Measuring the Finite Speed of Light

Martin John Madsen[†]

Purdue University

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We confirmed the theory that light has a finite speed by using the Foucault spinning mirror method. We took preliminary quantitative measurements of the speed of light. Error analysis given for measurements. The mean calculated value of the speed of light is within 7% of the accepted value, found by Baird et. al. Suggestions given for improvements of the experimental method.

I. INTRODUCTION

It has been known for some 300 years that the speed of light was a finite quantity, and it has been the task of physicists to try and measure that quantity. Starting with Galileo, the basic method for measuring the speed of light was based on the "time of flight" method. Given that the speed of light is finite, then it should be possible to measure the time it takes to travel a given distance. Galileo initially attempted to measure the speed of light using lanterns. He and an assistant stood on hills separated by a few kilometers. One would uncover his lantern, then the other would uncover his as soon as he saw the signal. They discovered that the speed of light was greater then what they were able to measure.[1] (p.165) The first quantitative measurements were astronomical in nature, using the motion of the moons around Jupiter to measure the speed of light. The first direct measurements of the speed of light were done using a toothed wheel to chop the light into pulses. Measurements taken by Fizeau using this method yielded results of 3.15×10^8m/s.[2] Foucault, Michelson, and others refined this method by using a rotating mirror instead of a chopper, with results of 2.98×10^8m/s, and 2.99774×10^8m/s respectively.[3,4] In more recent times, other methods have been used including measuring the speed of waves along wires, microwave cavity resonators, radar, quartz modulators, microwave interferometry, and, most recently, optical frequency measurement.[1] Our method follows the work of Foucault and Michelson in the use of a rotating mirror to measure the speed of light, primarily due to the available equipment.

II. THEORY

If a pulse of light, traveling off of a mirror, had an infinite speed, then that pulse of light would return instantaneously to the mirror. There would be no difference in the angle with which the light bounced off of the mirror if the mirror were spinning or not, because the light would return back to the mirror at the same instant that it left, therefore at the same angle. An increase in angular speed of the mirror would not change the deflection off of the mirror. On the other hand, if the speed of light were finite, then in the time that it takes a pulse of light to travel a distance and return to the mirror, the mirror would have turned a fraction. Therefore, the pulse would deflect off at a different angle then the angle at which it had originally had deflected:

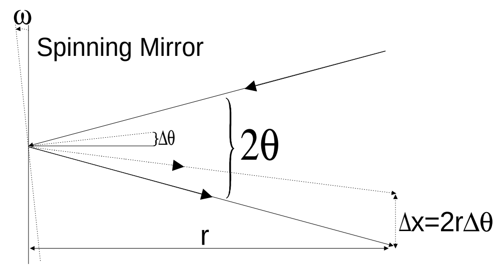

FIG. 1: Geometrical argument for derivation of equation calculating c as a function of ω

$$\Delta\theta = \omega \mathrm{t} = \omega\frac{OPL}{v}$$

where θ is the angle from the perpendicular, at which the incident beam strikes the mirror; ω is the angular velocity of the spinning mirror; v is the speed of light; and OPL is the optical path length between the spinning mirror, and itself, the distance that a pulse of light travels. In order to observe this change in angular reflection of the pulse of light, we look at a point far from the mirror. Displacements at that point would be equal to the radius (r) times the change in the angle. So, if we look at the spatial deviation at this point, far from the mirror, we measure the deflection which is 2θ, due to the law of reflection (see Fig.1). What we end up observing is as follows:

$$\Delta\theta = \frac{\Delta \mathrm{x}}{2r} \quad v = \frac{2(OPL)r\Delta\omega}{\Delta \mathrm{x}}$$

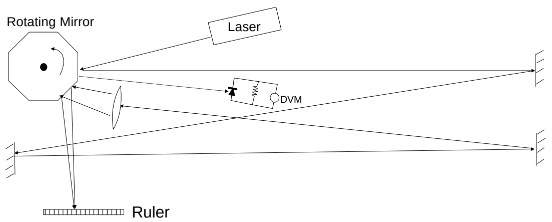

FIG. 2: Experimental Setup

where $\Delta\omega$ is the difference in angular velocity of the mirror which leads to a change in the displacement of the beam, Δx.

III. EXPERIMENTAL SETUP

We used a Spectra Physics Model 155 He-Ne laser, with $\lambda = 632.8nm$, power $P = 500\mu$W and with divergence angle $\theta_0 = 7\times10^{-4}rad$. The laser bounces off of one face of an eight sided, spinning mirror with diameter 5.5mm, each face measuring 2.3cm in width and 0.5cm in height (Fig.2). The mirror is powered by a Tenma DC power supply and driven internally by magnetic coils, connected to Hall sensors which in turn control the power fed to the coils. From the mirror, the beam goes along an optical path length and returns to an adjacent face of the mirror. The optical path length(OPL) was (38.0 ± 0.5)m and the radius from the face of the mirror to the detection point (r) was (1.1 ± 0.1)m. Before the beam hits the mirror again, however, we inserted a lens with a long focal length (f=121cm), following the method from Cornell.[5] This places the focal point at the detection point, after the light reflects off of the spinning mirror. The reason for using the lens before the mirror is due to the need to collect and focus the divergent laser beam. But, if we were to collect it after it had bounced off the mirror try to measure the displacement at the focal point, we would not see any displacement. This is due to the fact that the beams coming off the spinning mirror are close enough to being parallel rays, that they would focus at the same spot. We used an eyepiece with a 10mm ruling on the front face to measure deflection in the beam. In order to detect the frequency of the rotating mirror, we used a Thorlabs high speed silicon photo detector (response time of 20ns). The detector was placed in the path of the beam that was swept out as the laser struck a face of the spinning mirror. We connected the detector in parallel with a variable resistor and then measured a voltage across the resistor. Using a DVM with a frequency counter, we could also read off a frequency from the detector. We checked the accuracy of the frequency counter with an oscilloscope and found both measurements to agree within the experimental uncertainty.

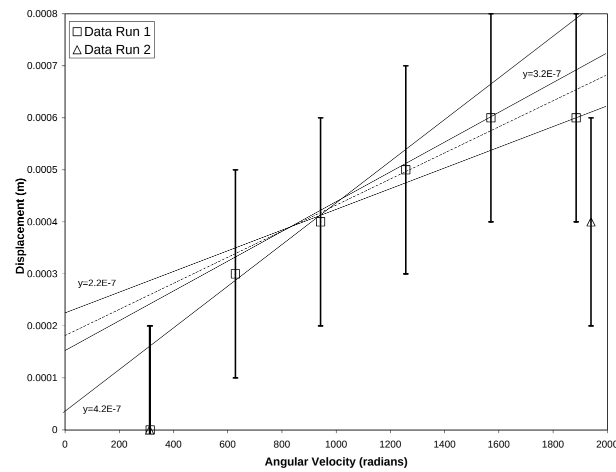

FIG. 3: Two initial sets of data, showing angular velocity vs. observed displacement. The dashed line shows the accepted value for the speed of light. The other lines show a best fit (the middle line) as well as two worst fit cases.

IV. EXPERIMENTAL DATA

We took two data runs in the available time. The data points, along with error bars is shown in Fig. 3. The calculation for c is as follows:

$$c\pm\delta\mathrm{c} = c[1\pm((\frac{\delta\mathrm{OPL}}{OPL}^{2)} + (\frac{\delta\mathrm{r}}{r})^2 + (\frac{\delta\Delta\omega}{\Delta\omega})^2 + (\frac{\delta\Delta\mathrm{x}}{\Delta\mathrm{x}})^2)^{1/2}]$$

Our data for the first run yield a value of $(\Delta\omega\pm\delta\Delta\omega) = (1571\pm8)$rad/s and $(\Delta\mathrm{x}\pm\delta\Delta\mathrm{x}) = (0.6\pm0.4\mathrm{mm})$. This gives a value of $c\pm\delta\mathrm{c}$ as:

$$2.2\times10^8[1\pm(2\times10^{-4}+8\times10^{-3}+3\times10^{-5}+0.45)^{1/2}]\mathrm{m/s}$$

or, $(c\pm\delta\mathrm{c}) = (2.2\times10^8\pm1.5\times10^8)$m/s. The second data run led to a value of $(c\pm\delta\mathrm{c}) = (3.4\times10^8\pm3.4\times10^8)$m/s. The uncertainties were calculated using the worst uncertainty propagation technique, which, from the graph of the data, appears to overestimate the uncertainty of the experiment. The data, within the error bars, falls on a line. Using the plot of the data points to do an analysis of the uncertainty, it is possible to do a best fit to the data, then plot different slopes that fit the data within the error bars. This method leads to a best fit slope of 3.2×10^{-7} with the worst fit slopes of 2.2×10^{-7} and 4.2×10^{-7}. This leads to a value of the speed of light:

$$(c\pm\delta\mathrm{c}) = 3.2\times10^8\pm1\times10^8\mathrm{m/s}$$

The value of (c) found by Baird, et. al., using frequency and wavelength measurements of CO_2, was 2.99792460×10^8m/s[6]. The mean value of our measurement of the speed of light is within 7% of the value found by Baird, et. al.

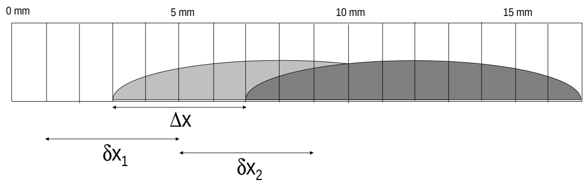

FIG. 4: Beam displacement detection. We used the end of the spot to measure displacements in the beam.

V. EXPERIMENTAL IMPROVEMENTS

The largest factor in the total uncertainty is the uncertainty in the measurement of the displacement ($\delta\Delta\text{x}/\Delta\text{x}$). This uncertainty comes from the difficulty in measuring an exact edge of the spot visible on the ruler (see Fig.4). The width of the spot was about 1mm, and it did not have a clear end to it, which we estimated to be an uncertainty of $\pm 0.4mm$. In order to reduce the contribution of the uncertainty in this measurement, it would be necessary to increase the displacement, as a function of the angular velocity, as compared to the uncertainty in the edge of the spot. If we were to triple the OPL and increase the focal length of the lens by five times (which leads to the length of r), we would expect a displacement of 6mm, with an uncertainty of 0.4mm. That would lead to a contributing factor of:

$$\frac{\delta\Delta\text{x}}{\Delta\text{x}} = \frac{0.4mm}{6mm} = 0.07$$

instead of the 70% and 100% uncertainties that currently come from this term. We set up the experiment with the thought that the OPL could be extended to 120m by using the long hallway in the Physics Building.

REFERENCES

† mjmadsen@purdue.edu
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