from it we restrict j_μ^a to the physical degrees of freedom $j_0^a = 0$; $\partial_i j_i^a = 0$ and apply the reduction formula with the wavefunction renormalization factors $Z^{1/2}$ as determined in that gauge.

Of course when we calculate in perturbation theory it is simpler to work in a covariant gauge rather than Coulomb gauge and usually we choose a Stückelberg type gauge condition

$$f_a(A_\mu) = \partial_\mu A^{a\mu}(x) - a^a(x)$$

with $a^a(x)$ some prescribed function.

Finally as can be seen when matter is present we arrive at the same conclusions by just repeating all of the preceding arguments; Thus the generating functional in a theory with local gauge symmetry group G having "matter" fields φ^α in representation $(T^a)_{\alpha\beta}$ of the Group interacting with the Yang-Mills gauge bosons A^a_μ is given by

$$\boxed{\begin{aligned} Z[J, J^a_\mu] = \int [dA^a_\mu][d\varphi^\alpha] \Delta_f [A_\mu, \varphi] \times \\ \times \delta[f_a(A_\mu, \varphi)]\, e^{i\int d^4x [\mathcal{L}_{inv}(\varphi, A) + J_\alpha \varphi^\alpha + J^a_\mu A^{a\mu}]} \end{aligned}}$$

where $\mathcal{L}_{inv}(\varphi, A) = \mathcal{L}(\varphi, D_\mu \varphi) - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu}$ with $\mathcal{L}(\varphi, D_\mu\varphi)$ globally (and locally) G-invariant. In addition we have allowed the gauge fixing condition to depend on the field φ^α (ex. $f_a = \partial_\mu A^{a\mu} + \xi v^\alpha (T^a)_{\alpha\beta} \varphi^\beta$) and hence Δ_f is also a functional of φ and A_μ with

$$\Delta_f^{-1}[A_\mu,\varphi] \equiv \int [dg]\, \delta[f_a(A_\mu^g, \varphi^g)]$$

and expanding about $g=e$ we have

$$\Delta_f[A_\mu,\varphi] = \det \left| \frac{\delta_Q(\omega) f_a(A_\mu,\varphi)(x)}{\delta \omega^b(y)} \right|$$

$$\equiv \det M_f^{ab}(x,y)$$

In general it is difficult to evaluate these integrals when the δ-functional and determinant appear explicitly in the integrand. We would like to represent these two quantities as exponentials and hence an effective Lagrangian — they are additional terms to add to $\mathcal{L}_{inv}$. Then we can more readily evaluate the integrals — in particular we may derive a perturbation expansion for the Green functions.

The first thing to note is that we may consider gauges of the form

$$f_a(A_\mu, \varphi) - a_a(x) \quad \text{where}$$

$a_a(x)$ is a given but arbitrary function of x.

Then as argued earlier the S-matrix elements are the same as calculated when we simply had the gauge condition $f_a(A_\mu, \varphi)$. Further

$$\Delta^{-1}_{f-a}[A_\mu, \varphi] = \int [dg]\, \delta[f_a(A^g_\mu, \varphi^g) - a_a(x)]$$

$$= \frac{1}{\det \left| \frac{\delta_Q(\omega)(f_a - a_a)}{\delta \omega^b} \right|}$$

$$= \frac{1}{\det \left| \frac{\delta_Q(\omega) f_a(A_\mu, \varphi)(x)}{\delta \omega^b(y)} \right|} = \Delta^{-1}_f [A_\mu, \varphi]$$

since a_a is field independent.

Hence we can average over the functions $a_a(x)$ with a Gaussian weight ~~&~~ the Z will lead to ~~the~~ same S matrix

Thus using

$$\int[da_a]\, e^{-\frac{i}{2}\int d^4x\, a_a(x)a_a(x)}\,\delta[f_a(A_\mu,\varphi)-a_a(x)]$$

$$= e^{-\frac{i}{2}\int d^4x\, f_a(A_\mu,\varphi)f_a(A_\mu,\varphi)}$$

Thus

$$\boxed{\begin{aligned} Z[J,J_\mu] &= \int[dA^a_\mu][d\varphi^\alpha]\,\Delta_f[A_\mu,\varphi]\,\times \\ &\quad\times e^{i\int d^4x\left[\mathcal{L}_{inv}(\varphi,A)+\mathcal{L}_f(\varphi,A)+J_\alpha\varphi^\alpha+J^a_\mu A^{a\mu}\right]} \\ &\text{where } \mathcal{L}_f(\varphi,A) = -\frac{1}{2}f_a(A_\mu,\varphi)f_a(A_\mu,\varphi) \end{aligned}}$$

(Note: there are many ways to represent the δ-function — another is to introduce another dummy variable $\lambda_a(x)$ and

$$\delta[f_a(A_\mu,\varphi)] = \int [d\lambda^a]\, e^{i\int d^4x\, \lambda^a(x) f_a(A_\mu,\varphi)}$$

hence we have added an additional "auxiliary" field to our model)

Next we can apply the trick of Faddeev & Popov to write the determinant of M_f as an integral over additional dummy variables $c_a(x), \bar{c}_a(x)$ where c_a is a complex anti-commuting scalar field and $\bar{c}_a = c_a^\dagger$ is the independent complex conjugate field. These are known as Faddeev-Popov fields. Then

$$\int [dc_a][d\bar{c}_a]\, e^{+i\int d^4x\, d^4y\, \bar{c}_a(x) M_f^{ab}(x,y) c_b(y)}$$

$$= \det M_f = \Delta_f[A_\mu,\varphi]$$

Thus we finally have the generating functional represented as a path integral over fields with a weight of exponent of i times the effective action

$$Z[J, J_\mu] = \int [dA_\mu^a][d\varphi^\alpha][dc_a][d\bar{c}_a] \times$$
$$\times e^{+i\int d^4x \left[\mathcal{L}(\varphi, A_\mu, c, \bar{c}) + J_\alpha \varphi^\alpha + J_\mu^a A^{a\mu}\right]}$$

where

$$\mathcal{L}(\varphi, A_\mu, c, \bar{c}) = \mathcal{L}_{inv}(\varphi, A_\mu) + \mathcal{L}_f(\varphi, A_\mu) + \mathcal{L}_{\phi\pi}(\varphi, A_\mu, c, \bar{c})$$

where

$$\mathcal{L}_{inv}(\varphi, A_\mu) = \mathcal{L}(\varphi, D_\mu\varphi) - \frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu}$$

$$\mathcal{L}_f(\varphi, A_\mu) = -\frac{1}{2} f_a(A_\mu, \varphi) f_a(A_\mu, \varphi)$$

$$\mathcal{L}_{\phi\pi}(\varphi, A_\mu, c, \bar{c}) = \int d^4y \, \bar{c}_a(x) M_f^{ab}(x,y) c_b(y)$$

Just as we can calculate Green functions involving the unphysical fields A_0^a and the A_i^L longitudinal field it is often useful to include Fadeev-Popov fields in the Green function. Thus we consider the more general generating functional $Z[J, J_\mu, \xi, \bar{\xi}]$ which includes sources for $c_a, \bar{c}_a$.

$$\boxed{Z[J, J_\mu, \xi, \bar{\xi}] = \int [dA_\mu^a][d\varphi^\alpha][dc_a][d\bar{c}_a] \times e^{+i\int d^4x \left[\mathcal{L}(\varphi, A_\mu, c, \bar{c}) + J_\alpha \varphi^\alpha + J_\mu^a A^{a\mu} + \bar{c}_a \xi^a + \bar{\xi}^a c_a\right]}}$$

Note that $Z[J, J_\mu] = Z[J, J_\mu, 0, 0]$