to quantization of Yang-Mills theories.

Along these lines, let's recall again the intuitive derivation of the functional integral representation of $Z$ also given by Faddeev and Popov.

They argued that if one considers the naive integral

$$Z[j^a_\mu] = \int [dA^a_\mu]\, e^{i\int d^4x\,[\mathcal{L}_{inv}(A) + j^a_\mu A^{a\mu}]}$$

in analogy with scalar field theory then consider the integration over all fields gauge equivalent to some particular $A^a_\mu$

$$A^g_\mu \equiv U(g) A_\mu U^{-1}(g) + U(g)\partial_\mu U^{-1}(g)$$

that is fields in the gauge orbit of $A^a_\mu$; then sum over gauge inequivalent fields. Since $\mathcal{L}_{inv}$ is invariant under such transformation the exponential damping factor is unchanged and we pick up a group volume factor "$\int [dg]$" (for $j^a_\mu = 0$ first). The ansatz of F-P is that we should not sum over these gauge equivalent fields since they describe the same physical situation and we would be overcounting, but in fact choose only one field configuration from each orbit and sum over these gauge inequivalent configurations only. Hence we have factored out this infinite group volume from the integral and we define the generating functional by the remaining sum.

In order to choose only one field configuration from each orbit we choose a hypersurface in field space that intersects each orbit only once and sum over these so chosen fields. Thus we specify some hypersurface by dim G equations

$$f_a(A^b_\mu) = 0 \qquad a = 1, \ldots, \dim G$$

It chooses only one field from each orbit; hence if we evaluate

$$f_a(A^g_\mu) = 0 \qquad \text{for a gauge}$$

equivalent field to some specific $A_\mu$ then $g$ is determined uniquely for this specific $A_\mu$

by this hypersurface condition.

In order to relate the choice of group element $g$ to the field configuration determined by $f_a(A_\mu^g) = 0$ we consider the sum over all group elements

$$\int [dg]\, \delta[f_a(A_\mu^g)] \equiv \Delta_f^{-1}[A_\mu]$$

with $[dg]$ the group invariant Hurwitz measure $[dg'] = [dgg']$ for $g, g' \in \mathcal{G}$.

Hence we have that $\Delta_f$ is gauge invariant

$$\Delta_f^{-1}[A_\mu^g] = \int [dg']\, \delta[f_a(A_\mu^{gg'})]$$

$$= \int [dgg']\, \delta[f_a(A_\mu^{gg'})]$$

$$= \int [dg'']\, \delta[f_a(A_\mu^{g''})] = \Delta_f^{-1}[A_\mu]$$

Then insert $1 = \Delta_f[A_\mu] \int [dg]\, \delta[f_a(A_\mu^g)]$

into the naive vacuum functional

$$\text{“}Z[0]\text{''} = \int [dA_\mu^a][dg]\, \delta[f_a(A_\mu^g)]\, \Delta_f[A_\mu] \times$$
$$\times\, e^{i\int d^4x\, \mathcal{L}_{inv}(A_\mu)}$$

Now perform a change of variables corresponding to a gauge transformation of $A_\mu$

$$A_\mu \to A_\mu^{g'} \quad \text{so} \quad [dA_\mu] = [dA_\mu^{g'}]$$

and

$$\text{"}Z[0]\text{"} = \int [dA_\mu^a][dg]\,\delta[f_a(A_\mu^{g'g})]\,\Delta_f[A_\mu^{g'}]\; e^{i\int d^4x\, \mathcal{L}_{inv}(A_\mu^{g'})}$$

but $\Delta_f[A_\mu^{g'}] = \Delta_f[A_\mu]$

$$\mathcal{L}_{inv}(A_\mu^{g'}) = \mathcal{L}_{inv}(A_\mu)$$

further we can choose $g' = g^{-1}$ so that

$$f_a(A_\mu^{g^{-1}g}) = f_a(A_\mu) \quad \text{hence}$$

$$\text{"}Z[0]\text{"} = \int [dg]\Big[\int [dA_\mu^a]\,\delta[f_a(A_\mu)]\,\Delta_f[A_\mu] \times e^{i\int d^4x\, \mathcal{L}_{inv}(A_\mu)}\Big]$$

Now the expression in brackets is independent of $g$; thus we have factored out the infinite group volume. The Faddeev-Popov ansatz is that we define the generating functional by the remaining sum

$$\boxed{Z[j_\mu^a] \equiv \int [dA_\mu^a]\, \Delta_f[A_\mu]\, \delta[f_a(A_\mu)] \times e^{i\int d^4x\,[\mathcal{L}_{inv}(A_\mu) + j_\mu^a A^{a\mu}]}}$$

(Normalized so that $Z[0]=1$)

In order to compute $\Delta_f[A_\mu]$ we note that we only need $\Delta_f[A_\mu]$ for fields $A_\mu^a$ such that $f_a(A_\mu)=0$ due to the delta-functional. Hence we only need to evaluate $f_a(A_\mu^g)$ for $g$ close to the identity. Now recall that close to the identity

$g = e^{-i\omega^a I^a}$ where $I^a$ obey the Lie algebra and are the abstract group generators.

Thus $[dg]=[d\omega^a]$ the Hurwitz measure close to the identity is just the integration over the angles of rotation.

Also

$$f_a(A_\mu^g) = f_a(A_\mu + \delta_Q(\omega)A_\mu)$$

$$= f_a(A_\mu) + \delta_Q(\omega) f_a(A_\mu)$$

close to the identity now using

$$\delta(f(\omega)) = \frac{\delta(\omega-\omega_0)}{\left|\frac{\partial f}{\partial \omega}(\omega_0)\right|} \quad \text{for } f(\omega_0)=0$$

we have for $f_a(A_\mu)=0$

$$\Delta_f^{-1}[A_\mu] \equiv \int [dg]\, \delta[f_a(A_\mu^g)]$$

$$= \int [d\omega^a]\, \delta(\omega^a) \frac{1}{\det\left|\frac{\delta_Q(\omega) f_a(A_\mu)(x)}{\delta\omega^b(y)}\right|}$$

Thus

$$\boxed{\Delta_f[A_\mu] = \det\left|\frac{\delta_{Q(\omega)} f_a(A_\mu)(x)}{\delta\omega^b(y)}\right|}$$

Now we define

$$\boxed{M_f^{ab}(x,y) \equiv \frac{\delta_{Q(\omega)} f_a(A_\mu)(x)}{\delta\omega^b(y)}}$$

Hence the Faddeev-Popov ansatz becomes

$$\boxed{Z[j_\mu^a] = \int [dA_\mu^a](\det M_f)\,\delta[f_a(A_\mu)] \times \times e^{i\int d^4x[\mathcal{L}_{inv}(A) + j_\mu^a A^{\mu a}]}}$$