We begin our orderly reformulation of field theory by considering the set of assumptions known as the LSZ axioms!

Since we are dealing with relativistic systems we must first postulate the principles of rel. Q.M.

Axiom I: Relativistic Quantum Theory

a) The states of the theory are described by unit rays in a (separable) Hilbert space $\mathcal{H}$ and the observables are represented by self-adjoint operators in $\mathcal{H}$!

b) The relativistic transformation law of the states is given by a continuous unitary representation of the covering group of the Poincaré group, that is $ISL(2,C)$! That is corresponding to each $ISL(2,C)$ transformation $\{a, S\}$, where $x' = SxS^{\dagger} + a$, associated with corresponding Poincaré transf. $x'^{\mu} = \Lambda^{\mu\nu}(S)x_{\nu} + a^{\mu}$, there is a unitary operator in $\mathcal{H}$, $U(a,A)$ representing it.

I b) Since $U(a,1)$ is unitary it can be written as $U(a,1) = e^{+i a_\mu P^\mu}$ where P^μ is an unbounded Hermitian operator, interpreted as the E-M operator of the theory.

We postulate that the eigenvalues of P^μ lie in or on the forward light cone $\bar{V}^+$, that is $p^2 \geq 0$; $p^0 \geq 0$ (This is known as the Spectral Condition).

We also assume that P^μ forms a complete set of observables so the eigenstates of P^μ span $\mathcal{H}$. Thus we can label the states by $|k^\mu;\alpha\rangle$ with $P^\mu|k;\alpha\rangle = k^\mu|k;\alpha\rangle$ and α comprising the eigenvalues of all other ~~observ~~ observables which commute with P^μ in order to make a CSCO.

If for fixed α but variable k^μ we find only one possible value for the invariant

$$m^2 = P^\mu P_\mu = k^2 \quad , \quad \text{the}$$

state $|k^\mu; \alpha\rangle$ is a <u>one</u> particle state

If for fixed α, P^2 is continuous, these are called the 2, 3, ..., particle <u>scattering states</u> (collision).

c) There is an invariant state $|0\rangle$, the vacuum state,

1) $$U(a,\Lambda)|0\rangle = |0\rangle \Rightarrow \begin{array}{l} P^\mu|0\rangle = 0 \\ M^{\mu\nu}|0\rangle = 0 \end{array}$$

2) $$U(\alpha)|0\rangle = |0\rangle$$

$U(\alpha)$ = internal symmetries $\Rightarrow Q|0\rangle = 0$

unique (up to a constant phase factor) and normalized to 1

$$\langle 0|0\rangle = 1.$$

Consequences: 1) Heisenberg picture since we are dealing with eigenstates of P^μ

$$\langle k' | B(x') | p' \rangle = \langle k | B(x) | p \rangle$$

$$\| $$

$$\langle k | U^{-1}(a,1) B(x') U(a,1) | p \rangle$$

$\Rightarrow$

$$B(x') = U(a,1) B(x) U^{-1}(a,1)$$ since $\{|p\rangle\}$ are complete

So

$$B(x+a) = e^{i a_\mu P^\mu} B(x) e^{-i a_\mu P^\mu}$$

1) let $a^0 \neq 0$ $\Rightarrow$ $B(\vec{x},t) = e^{itH} B(\vec{x},0) e^{-iHt}$

$a^0 = t$

$H = P^0$

Heisenberg operator eq. of motion

2) Infinitesimal a^μ & Taylor expand

$$B(x) + a^\mu \partial_\mu B(x) = B(x) + i a^\mu [P_\mu, B(x)]$$

$$\Rightarrow \quad [P_\mu, B(x)] = -i \partial_\mu B(x)$$

So we have differentiated form of Heisenberg's eq. of motion

$$-i \frac{\partial}{\partial t} B(x) = [H, B(x)] .$$

As we saw ~~in~~ the introduction we would like the system to reduce to freely moving particles as $t \to \pm\infty$ of a specific number with definite momentum and spin (α) and that ~~these~~ states would be a complete set of eigenstates of our observables.

Hence the second axiom states

Axiom 2: **Asymptotic Completeness**

$$\mathcal{H}_{in} = \mathcal{H} = \mathcal{H}_{out}$$

When $t \to -\infty$,
the theory consists of freely moving particles of given # each with specific mass, spin, charges α etc. Each of which correspond to the single particle states of the theory. The totality of these