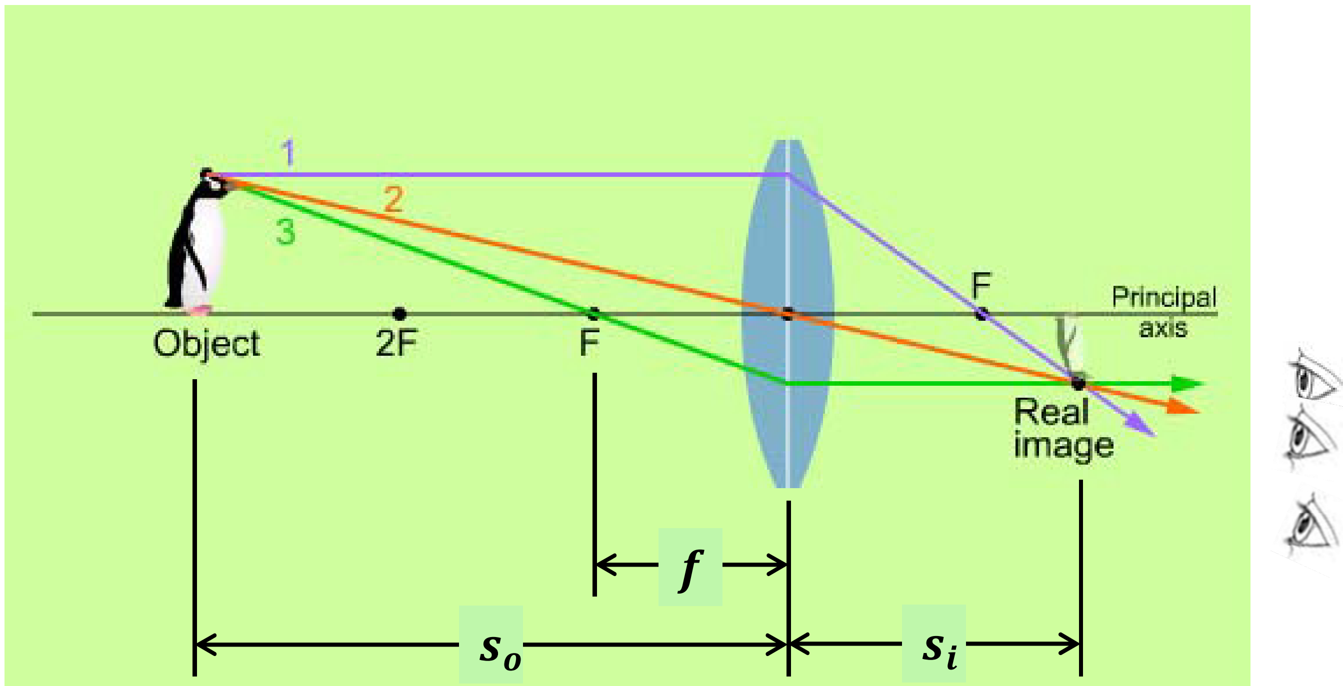
For a diverging lens, parallel light rays diverge and can be traced ***backwards*** until they intersect at a point. This intersection point is known as the ***focal point*** (**F**) of a diverging lens.

The action of a lens is symmetric. This means that each lens has two focal points – one on each side of the lens.

The distance from the center of the lens to the focal point is known as the ***focal length*** (denoted by f).

Points that are a distance $2f$ from the lens are labeled **2F**.

Image Formation

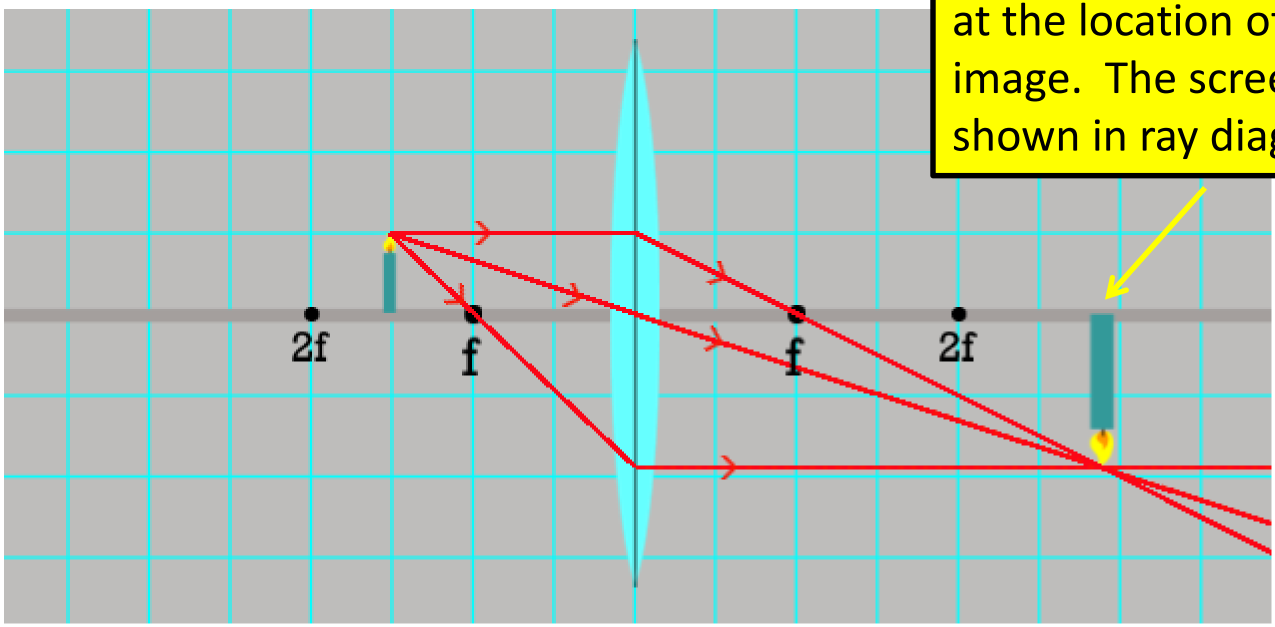


1. A ray parallel to the principal axis is refracted through the focal point.
2. A ray through the center of the lens is not refracted.
3. A ray through the focal point is refracted parallel to the principal axis.

An image is formed because light reflected from a point on the object is observed no matter where the eye is positioned.

Interactive Demonstration

A screen is often placed at the location of the image. The screen is NOT shown in ray diagrams.



http://www.physicsclassroom.com/Physics-Interactives/Refraction-and-Lenses

Thin Lens Equation

- Terminology:
  - Object distance: s_o (always positive)
  - Image distance: s_i (positive or negative)
  - Focal length: f (+ for converging, - for diverging)
  - Object height: h_o (always positive)
  - Image height: h_i (positive or negative)
  - Magnification: m
- Thin lens equation:

$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

- Magnification:

$$m = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

These are exactly the same as the corresponding equations for spherical mirrors.

Sign Conventions are Important

(Different text books may use different sign conventions)

Quantity	Symbol	Positive sign means	Negative sign means
Focal length	f	Converging lens	Diverging lens
Object distance	s_o	Object side of lens (real)	Behind lens (virtual)
Image distance	s_i	Other side of lens (real)	Same side as object (virtual)
Magnification	m	Upright image	Inverted image
Image height	h_i	“	“

Be able to distinguish real and virtual images. Remember the definition?
Light rays actually pass through a real image.
They only appear to originate from a virtual image.

Each symbol can be assigned a positive or negative value.

Same equations as for spherical mirrors: $\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$ $\quad m = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$

Example

- What is the significance of the 2F point?
- Suppose we place an object at the 2F point.
  - Where is the image formed?
  - What is the magnification?
- Object distance: $s_o = 2f$
- Thin lens equation:

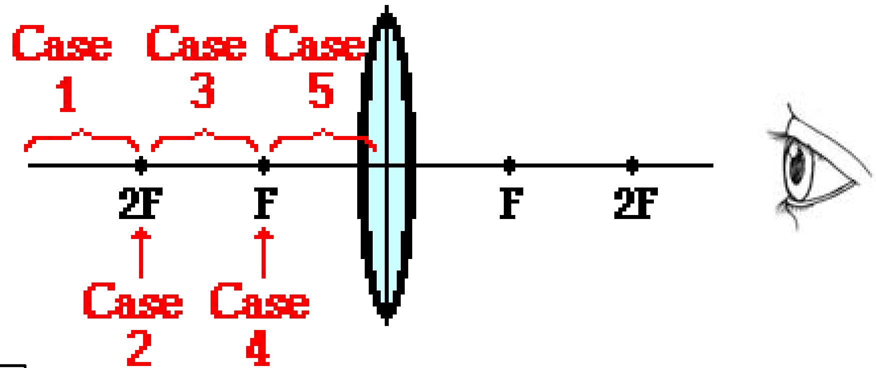
$$\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f} \Rightarrow \frac{1}{s_i} = \frac{1}{f} - \frac{1}{2f} = \frac{1}{2f} \Rightarrow s_i = 2f$$

$$m = -\frac{s_o}{s_i} = -1$$

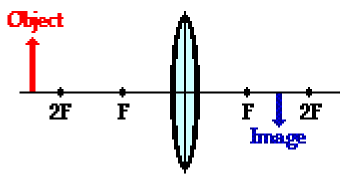
- The image is formed at the other 2F point.
- It is the same size as the object, but is inverted.

Converging Lenses

All possible cases:

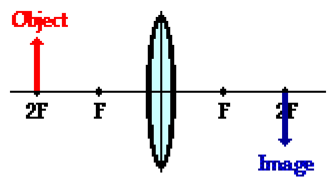


Case 1: s_o and s_i positive



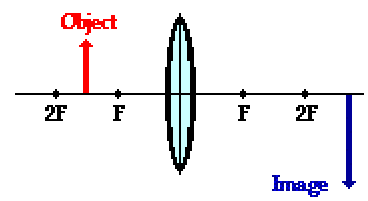
Inverted smaller image

Case 2: s_o and s_i positive

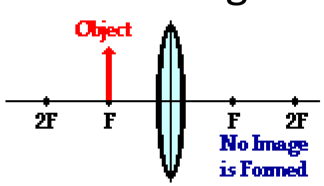


Inverted image – same size

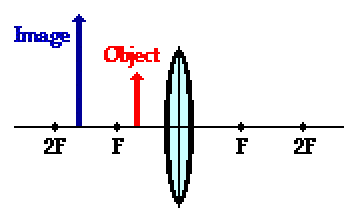
Case 3: s_o and s_i positive



Case 4: No image

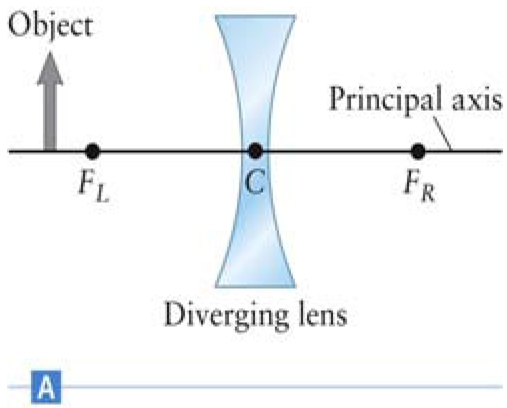


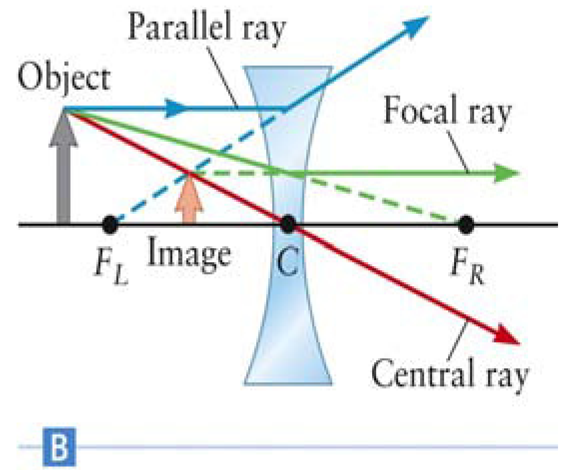
Case 5: $s_o > 0$ and $s_i < 0$



Upright larger image (virtual)

Diverging Lens





Object locations

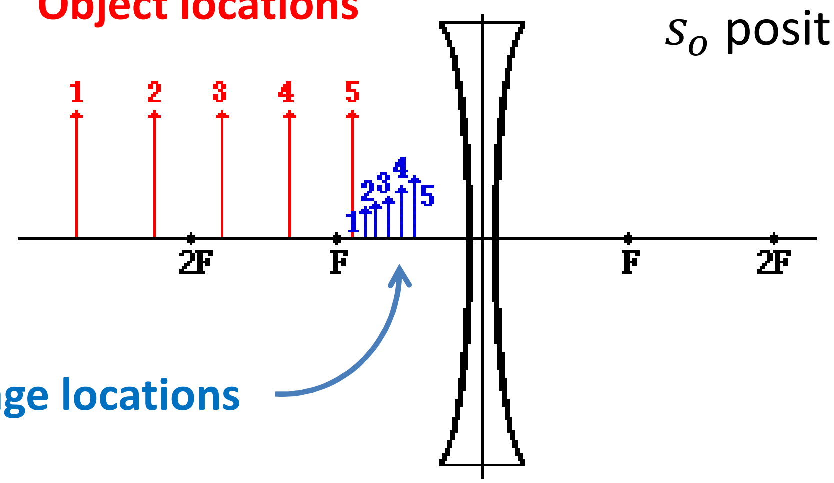
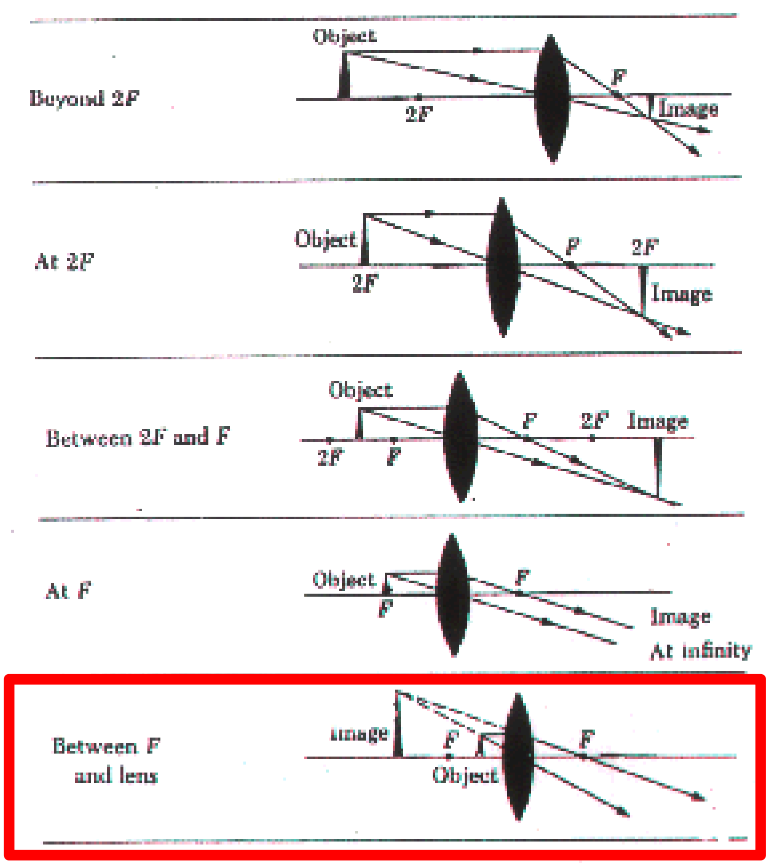


Image locations

s_o positive and s_i negative.

Example #1: A 6.0-cm tall candle is placed a distance of 7.0-cm from a convex lens which has a focal length of 16.0-cm. Determine the image distance and the image size.

Five cases – which one applies?



$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$m = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

$\boldsymbol{s_o = 7\ cm}$ $\boldsymbol{f = 16\ cm}$ $\boldsymbol{h_o = 6\ cm}$

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o}$$

$$= \frac{1}{16\ cm} - \frac{1}{7\ cm}$$

$$= -0.080\ cm^{-1}$$

$$s_i = \frac{1}{-0.080\ cm^{-1}}$$

$$\boldsymbol{s_i = -12.44\ cm}$$

$$m = -\frac{s_i}{s_o}$$

$$= -\frac{-12.44\ cm}{7\ cm}$$

$$= +1.78$$

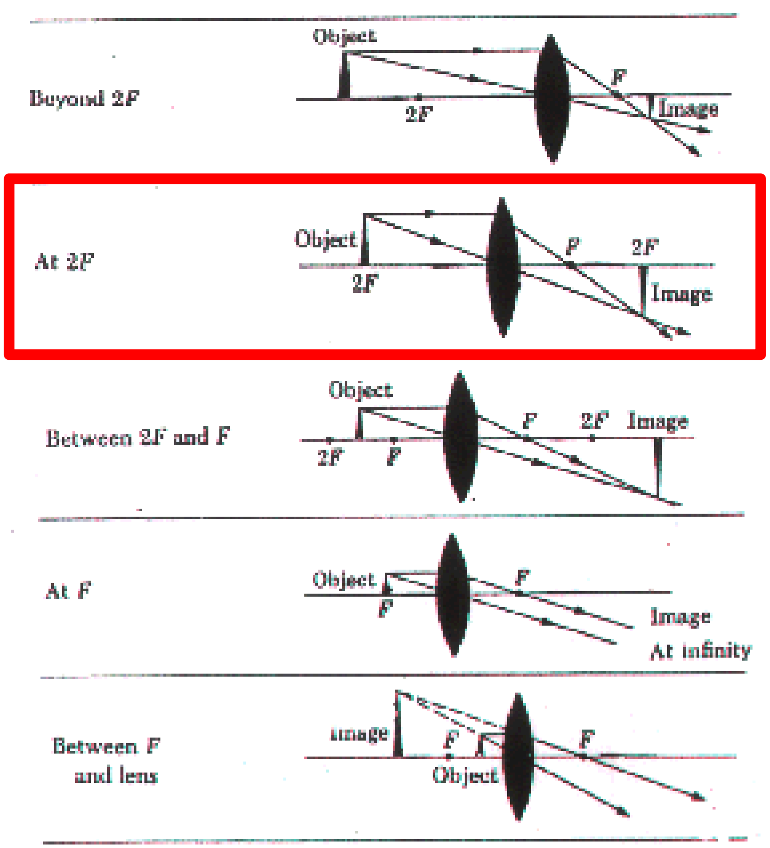
$$h_i = h_o \times m$$

$$= (6\ cm) \times (1.78)$$

$$\boldsymbol{h_i = +10.66\ cm}$$

Example #2: A 6.0-cm tall candle is placed a distance of 32.0-cm from a convex lens which has a focal length of 16.0-cm. Determine the image distance and the image size.

Five cases – which one applies?



$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$m = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

$\boldsymbol{s_o = 32\ cm}$ $\boldsymbol{f = 16\ cm}$ $\boldsymbol{h_o = 6\ cm}$

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o}$$

$$= \frac{1}{16\ cm} - \frac{1}{32\ cm}$$

$$= 0.03125\ cm^{-1}$$

$$s_i = \frac{1}{0.03125\ cm^{-1}}$$

$$\boldsymbol{s_i = 32.0\ cm}$$

$$m = -\frac{s_i}{s_o}$$

$$= -\frac{32\ cm}{32\ cm}$$

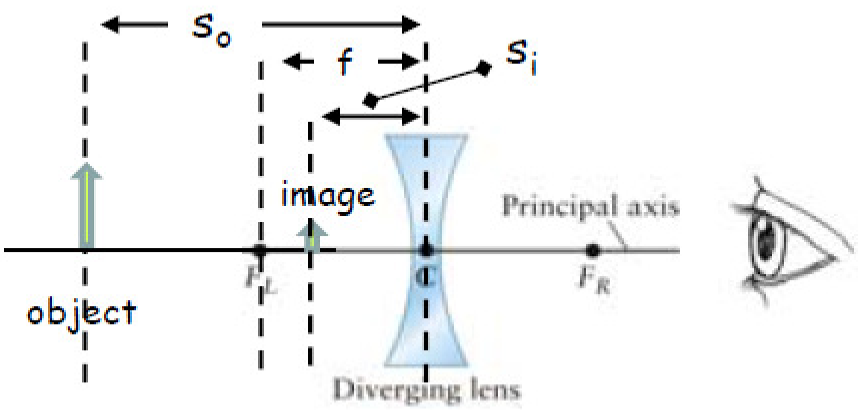
$$= -1$$

$$h_i = h_o \times m$$

$$= (6\ cm) \times (-1)$$

$$\boldsymbol{h_i = -6\ cm}$$

Example #3: A 2.5-cm tall nail is placed a distance of 40.0-cm from a diverging lens which has a focal length of -15.0-cm. Determine the image distance and the image size.



$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$m = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

$\boldsymbol{s_o = 40\ cm}$ $\boldsymbol{f = -15\ cm}$ $\boldsymbol{h_o = 2.5\ cm}$

$$\frac{1}{s_i} = \frac{1}{f} - \frac{1}{s_o}$$

$$= \frac{1}{-15\ cm} - \frac{1}{40\ cm}$$

$$= -0.092\ cm^{-1}$$

$$s_i = \frac{1}{-0.092\ cm^{-1}}$$

$$\boldsymbol{s_i = -10.91\ cm}$$

$$m = -\frac{s_i}{s_o}$$

$$= -\frac{-10.91\ cm}{40\ cm}$$

$$= +0.27$$

$$h_i = h_o \times m$$

$$= (2.5\ cm) \times (0.27)$$

$$\boldsymbol{h_i = +0.68\ cm}$$

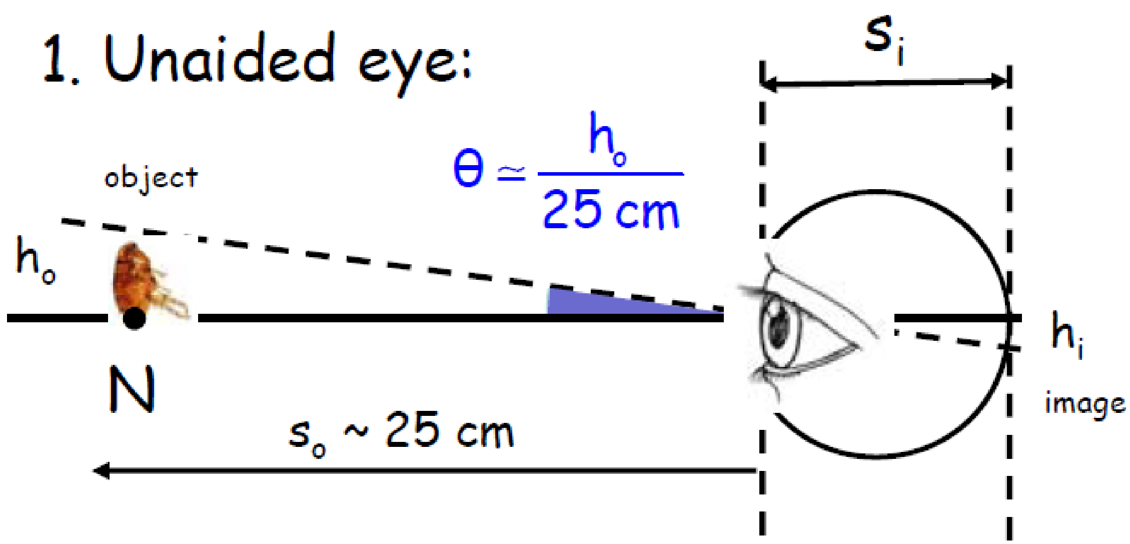
The image is a virtual image (why?)

The image is upright.

The magnifying glass

- The closer an object is to your eye, the more detail you can see.
- For a normal eye, the shortest distance for clear vision is about N=25 cm.

1. Unaided eye:



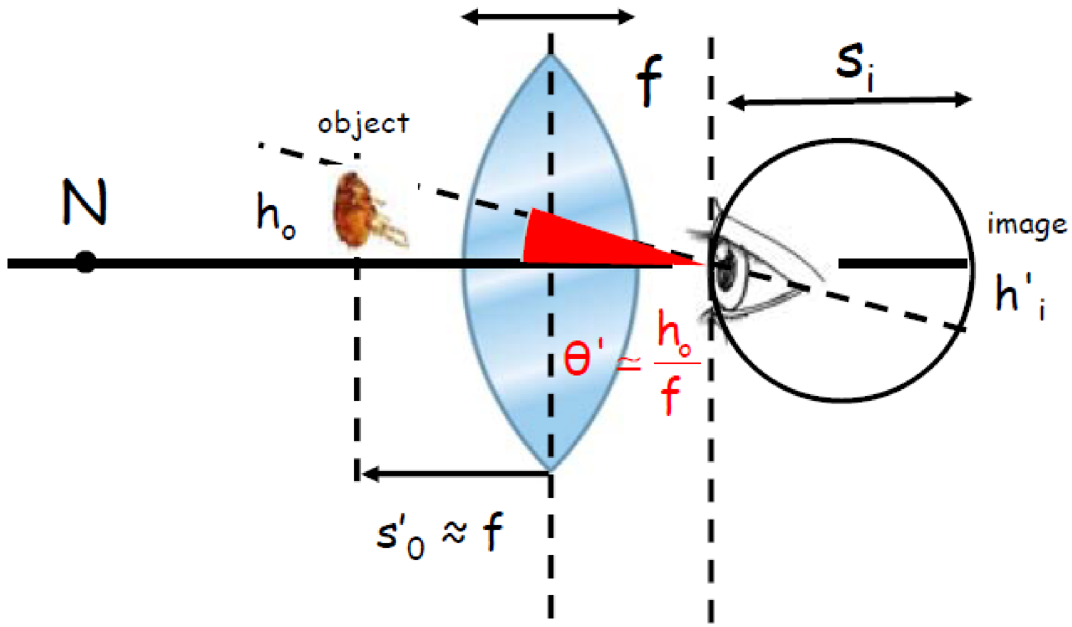
$$m = \frac{h_i}{h_o} = -\frac{s_i}{s_o}$$

$$m\frac{h_o}{s_i} = \frac{h_i}{s_i} = -\frac{h_o}{s_o} \simeq \theta$$

The magnifying glass

- Converging lens with short focal length placed close to eye
- Object is placed near focal length of lens.

2. Lens with short focal length:



$$m' = \frac{h'_i}{h_o} = -\frac{s_i}{s'_o}$$

$$m' \times \frac{h_o}{s_i} = \frac{h'_i}{s_i} = -\frac{h_o}{s'_o} \simeq \theta'$$

$$M = \frac{m' \times \frac{h_o}{s_i}}{m \times \frac{h_o}{s_i}} \simeq \frac{\theta'}{\theta} = \frac{h_o / f}{h_o / 25\ \text{cm}} = \frac{25\ \text{cm}}{f\ (\text{in cm})}$$