Standard Model Summary:

The Standard Model of electroweak and strong interactions based on the gauge symmetry group $SU(3) \times SU(2) \times U(1)$ has dynamics described by the gauge invariant Lagrangian

$$\mathcal{L}_{SM} = \mathcal{L}_{YM} + \mathcal{L}_F + \mathcal{L}_\phi + \mathcal{L}_{Yuk}$$

1) $$\mathcal{L}_{YM} = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} G^m_{\mu\nu} G^{m\mu\nu}$$

where the anti-symmetric covariant field strength tensors are

$$F^i_{\mu\nu} = \partial_\mu A^i_\nu - \partial_\nu A^i_\mu + g_2 \epsilon_{ijk} A^j_\mu A^k_\nu \quad ; \; i = 1,2,3$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

$$G^m_{\mu\nu} = \partial_\mu G^m_\nu - \partial_\nu G^m_\mu + g_3 f_{mnp} G^n_\mu G^p_\nu \quad , \; m = 1,2,\ldots,8$$

with f_{mnp} the $SU(3)$ structure constants.

2) $\mathcal{L}_F = \bar{l}_L^w i\not{D} l_L^w + \bar{q}_L^w i\not{D} q_L^w + \bar{e}_R^w i\not{D} e_R^w + \bar{u}_R^w i\not{D} u_R^w$
$+ \bar{d}_R^w i\not{D} d_R^w$

where the covariant derivatives are

$$D_\mu l_L^w = \left[\partial_\mu - \frac{ig_2}{2}\vec{\tau}\cdot\vec{A}_\mu + \frac{ig_1}{2}B_\mu\right] l_L^w$$

$$D_\mu^{ab} q_L^{wb} = \left[\left(\partial_\mu - \frac{ig_2}{2}\vec{\tau}\cdot\vec{A}_\mu - \frac{ig_1}{6}B_\mu\right)\delta^{ab} - \frac{ig_3}{2}(\vec{\lambda}\cdot\vec{G}_\mu)_{ab}\right] q_L^{wb}$$

$$D_\mu e_R^w = (\partial_\mu + ig_1 B_\mu) e_R^w$$

$$D_\mu^{ab} u_R^{wb} = \left[\left(\partial_\mu - \frac{2i}{3}g_1 B_\mu\right)\delta^{ab} - \frac{ig_3}{2}(\vec{\lambda}\cdot\vec{G}_\mu)_{ab}\right] u_R^{wb}$$

$$D_\mu^{ab} d_R^{wb} = \left[\left(\partial_\mu + \frac{i}{3}g_1 B_\mu\right)\delta^{ab} - \frac{ig_3}{2}(\vec{\lambda}\cdot\vec{G}_\mu)_{ab}\right] d_R^{wb}$$

where $a, b = 1, 2, 3$ = color indices of the quarks.

---

3) $\mathcal{L}_\phi = (D_\mu\phi)^\dagger(D^\mu\phi) - V(\phi^\dagger\phi)$

$$D_\mu\phi = \left(\partial_\mu - \frac{ig_2}{2}\vec{\tau}\cdot\vec{A}_\mu - \frac{ig_1}{2}B_\mu\right)\phi$$

& $V(\phi^\dagger\phi) = -\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$

$\phi = \begin{bmatrix}\phi^+ \\ \phi^0\end{bmatrix}$ complex scalar doublet. $(\phi^+)^\dagger = \phi^-$
$(\phi^0)^\dagger = \phi^{0\dagger}$

4) $\mathcal{L}_{Yuk} = \Gamma^e_{mn} \bar{l}^w_{mL} \phi\, e^w_{nR} + \Gamma^d_{mn} \bar{q}^w_{mL} \phi\, d^w_{nR}$
$$+ \Gamma^u_{mn} \bar{q}^w_{mL} \tilde{\phi}\, u^w_{nR} \quad + h.c.$$

with
$$\tilde{\phi} = i\sigma^2 \phi^* = \begin{bmatrix} \phi^{o+} \\ -\phi^- \end{bmatrix} \quad \text{a } (2, -\tfrac{1}{2}) \text{ representation}$$

Since $-\mu^2 = m^2 < 0$ the $SU(2) \times U(1)$ symmetry is spontaneously broken to $U(1)_{em}$ with the unbroken electric charge given by

$$Q = T^3 + y \quad \text{for each field.}$$

Hence the minimum of V is at $\langle 0|\phi|0\rangle = \begin{bmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{bmatrix}$

And we can express the SM in the unitary gauge where
$$\phi = \begin{bmatrix} 0 \\ \frac{v+\eta}{\sqrt{2}} \end{bmatrix}$$

$\eta(x) = H(x)$ the Higgs field.

& $W^\pm_\mu \equiv \frac{1}{\sqrt{2}}\left(A^1_\mu \mp i A^2_\mu\right)$; $A^1_\mu = \frac{1}{\sqrt{2}}\left(W^+_\mu + W^-_\mu\right)$
$A^2_\mu = \frac{i}{\sqrt{2}}\left(W^+_\mu - W^-_\mu\right)$

and
$$\begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix} \equiv \begin{bmatrix} \cos\theta_w & -\sin\theta_w \\ \sin\theta_w & \cos\theta_w \end{bmatrix} \begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix}, \quad \tan\theta_w = \frac{g_1}{g_2}$$

with $\begin{pmatrix} A^3_\mu \\ B_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & \sin\theta_W \\ -\sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$.

So $\mathcal{L}_{SM}$ in the unitary gauge becomes

1) $\mathcal{L}_{YM}$;

$$F^i_{\mu\nu} = \begin{cases} \frac{1}{\sqrt{2}}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu) + \frac{1}{\sqrt{2}}(\partial_\mu W^-_\nu - \partial_\nu W^-_\mu) \\ \quad + \frac{ig_2}{\sqrt{2}}\left[(W^+_\mu - W^-_\mu)(\cos\theta_W Z_\nu + s\theta_W A_\nu) - (\mu \leftrightarrow \nu)\right] & i=1 \\ \frac{i}{\sqrt{2}}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu) - \frac{i}{\sqrt{2}}(\partial_\mu W^-_\nu - \partial_\nu W^-_\mu) \\ \quad + \frac{g_2}{\sqrt{2}}\left[(c\theta_W Z_\mu + s\theta_W A_\mu)(W^+_\nu + W^-_\nu) - (\mu \leftrightarrow \nu)\right] & i=2 \\ \cos\theta_W(\partial_\mu Z_\nu - \partial_\nu Z_\mu) + \sin\theta_W(\partial_\mu A_\nu - \partial_\nu A_\mu) \\ \quad + ig_2\left[W^-_\mu W^+_\nu - W^+_\mu W^-_\nu\right] & i=3 \end{cases}$$

$$B_{\mu\nu} = -\sin\theta_W(\partial_\mu Z_\nu - \partial_\nu Z_\mu) + \cos\theta_W(\partial_\mu A_\nu - \partial_\nu A_\mu)$$

$$2)\ \mathcal{L}_F = \bar{\nu}_L i\not{\partial}\nu_L + \bar{e}i\not{\partial}e + \bar{u}i\left[\not{\partial} - \frac{ig_3}{2}(\vec{\lambda}\cdot\vec{\not{G}})\right]u + \bar{d}i\left[\not{\partial} - \frac{ig_3}{2}(\vec{\lambda}\cdot\vec{\not{G}})\right]d$$
$$+ eA_\mu J^\mu_{em} + \frac{e}{2\sqrt{2}\sin\theta_W}\left[J^\mu_W W^-_\mu + J^{\mu+}_W W^+_\mu\right] + \frac{e}{\sin 2\theta_W} J^\mu_Z Z_\mu$$

2) where $e = g_2 \sin\theta_W = g_1 \cos\theta_W$

and 1) The electromagnetic current

$$J^\mu_{em} = q_M \bar{\psi}_M \gamma^\mu \psi_M$$

M sums over all fields with q_M their electric charge

$$= +\tfrac{2}{3} \bar{u}_m \gamma^\mu u_m - \tfrac{1}{3} \bar{d}_m \gamma^\mu d_m - \bar{e}_m \gamma^\mu e_m$$

2) Charged Weak Current

$$J^\mu_W = \bar{e} \gamma^\mu (1-\gamma_5) \nu + \bar{d} \gamma^\mu (1-\gamma_5) A_{CKM} u$$

3) Neutral Weak Current

$$J^\mu_Z = \bar{\psi}_M \gamma^\mu T^3_M (1-\gamma_5) \psi_M - 2 q_M \bar{\psi}_M \gamma^\mu \psi_M$$

$$= \bar{\nu}_L \gamma^\mu \nu_L - \bar{e}_L \gamma^\mu e_L + \bar{u}_L \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L + 2\sin^2\theta_W \left(\bar{e} \gamma^\mu e - \tfrac{2}{3} \bar{u} \gamma^\mu u + \tfrac{1}{3} \bar{d} \gamma^\mu d \right)$$

where A_{CKM} is the Cabbibo-Kobayashi-Maskawa matrix

2) Let

$$V = A_{CKM}^{\dagger} = (A_L^{d\dagger} A_L^{u})^{\dagger}$$

$$= (A_L^{u\dagger} A_L^{d})$$

$$= \begin{array}{c c} & \begin{array}{ccc} d & s & b \end{array} \\ \begin{array}{c} u \\ c \\ t \end{array} & \begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix} \end{array}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_2 & -S_2 \\ 0 & S_2 & C_2 \end{pmatrix} \begin{pmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_3 & S_3 \\ 0 & -S_3 & C_3 \end{pmatrix}$$

$$= \begin{pmatrix} C_1 & -S_1 C_3 & -S_1 S_3 \\ S_1 C_2 & C_1 C_2 C_3 - S_2 S_3 e^{i\delta} & C_1 C_2 S_3 + S_2 C_3 e^{i\delta} \\ S_1 S_2 & C_1 S_2 C_3 + C_2 S_3 e^{i\delta} & C_1 S_2 S_3 - C_2 C_3 e^{i\delta} \end{pmatrix}$$

$$= \begin{bmatrix} 0.9738-0.9750 & 0.218-0.224 & 0.001-0.007 \\ 0.218-0.224 & 0.9734-0.9752 & 0.030-0.058 \\ 0.003-0.019 & 0.029-0.058 & 0.9983-0.9996 \end{bmatrix}$$

3)
$$\mathcal{L}_\phi = \frac{1}{2}\partial_\mu\eta\partial^\mu\eta - \frac{1}{2}m_H^2\eta^2 - \frac{\lambda}{4}(\eta^4 + 4v\eta^3)$$
$$+ M_W^2 W_\mu^+ W^{-\mu} + \frac{1}{2}M_Z^2 Z_\mu Z^\mu$$
$$+ g_2\left[M_W\eta + \frac{1}{4}g_2\eta^2\right]W_\mu^+ W^{-\mu}$$
$$+\frac{1}{2}\left[\frac{g_2 M_W}{\cos^2\theta_W}\eta + \frac{1}{4}\frac{g_2^2}{\cos^2\theta_W}\eta^2\right]Z_\mu Z^\mu$$

where
$$M_W = \frac{g_2 v}{2} \quad \left(\approx \frac{37}{\sin\theta_W}\,\text{GeV}\right)$$
$$M_Z = \frac{M_W}{\cos\theta_W} \quad \left(\approx \frac{75}{\sin 2\theta_W}\,\text{GeV}\right)$$
$$m_H^2 = 2\mu^2 = 2\lambda v^2$$

---

4)
$$\mathcal{L}_{Yuk} = -\left[1 + \frac{g_2}{2M_W}\eta\right]\left[m_u\bar{u}u + m_c\bar{c}c\right.$$
$$+ m_t\bar{t}t + m_d\bar{d}d + m_s\bar{s}s + m_b\bar{b}b$$
$$\left. + m_e\bar{e}e + m_\mu\bar{\mu}\mu + m_\tau\bar{\tau}\tau\right]$$

Spinor Interlude:

In what was done we separated Dirac fermions into left & Right handed spinors

$$\psi_D = (\gamma_+ + \gamma_-)\psi_D = \underbrace{\gamma_+\psi_D}_{\equiv \psi_R} + \underbrace{\gamma_-\psi_D}_{\equiv \psi_L}$$

And they interacted differently under $SU(2) \times U(1)$. The 3-2-1 representations of the matter field is ($a = 1, 2, 3$ = color = R, G, B)

Field	Families ($m = 1,2,3$)	$(SU(3), SU(2), U(1))$	Family Electro-Weak Multiplets
$l_{mL} = \begin{pmatrix} \nu_{mL} \\ e_{mL} \end{pmatrix}$	3	$(1, 2, -\frac{1}{2})$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$
$q^a_{mL} = \begin{pmatrix} u_{mL} \\ d_{mL} \end{pmatrix}$	3	$(3, 2, +\frac{1}{6})$	$\begin{pmatrix} u^a_L \\ d^a_L \end{pmatrix}, \begin{pmatrix} c^a_L \\ s^a_L \end{pmatrix}, \begin{pmatrix} t^a_L \\ b^a_L \end{pmatrix}$
e_{mR}	3	$(1, 1, -1)$	e_R, μ_R, τ_R
u^a_{mR}	3	$(3, 1, +\frac{2}{3})$	u^a_R, c^a_R, t^a_R
d^a_{mR}	3	$(3, 1, -\frac{1}{3})$	d^a_R, s^a_R, b^a_R
ϕ		$(1, 2, +\frac{1}{2})$	$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$
$(\tilde{\phi} = i\sigma^2\phi^*)$		$(1, 2, -\frac{1}{2})$	$\tilde{\phi} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$

It is useful in GUTS & SUSY to deal with fields with the same chirality. For instance in a SU(5) GUT the RH down quarks and the left handed lepton e, ν_e are put in the same $\bar{5}$ representation — they must have the same chirality. To accomplish this we can use the charge conjugate fields instead of the RH fields as they will be LH. i.e. e_R, u^a_R, d^a_R will become LH under charge conjugation. Recall Charge conjugation:

$$\psi \to \mathcal{C}\psi\mathcal{C}^{-1} \equiv \psi^c = C\bar{\psi}^T$$

$$\bar{\psi} \to \mathcal{C}\bar{\psi}\mathcal{C}^{-1} \equiv \overline{\psi^c} = -\psi^T C^{-1} \quad (= \psi^{c\dagger}\gamma^0)$$

where $C^{-1}\gamma_\mu C = -\gamma_\mu^T$ & for our representation

$$C = -C^{-1} = -C^\dagger = -C^T = i\gamma^2\gamma^0$$

So
$$\mathcal{C}\psi_{\substack{L\\R}}\mathcal{C}^{-1} = \gamma_{\mp}\mathcal{C}\psi\mathcal{C}^{-1}$$

$$= \gamma_{\mp}\psi^c = \gamma_{\mp}C\bar{\psi}^T$$

$$= C\gamma_{\mp}\bar{\psi}^T = C(\bar{\psi}\gamma_{\mp})^T \qquad (\gamma_5^T = \gamma_5)$$

$$= C\,\overline{\psi_{\substack{R\\L}}}^{\,T} \equiv \psi^c_{\substack{L\\R}}$$

So $\boxed{\psi^c_{\substack{L\\R}} = C\bar{\psi}_{\substack{R\\L}}^{\,T}} \Rightarrow \bar{\psi}_{\substack{R\\L}} C^T = (\psi^c_{\substack{L\\R}})^T$

$$\Rightarrow \bar{\psi}_{\substack{R\\L}} = (\psi^c_{\substack{L\\R}})^T C$$

Similarly

$$\mathcal{C}\bar{\psi}_{\substack{L\\R}}\mathcal{C}^{-1} = \mathcal{C}\bar{\psi}\mathcal{C}^{-1}\gamma_{\pm} = -\psi^T C^{-1}\gamma_{\pm}$$

$$= -\psi^T\gamma_{\pm} C^{-1} = -(\gamma_{\pm}\psi)^T C^{-1}$$

$$\boxed{= -\psi_{\substack{R\\L}}^T C^{-1} = \overline{\psi^c_{\substack{L\\R}}}} \quad \left(= \psi^{c\dagger}_{\substack{L\\R}}\gamma^0\right)$$

(That is $\overline{\psi^c_{\substack{L\\R}}} = \psi^{c\,\dagger}_{\substack{L\\R}}\gamma^0 = \bar{\psi}^*_{\substack{R\\L}} C^\dagger\gamma^0 = \psi^T_{\substack{R\\L}}\gamma^{0*}C^{-1}\gamma^0$

$$= -\psi^T_{\substack{R\\L}} C^{-1} \checkmark \Big)$$

hence $\left(\overline{\psi^c_{\substack{L\\R}}}\right)^T = -C^{-1T}\psi_{\substack{R\\L}}$

$$= -C\psi_{\substack{R\\L}}$$

and so $\boxed{\psi_{\substack{R\\L}} = C\left(\overline{\psi^c_{\substack{L\\R}}}\right)^T}$

& from above $\boxed{\bar{\psi}_{\substack{R\\L}} = \psi^{c\,T}_{\substack{L\\R}} C}$

So instead of (ψ_L, ψ_R) as fundamental fields, we can use the equivalent pairs

$$(\psi_L, \psi_L^c) \text{ or } (\psi_R, \psi_R^c) \text{ or } (\psi_R^c, \psi_L^c)$$

we will replace $\underline{\psi_R \text{ with } \psi_L^c}$ in the SM

So (ψ_L, ψ_L^c) will become the fundamental fields. So for example we will replace e_R^-, u_R^a, d_R^a with the fields

$$e_L^+, u_L^{ca}, d_L^{ca}$$

where $e_L^+ = C\bar{e}_R^{-T} = e_L^{-c}$ (making electric charge explicit)
or leaving off $\pm$ $e_L^c, u_L^{ca}, d_L^{ca}$

Now the charge conjugation takes the fields to the complex conjugation group representation and opposite U(1) charge

So for example d_R^a is a $(3, 1, -\frac{1}{3})$

but $d_L^c = C\bar{d}_R^T$ transforms as

a $(\bar{3}, 1, +\frac{1}{3})$ under $SU(3) \times SU(2) \times U(1)$.

since the $\bar{d}_R$ transforms according to

U^{-1} taking a 3 to a $\bar{3}$ & $-\frac{1}{3}$ to $+\frac{1}{3}$.

Likewise $e^c_L = C(\overline{e_R})^T$ The $\overline{e_R}$ flips the hypercharge of e_R from -1 to $+1$. So we have the 3-2-1 table

Field	(SU(3), SU(2), U(1))	Family Multiplets
$l_{mL} = \begin{pmatrix} \nu_{mL} \\ e_{mL} \end{pmatrix}$	$(1, 2, -\frac{1}{2})$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}, \begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}, \begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$
$q^a_{mL} = \begin{pmatrix} u_{mL} \\ d_{mL} \end{pmatrix}$	$(3, 2, +\frac{1}{6})$	$\begin{pmatrix} u^a_L \\ d^a_L \end{pmatrix}, \begin{pmatrix} c^a_L \\ s^a_L \end{pmatrix}, \begin{pmatrix} t^a_L \\ b^a_L \end{pmatrix}$
$e^+_{mL} = e^c_{mL}$	$(1, 1, +1)$	e^c_L, μ^c_L, τ^c_L
u^{ca}_{mL}	$(\bar{3}, 1, -\frac{2}{3})$	$u^{ca}_L, c^{ca}_L, t^{ca}_L$
d^{ca}_{mL}	$(\bar{3}, 1, +\frac{1}{3})$	$d^{ca}_L, s^{ca}_L, b^{ca}_L$
ϕ	$(1, 2, +\frac{1}{2})$	$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$
$(\tilde{\phi} = i\sigma^2 \phi^*)$	$(1, 2, -\frac{1}{2})$	$\tilde{\phi} = \begin{pmatrix} \phi^{0*} \\ -\phi^- \end{pmatrix}$

Note:

$i\sigma^2 l_L$	$(1, \bar{2}, -\frac{1}{2})$	$i\sigma^2 l_L = \begin{pmatrix} e^-_L \\ -\nu_{eL} \end{pmatrix}$

The invariant fermion kinetic energy terms become for example

$$\bar{u}_R i \not{D} u_R = \bar{u}_R^a i \Big((\partial_\mu - \tfrac{2i}{3} g_1 B_\mu)\delta^{ab} - \tfrac{ig_3}{2}(\vec{\lambda}\cdot\vec{G}_\mu)_{ab} \Big) \gamma^\mu u_R^b$$

chain Rule Differentiation throw away total derivatives since action $\sim \int d^4x$

Sign flip

$$= -i\Big[(\partial_\mu + \tfrac{2i}{3} g_1 B_\mu)\delta^{ba} + \underbrace{\tfrac{ig_3}{2}(\vec{\lambda}\cdot\vec{G}_\mu)^T}_{3 \to \bar{3}} \Big] \bar{u}_R^a \cdot \gamma^\mu u_R^b$$

$$= -i\Big[(\partial_\mu + \tfrac{2i}{3} g_1 B_\mu)\delta^{ba} + \tfrac{ig_3}{2}(\vec{\lambda}\cdot\vec{G}_\mu)^T_{ba} \Big] u_L^{ca\,T} C \cdot \gamma^\mu C (\bar{u}_L^{cb})^T$$

$$= + \bar{u}_L^{cb} C^T \gamma^{\mu T} C^T i\Big[(\partial_\mu + \tfrac{2i}{3} g_1 B_\mu)\delta^{ba} + \tfrac{ig_3}{2}(\vec{\lambda}\cdot\vec{G}_\mu)^T_{ba} \Big] u_L^{ca}$$

(Now $C^T \gamma^{\mu T} C^T = \gamma^\mu$)

$$= \bar{u}_L^{c\,b} \gamma^\mu i \Big[\underbrace{(\partial_\mu + \tfrac{2i}{3} g_1 B_\mu)\delta^{ba} + \tfrac{ig_3}{2}(\vec{\lambda}\cdot\vec{G}_\mu)^T_{ba}}_{\text{covariant derivative for } (\bar{3},1,-\frac{2}{3})} \Big] u_L^{c\,a}$$

$$\Rightarrow \boxed{\bar{u}_R i \not{D} u_R = \bar{u}_L^c \, i \not{D} u_L^c}$$

Likewise

$$\bar{e}_R i \not{D} e_R = \bar{e}^c_L i \not{D} e^c_L$$

&

$$\bar{d}_R i \not{D} d_R = \bar{d}^c_L i \not{D} d^c_L$$

with

$$D_\mu e^c_L = (\partial_\mu - i g_1 B_\mu) e^c_L$$

$$D^{ab}_\mu u^{cb}_L = \left[\left(\partial_\mu + \frac{2i}{3} g_1 B_\mu\right)\delta^{ab} + \frac{i g_3}{2}\left(\vec{\lambda}\cdot\vec{G}_\mu\right)^T_{ab}\right] u^{cb}_L$$

$$D^{ab}_\mu d^{cb}_L = \left[\left(\partial_\mu - \frac{i}{3} g_1 B_\mu\right)\delta^{ab} + \frac{i g_3}{2}\left(\vec{\lambda}\cdot\vec{G}_\mu\right)^T_{ab}\right] d^{cb}_L$$

and

$$\mathcal{L}_F = \bar{l}_L i \not{D} l_L + \bar{q}_L i \not{D} q_L + \bar{e}^c_L i \not{D} e^c_L + \bar{u}^c_L i \not{D} u^c_L + \bar{d}^c_L i \not{D} d^c_L$$

Finally the Yukawa terms must be re-expressed in terms of the left handed charge conjugate fields.

These are Fermion bilinears of the form

$$\bar{l}_L e_R = \bar{l}_L C (\overline{e^c_L})^T$$

$$\bar{e}_R l_L = e^{c\,T}_L C l_L$$

$$\bar{q}_L d_R = \bar{q}_L C (\overline{d^c_L})^T \qquad \bar{q}_L u_R = \bar{q}_L C \overline{u^c_L}^T$$

$$\bar{d}_R q_L = d^{c\,T}_L C q_L \qquad \bar{u}_R q_L = u^{c\,T}_L C q_L$$

So

$$\boxed{\begin{aligned} \mathcal{L}_{yuk} = {} & \Gamma^e_{mn} \bar{l}_{mL} \phi C \overline{e^c_{nL}}^T + \Gamma^{e\dagger}_{mn} e^{c\,T}_{mL} \phi^\dagger C l_{nL} \\ & + \Gamma^d_{mn} \bar{q}_{mL} \phi C \overline{d^c_{nL}}^T + \Gamma^{d\dagger}_{mn} d^{c\,T}_{mL} \phi^\dagger C q_{nL} \\ & + \Gamma^u_{mn} \bar{q}_{mL} \tilde{\phi} C \overline{u^c_{nL}}^T + \Gamma^{u\dagger}_{mn} u^{c\,T}_{mL} \tilde{\phi}^\dagger C q_{nL} \end{aligned}}$$

As usual the mass & Higgs couplings are L-R types of coupling

$$\overline{\psi^c_R} \phi_L = \psi^T_L C \phi_L$$

& $\bar{\phi}_L \psi^c_R = \bar{\phi}_L C \bar{\psi}_R^T$ as above.